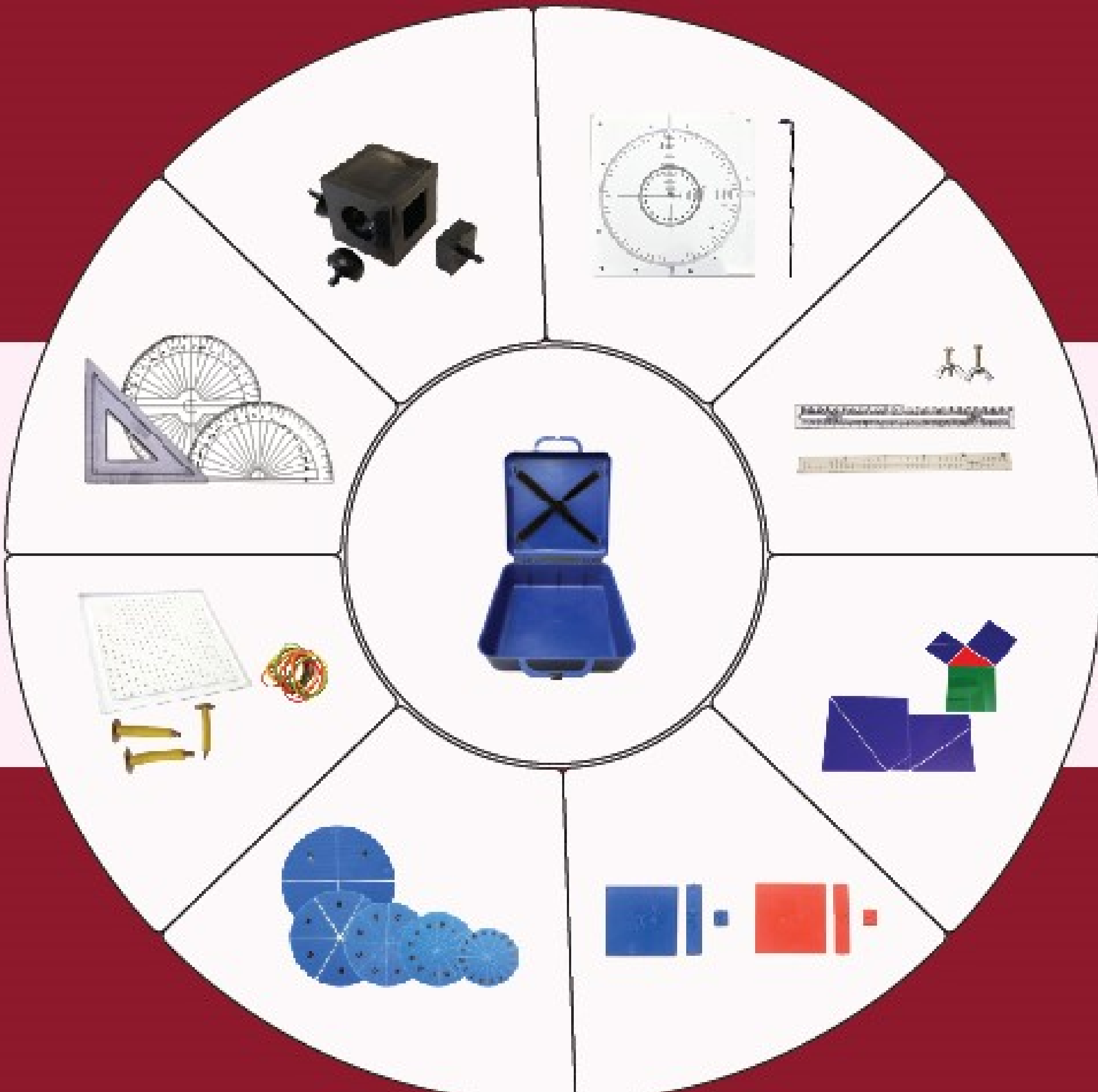
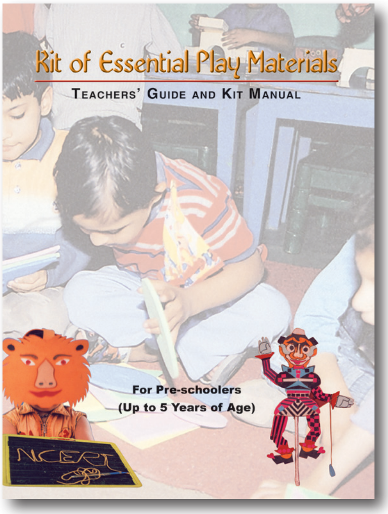
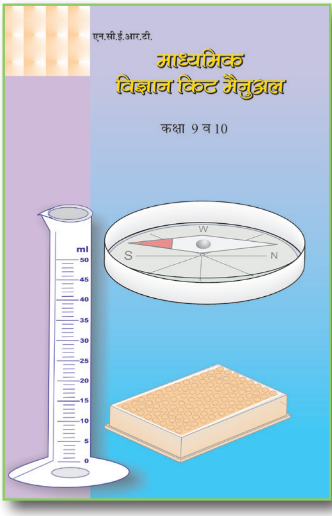
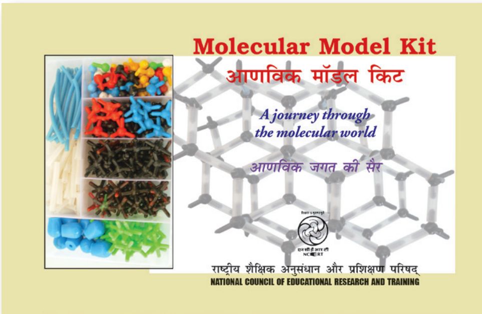
Manual for
Secondary Stage Mathematics Kit

