AREAS RELATED TO CIRCLES 12

12.1 Introduction

You are already familiar with some methods of finding perimeters and areas of simple plane figures such as rectangles, squares, parallelograms, triangles and circles from your earlier classes. Many objects that we come across in our daily life are related to the circular shape in some form or the other. Cycle wheels, wheel barrow (*thela*), dartboard, round cake, *papad*, drain cover, various designs, bangles, brooches, circular paths, washers, flower beds, etc. are some examples of such objects (see Fig. 12.1). So, the problem of finding perimeters and areas related to circular figures is of great practical importance. In this chapter, we shall begin our discussion with a review of the concepts of perimeter (circumference) and area of a circle and apply this knowledge in finding the areas of two special 'parts' of a circular region (or briefly of a circle) known as *sector* and *segment*. We shall also see how to find the areas of some combinations of plane figures involving circles or their parts.

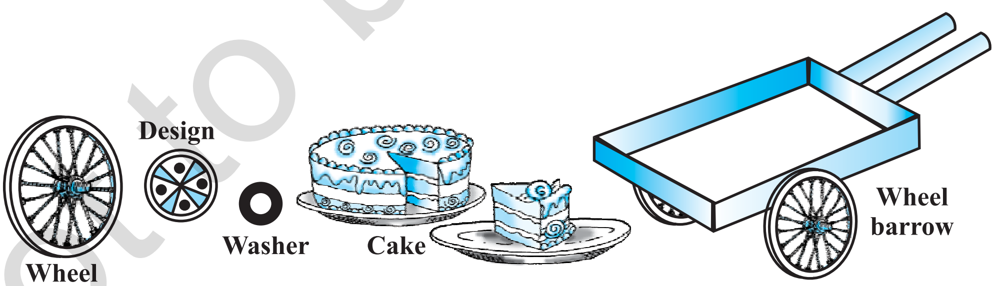

Fig. 12.1

12.2 Perimeter and Area of a Circle — A Review

Recall that the distance covered by travelling once around a circle is its *perimeter*, usually called its *circumference*. You also know from your earlier classes, that circumference of a circle bears a constant ratio with its diameter. This constant ratio is denoted by the Greek letter π (read as 'pi'). In other words,

$$\frac{\text{circumference}}{\text{diameter}} = \pi$$

or,
$$\text{circumference} = \pi \times \text{diameter}$$
$$= \pi \times 2r \quad (\text{where } r \text{ is the radius of the circle})$$
$$= 2\pi r$$

The great Indian mathematician Aryabhatta (C.E. 476 – 550) gave an approximate value of π. He stated that $\pi = \frac{62832}{20000}$, which is nearly equal to 3.1416. It is also interesting to note that using an identity of the great mathematical genius Srinivas Ramanujan (1887–1920) of India, mathematicians have been able to calculate the value of π correct to million places of decimals. As you know from Chapter 1 of Class IX, π is an irrational number and its decimal expansion is non-terminating and non-recurring (non-repeating). However, for practical purposes, we generally take the value of π as $\frac{22}{7}$ or 3.14, approximately.

You may also recall that area of a circle is πr^2, where r is the radius of the circle. Recall that you have verified it in Class VII, by cutting a circle into a number of sectors and rearranging them as shown in Fig. 12.2.

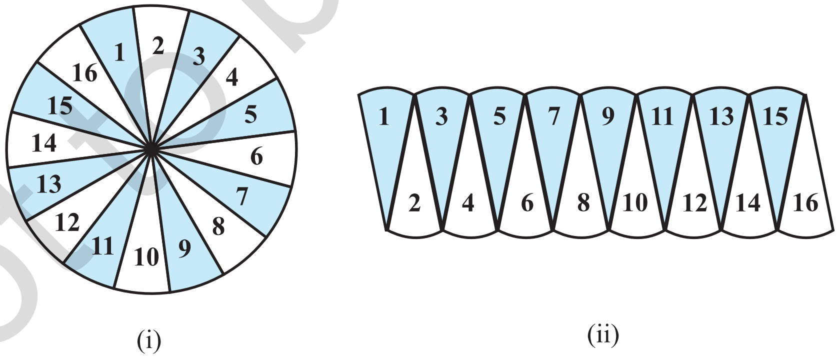

Fig 12.2

You can see that the shape in Fig. 12.2 (ii) is nearly a rectangle of length $\frac{1}{2} \times 2\pi r$ and breadth r. This suggests that the area of the circle = $\frac{1}{2} \times 2\pi r \times r = \pi r^2$. Let us recall the concepts learnt in earlier classes, through an example.

Example 1 : The cost of fencing a circular field at the rate of ₹ 24 per metre is ₹ 5280. The field is to be ploughed at the rate of ₹ 0.50 per m^2. Find the cost of ploughing the field (Take $\pi = \frac{22}{7}$).

Solution : Length of the fence (in metres) = $\frac{\text{Total cost}}{\text{Rate}} = \frac{5280}{24} = 220$

So, circumference of the field = 220 m

Therefore, if r metres is the radius of the field, then

$$2\pi r = 220$$

or, $$2 \times \frac{22}{7} \times r = 220$$

or, $$r = \frac{220 \times 7}{2 \times 22} = 35$$

i.e., radius of the field is 35 m.

Therefore, area of the field = $\pi r^2 = \frac{22}{7} \times 35 \times 35 \text{ m}^2 = 22 \times 5 \times 35 \text{ m}^2$

Now, cost of ploughing 1 m^2 of the field = ₹ 0.50

So, total cost of ploughing the field = ₹ $22 \times 5 \times 35 \times 0.50$ = ₹ 1925

EXERCISE 12.1

Unless stated otherwise, use $\pi = \frac{22}{7}$.

1. The radii of two circles are 19 cm and 9 cm respectively. Find the radius of the circle which has circumference equal to the sum of the circumferences of the two circles.
2. The radii of two circles are 8 cm and 6 cm respectively. Find the radius of the circle having area equal to the sum of the areas of the two circles.
3. Fig. 12.3 depicts an archery target marked with its five scoring regions from the centre outwards as Gold, Red, Blue, Black and White. The diameter of the region representing Gold score is 21 cm and each of the other bands is 10.5 cm wide. Find the area of each of the five scoring regions.

Fig. 12.3