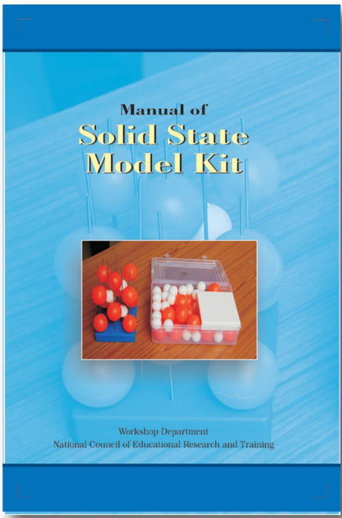
Grades 9 and 10











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Manual
for
Secondary Stage
Mathematics Kit
Grades 9 and 10

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

13132 – MANUAL FOR SECONDARY STAGE MATHEMATICS KIT

ISBN 978-93-5007-752-8

First Edition

August 2016 Ashadha 1938

Reprinted

February 2018 Magha 1939
October 2018 Ashwina 1940
February 2021 Magha 1942

PD 10T RPS

OFFICES OF THE PUBLICATION DIVISION, NCERT

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Printed on 80 GSM paper with NCERT watermark

Published at the Publication Division by the Secretary, National Council of Educational Research and Training, Sri Aurobindo Marg, New Delhi 110 016.

Preface

One of the most significant recommendations of the *National Curriculum Framework* (NCF)-2005 is the mathematisation of the child's thought processes. In achieving this goal, concrete mathematical experiences play a major role. A child is motivated to learn mathematics by getting involved in handling various concrete manipulants in various activities. In addition to activities, games in mathematics also help the child's involvement in learning by strategising and reasoning. For learning mathematical concepts through the above-mentioned approach, a child-centred Mathematics kit has been developed for the students of secondary stage based on some of the concepts from the newly developed NCERT mathematics textbooks. The kit includes various items along with a manual for performing activities. The kit broadly covers the activities in the areas of geometry, algebra, trigonometry and mensuration. The kit has the following advantages :

- Availability of necessary materials at one place
- Multipurpose use of items
- Economy of time in doing the activities
- Portability from one place to another
- Provision for teacher's innovation
- Low-cost material and use of indigenous resources

Here are some of the special features of the kit items :

Two varieties of plastic strips with slots and markings have been provided. They help in creating angles, triangles, quadrilaterals and determination of values of trigonometric ratios. The full or half protractor can be fixed on the strips for measuring the angles in the activities related to angles, triangles, and quadrilaterals.

A Circular Board is designed in such a manner that it can be used to verify results related to a circle as well as trigonometric ratios.

A Geoboard of dimensions $19\,\text{cm} \times 19\,\text{cm} \times 1\,\text{cm}$ having holes drilled on a side of it at a distance of 1cm each. Geoboard pins can be fitted in the holes and with the help of rubber bands different geometrical shapes can be formed.

Cut-outs of corrugated sheets in the form of parallelogram, triangle, trapezium and circle help in learning concepts related to areas.

A cube with adjusting cut-outs of cuboid, cylinder, cone and hemisphere have been given to explain the concepts of surface area and volume.

Cut-outs of plastic cardboard in the form of triangles, quadrilaterals and rectangles etc. have been given to verify Pythagoras theorem and algebraic identities like $a^2 - b^2 = (a-b)(a+b)$.

Another interesting item, "Algebraic Tiles" has also been provided. They are provided in two different colours and three different sizes. They can be used for concretisation of the concept of factorisation of quadratic equations.

The kit items, apart from being academically useful, are also designed in attractive manner. It is hoped that this kit will generate enough interest for learning mathematics at secondary stage. It will prove to be an important part of the mathematics resource room in the schools across the country.

This manual has been revised in light of NEP 2020 and accordingly renamed also.

R. K. Parashar
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Acknowledgements

Special thanks are due to Professor H.O. Gupta (Retd.), NCERT, New Delhi for suggestion and support to DEK.

The Council acknowledges with thanks the contributions of DEK staffs, Shri V.B Patil, *Technical Officer*, Shri Anil Nayal, *Draftsmen* for making illustrations, Shri Satish Kumar, Afsana, *JPF*, Nargis Islam and Sumit Kumar, *DTP Operators*.

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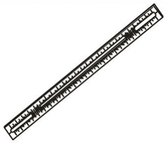
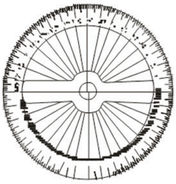
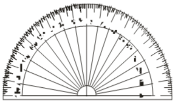
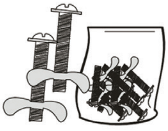
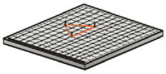
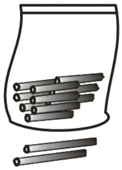
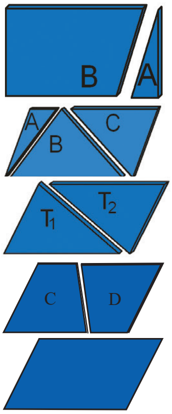
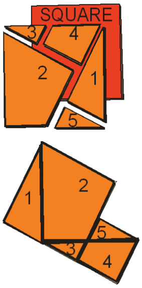
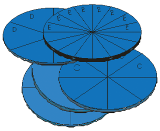
SECONDARY MATHEMATICS KIT

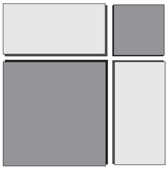
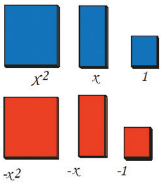
Grades 9 and 10

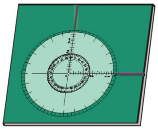
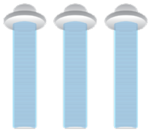
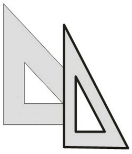
Technical Specification

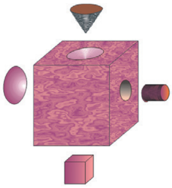
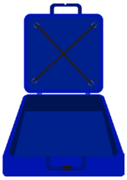
S. No.	Item	Figure/ Name	Specification Shapes	Unit
1.	Plastic Strip (A Type)		Cuboidal perspex having 3 slots of 5 mm width at (0-3) cm, (5-20) cm and (22-25) cm.	08
2.	Plastic Strip (B Type)		Cuboidal perspex having 3 slots of 5 mm width at (0-0.3) dm, (0.5-2.0)dm & (2.2-2.5) dm.	03
3.	Full Protractor		A 3 mm thick circular transparent plastic sheet of diameter 120 mm marked in (0-360) degrees.	04
4.	Half Protractor		A 3 mm thick semi circular transparent plastic sheet of diameter 90 mm marked in (0-180) degrees.	04
5.	Fly nut and screw		It is a combination of nut with wing and chromium plated screw with metric thread (M4) of length 20 mm and diameter 4 mm made up of mild steel having slotted round head or HDPE molded.	15 set

6.	Geo-board		ABS material. On one face is an array of square grids formed by blind holes of 2 mm diameter and 7 mm depth. The other face has an array of concentric cirles formed by in-built dowels of 2.5 mm diameter and 10 mm length.	01
7.	Geo-board Pins		Solid cylindrical pins of length 15 mm and diameter 2 mm made of stainless steel.	20
8.	Rubber bands		Rubber of assorted colours and sizes.	20
9.	Cutouts (For area of polygons)		Different shapes cut-outs of a 3 mm thick blue-coloured corrugated plastic sheets. The shapes are as follows. (1) A triangle and a trapezium marked as A and B respectively (2) Three triangles marked as A, B and C (3) Two triangles marked as T1 and T2 respectively (4) Two trapeziums marked as C and D respectively (5) A parallelogram	01 set each

10.	Cut–outs (For Pythagoras Theorem)		Made up of 5 mm thick cardboard and includes: (1) Square of side 125 mm. (2) 5 cut-outs of different shapes obtained from another square of side 125 mm. (3) Triangular cut-out of hypotenuse 125 mm. (4) Two squares of sides equal to height and base of triangular cut-out.	01 set each
11.	Cut-outs (For area of a circle)		Five blue-coloured circular corrugated sheets having 5 mm thickness and diameter 160 mm, divided into 4, 6, 8, 12, and 16 equal sectors .	01 set each

12.	Cut–outs (For Algebraic identities)	 	Made up of 5 mm thick plastic cardboard and includes: (1) Square cut-out of side 76 mm. (2) Three cut-outs obtained from another square of side 76 mm out of which one is a square of side 38 mm and the remaining two are trapezium of dim. (3) Square cut-outs of side 80 mm and 45 mm. (4) Two rectangular cut-out of dimensions 80 × 46 mm².	01 set each
13.	Algebraic Tiles (a) x^2, x, 1 (b) $-x^2$, $-x$, -1		Algebraic tiles x^2, x, 1; $-x^2$,-x, -1 engraved onto 5 mm thick blue-coloured plastic cardboard of the following sizes: (1) $-x^2$, x^2: 50 mm × 50 mm (5 tiles × 2). (2) -x, x: 50 mm × 10 mm (20 tiles × 2). (3) -1, 1: 10 mm × 10 mm (20 tiles × 2). $-x^2$, -x and -1 are red coloured, while x^2, x and 1 are blue coloured.	01 set each

14.	Trigono-metric Circular Board		Trigonometric circular hole: 260 mm x 260 mm x 12 mm plastic board with 5 mm wide, 6 mm deep circular groove (diameter: 200 mm), including coordinate axis markings. Inner circle (diameter: 92 mm) marked in degrees. Centre blind hole is 5 mm diameter and 10 mm depth. Three rectangular grooves (5 mm wide and 10 mm depth) touch circular groove. The board has fifteen 6 mm diameter blind holes.	01
15.	Connectors (For circular board)		Stainless steel rubber sleeved screw having 25 mm length, 5 mm diameter and a round head.	15
16.	Connectors (For strip)		Flat round headed screw of length 20 mm, diameter 4 mm with split ends and rubber sleeve.	10
17.	Set Square		As per standard medium size.	01 set each
18.	Rotating Needle		Steel rod of length 200 mm and diameter 3 mm. with one **'L'** band at one side of rod.	01

19.	Cube with cut-outs (a) Cone (b) Cuboid (c) Cylinder (d) Hemisphere		Solid cube of side 60mm having following cut-outs fitted in it - (1) Cuboid of dimensions $30 \times 30 \times 15 \, mm^3$. (2) Cone of height 30mm diameter 30mm (3) Hemisphere of dia. 30mm. (4) Cylinder of height 20mm and diameter 20mm.	01 set each
20.	Kit Box along with carton		Box with suitable hinges and lock system provided with pockets inside to keep the kit items.	01
21.	Plastic Box		80 mm x 70 mm x 40 mm	01

Activity 1

Measurement of Angles

Objective

To form different angles and measure them.

Material required

Two plastic strips, full protractor, and fly screws.

How to proceed?

1. Take two plastic strips and a full protractor.
2. Fix the strips along with the protractor at their end points with a fly screw.
3. Fix one of the strips along the 0°-180° marked line of the protractor as shown below in Fig.1:

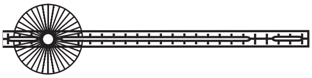


Fig.1. Zero angle

4. By moving the other strip (in anticlockwise direction), try to make angles of different measures (Fig.1, Fig.2, Fig.3, Fig.4, Fig.5, Fig.6 and Fig.7).

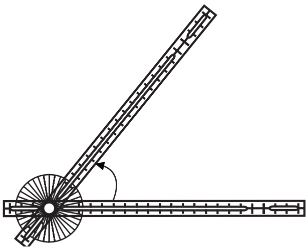


Fig. 2. Acute angle

Note

1. All angles are to be measured in anticlockwise direction from first strip.

2. Use the markings of the scale of the protractor carefully.

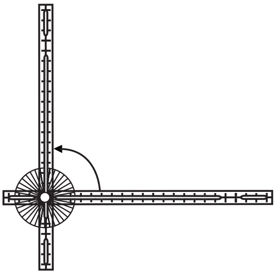


Fig. 3. Right angle

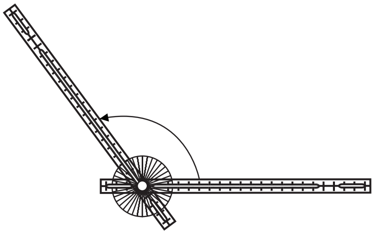


Fig. 4. Obtuse angle

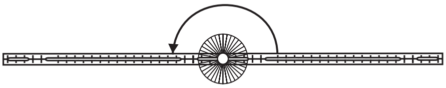


Fig. 5. Straight angle

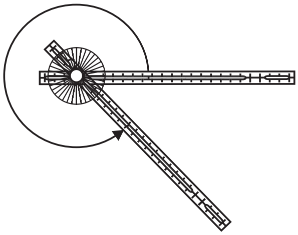


Fig. 6. Reflex angle

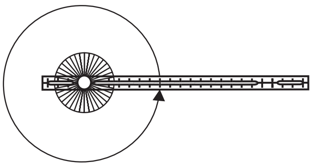


Fig. 7. Complete angle

Fill in the blanks after observing the measures of various angles so formed with a protractor and plastic strips.

1. Right angle is formed when the measure is ________________.
2. Straight angle is formed when the measure is ______________.
3. Complete angle is formed when the measure is _____________.
4. Acute angle is formed when the measure is _______________.
5. Obtuse angle is formed when the measure is ______________.
6. Reflex angle is formed when the measure is _______________.

Measuring angles with starting point of any degree other than zero degree.

Fix the first strip along with the 30° marked line of the protractor and second strip along with 70° marked line of the protractor.

- What is the measure of the angle formed?
- What type of angle is it?

The angle so formed is an acute angle of measure $70° - 30° = 40°$.

Similarly, take the two strips at different marked lines of the protractor and then complete the following table:

S. No.	Position of first strip (A)	Position of second strip (B)	Measure of angle (B–A)	Type of angle
1.	10°	50°	40°	Acute
2.	25°	60°	------	------
3.	------	170°	135°	------
4.	50°	200°	------	------
5.	-------	115°	------	Right
6.	-------	230°	180°	------
7.	30°	280°	------	------

Now, fix the first strip at 40°. Give measures of some more angles by which the second strip will be moved in anticlockwise direction to complete the following table:

S. No.	Position of first strip	Position of second strip	Measure of angle	Type of angle
1.	40°			Acute
2.	40°			Obtuse
3.	40°			Right
4.	40°			Straight
5.	40^0			Reflex

Activity 2

Two Parallel Lines and a Transversal

Objective

To verify the relation between different pairs of angles formed by a transversal with two parallel lines.

Material required

Three plastic strips, two full protractors, and fly screws.

How to proceed?

1. Take three strips and two full protractors and fix them with the help of fly screws in such a manner that the two strips are parallel to each other and the third strip is a transversal to them as shown in Fig. 1.
2. Measure all the angles so formed numbered from 1 to 8 and complete the following tables.

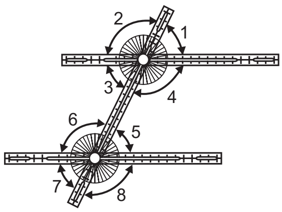


Fig 1

> ***Think!***
> *How would you check whether two strips are parallel or not?*

Table A: Corresponding angles

S. No.	Name of the angle	Measure of the angle	Name of he angle	Measure of the angle	Observation (Relationship)
1.	∠1	52°	∠5	52°	Equal
2.	∠2	52°	∠6	52°	Equal
3.	∠3	52°	∠7	52°	Equal
4.	∠4	52°	∠8	52°	Equal

Inference : ....

Table B: Alternate interior and exterior angles

S. No.	Name of the angle	Measure of the angle	Name of the angle	Measure of the angle	Observation (Relationship)
1.	∠3	52°	∠5	52°	Equal
2.	∠4	52°	∠6	52°	Equal
3.	∠1	52°	∠7	52°	Equal
4.	∠2	52°	∠8	52°	Equal

Inference : ....

Table C: Interior angles on the same side of the transversal

S. No.	Name of the angle	Measure of the angle	Name of the angle	Measure of the angle	Observation
1.	∠4	128°	∠5	52°	
2.	∠3	52°	∠6	52°	

Inference : ....

Now, fix these strips and two protractors in such a manner that the two strips are not parallel to each other and the third strip is transversal to them as shown in Fig. 2.

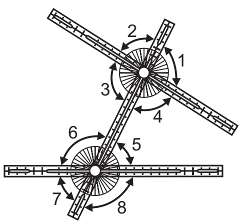
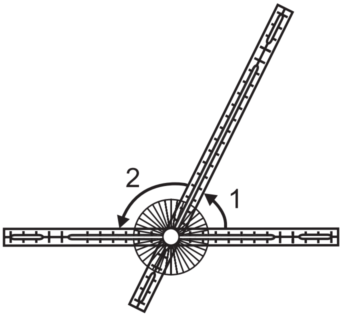


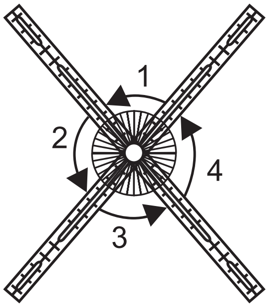
Fig. 2

Repeat the activity and complete the Tables A, B, and C as done earlier and write the inference in each case.

You can also observe the properties of a linear pair and vertically opposite angles using the set up as shown here:



(i) For linear pair



(ii) For vertically opposite angle

Fig. 3

Q1: Verify the results for a linear pair and vertically opposite angles using Fig. 3.

Q2: Can you correlate this activity with Euclid's 5th postulate, *if a straight line falling on two straight lines makes the interior angles on the same side of it taken together less than two right angles, then the two straight lines if produced indefinitely, meet on that side on which the sum of angles is less than two right angles.*

Activity 3

PROPERTIES OF A TRIANGLE

OBJECTIVE

To explore the properties of a triangle.

MATERIAL REQUIRED

Three plastic strips, three half protractors or full protractors, and fly screws.

HOW TO PROCEED?

1. Fix the strips along with protractors as shown in Fig.1. Two strips can be joined, if needed.

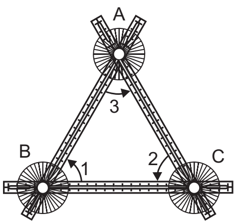


Fig. 1

2. Form different triangles by moving the strips. Also, measure the angles (interior as well as exterior taken in an order) and the sides of the triangles so formed and explore its properties.

Angle Sum Property of a Triangle

Vary the angles of the triangle, and note their measurements to find out the relationship between it angles.

S.No.	Angle 1	Angle 2	Angle 3	$\angle 1 + \angle 2 + \angle 3$
1.				

2.				
3.				

Inference : ....

Exterior Angle Property

Look at the exterior angles formed by the extended side and the respective interior opposite angles of the triangle. Note down their measurements to find out the relationship.

S. No.	Exterior angle	Interior opposite angles	Sum of interior opposite angles
1.			
2.			
3.			

Inference : ....

Isosceles Triangle

Form different triangles having two equal sides (Fig.2) by moving the strips and note down the measure of sides and angles of the triangles so formed.

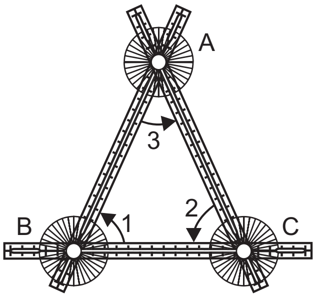


Fig.2

S. No.	Length of the side			Measure of the angle			Equal sides	Equal angles
	AB	BC	AC	∠1	∠2	∠3		
1.								
2.								
3.								

Inference : ....

Equilateral Triangle

Form different triangles having all three sides equal (Fig. 3) by moving the strips and note down the measures of sides and angles of the triangles, so formed, in the following table:

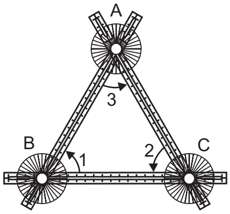


Fig.3

S. No	Length of the side			Measure of the angle		
	AB	BC	AC	∠1	∠2	∠3
1.						
2.						
3.						

Inference : ....

Scalene Triangle

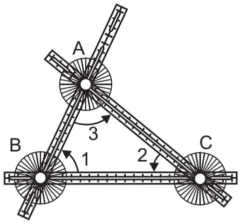


Fig.4

Form different scalene triangles (Fig. 4) by moving the strips and note down the measure of sides and angles of the triangles, so formed, in the following table:

S. No	Length of the side			Measure of the angle		
	AB	BC	AC	∠1	∠2	∠3
1.						
2.						
3.						

Inference : ....

Angle Opposite to Longer Side in a Triangle

Vary the length of one side so that it becomes longer than the other side. Measure sides, angles and note down in the table. Similarly, vary the angle to make it bigger than the other angle and note down in the table. Explore the relationship between two angles and sides opposite to them.

S. No.	Length of the side			Measure of the angle			Longer side	Greater angle
	AB	BC	AC	∠1	∠2	∠3		
1.								
2.								
3.								

Inference : ....

Sum of any two sides in a triangle

Make triangles of different side lengths and note their measurements in the table. Explore the relationship of sum of two sides with the third side.

S. No.	Length of the side			AB + BC	BC + AC	AB + AC
	AB	BC	AC			
1.						
2.						
3.						

Inference : ....

Right Triangle

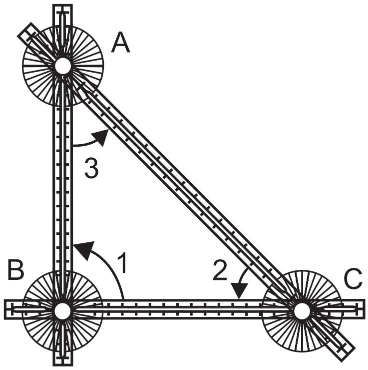
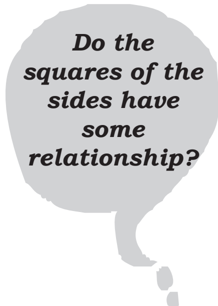


Fig.5



Make different right triangles (Fig. 5) by moving the strips and complete the following table:

S. No.	Length of the side			Measure of the angle			Longest side	Square of the length of the sides		
	AB	BC	AC	$\angle 1$	$\angle 2$	$\angle 3$		AB^2	BC^2	AC^2
1.										
2.										
3.										

Inference : ....

Activity 4

MID-POINT THEOREM

OBJECTIVE

To verify the mid-point theorem "A line segment joining the mid-points of any two sides of a triangle is parallel to the third side and is half of it".

MATERIAL REQUIRED

Four plastic strips, two half protractors, and fly screws.

HOW TO PROCEED?

1. Fix 3 strips to form a triangle ABC and one half protractor at one vertex (say B) as shown in Fig. 1.

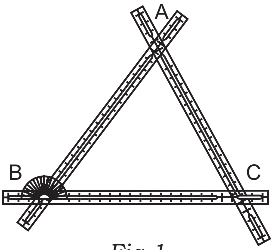


Fig. 1

2. Fix one more strip at the mid-point, say D, of one of the sides (say AB) of the triangle alongwith a half protractor.
3. Now adjust this strip so that it also passes through the mid-point, say E, of the other side AC as shown in Fig. 2.

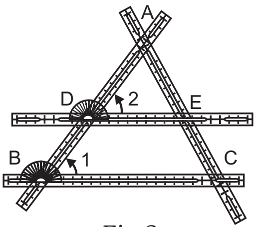


Fig.2

4. Now measure the angles shown by the two protractors.
5. Also, measure lengths of the sides BC and DE.
6. Repeat the above activity by forming different types of triangles with the help of strips in different orientations and complete the following table:

S. No.	$\angle 1$	$\angle 2$	Is $\angle 1 = \angle 2$?	Length of BC	Length of DE	Is DE = $\frac{1}{2}$ BC?
1.						
2.						
3.						

Inference : Since $\angle 1 = \angle 2$, so, DE.......... BC and DE $= \frac{1}{2} \times$........

Activity 5

Converse of Mid-point Theorem

Objective

To verify that a line drawn through the mid-point of one side of a triangle and parallel to the second side bisects the third side.

Material required

Four plastic strips, two half protractors, and fly screws.

How to proceed?

1. Fix 3 strips to form a triangle ABC and a half protractor at one of the vertices, say B, as shown in the Fig. 1.

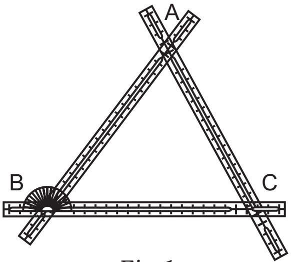


Fig. 1

2. Fix another strip and a protractor at the mid-point, say D, of one of the sides, say AB, of the triangle so that it intersects the third side of triangle at E.
3. Now adjust this strip so that the angles shown on the two protractors are equal as shown in the Fig. 2.

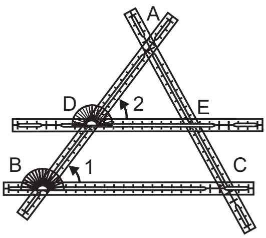


Fig. 2

4. These are corresponding angles. So, strip DE is parallel to BC.
5. Now measure the lengths AE and EC.
6. Repeat the above activity by forming different types of triangles and complete the following table:

S. No.	AE	EC	Is AE = EC?
1.			
2.			
3.			

Inference : ....

Activity 6

Basic Proportionality Theorem

Objective

To verify the Basic Proportionality Theorem: "If a line, drawn parallel to one side of a triangle, intersects the other two sides, then it divides them in the same proportion".

Material required

Four plastic strips, two half protractors, and fly screws.

How to proceed?

1. Fix 3 strips to form a triangle ABC and one half protractor at one of the vertices, say B, as shown in Fig. 1.

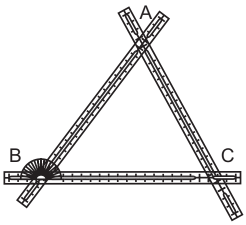


Fig. 1

2. Fix another strip along with a half protractor at any convenient point, say D, on one of the sides of the triangle so that the ratio of AD and DB can be calculated easily.
3. Now adjust this strip so that the angles on the two protractors are equal as shown in the Fig. 2.

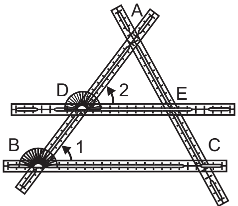


Fig. 2

4. Now measure the lengths of AD, DB, AE and EC.
5. Repeat the above activity by forming different types of triangles and complete the following table:

S. No.	AD	DB	AD : DB	AE	EC	AE : EC	Is AD:DB=AE:EC?
1.							
2.							
3.							

Inference : Since $\angle 1 = \angle 2$, so DE$\|$ .... If a line is drawn parallel to one side of a triangle, intersects the other two sides, then it divides them ....

Activity 7

Converse of Basic Proportionality Theorem

Objective

To verify that a line dividing two sides of a triangle in the same ratio is parallel to the third side.

Material required

Four plastic strips, two half protractors, and fly screws.

How to proceed?

1. Fix 3 strips to form a triangle ABC having a half protractor at one of the vertices, say B, as shown in Fig.1.

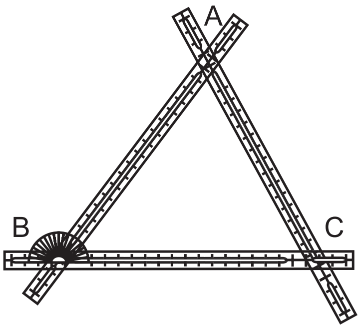


Fig. 1

2. Fix another strip at a convenient point, say D, on side AB and point, say E, on side AC, such that the ratios AD : DB and AE : EC are same. Also, fix a half protractor at D as shown in Fig. 2.

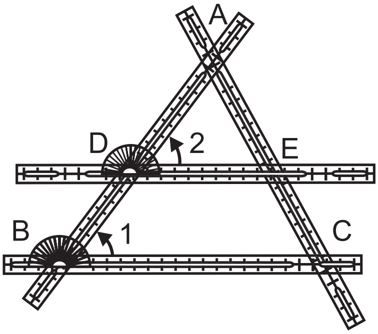


Fig.2

3. Now measure the angles at B and D as shown on the two half protractors.
4. Repeat the above activity by:
   - Forming different types of triangles.
   - Keeping a triangle fixed and varying the positions of the strip on the two sides AB and AC of triangle ABC such that the ratio AD : DB = AE : EC.
5. Now complete the following table:

S. No.	∠1	∠2	Is ∠1=∠2?
1.			
2.			
3.			

Inference : Since ∠1 = ∠2, so DE ∥ ....

Activity 8

PROPERTIES OF A QUADRILATERAL

OBJECTIVE

To explore properties of different types of quadrilaterals.

MATERIAL REQUIRED

Six plastic strips, four half protractors, one full protractor, and fly screws.

HOW TO PROCEED?

Angle Sum Property of a Quadrilateral

1. Fix four strips along with the half protractors using fly screws as shown in Fig. 1.

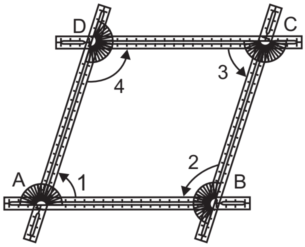


Fig. 1

2. Similarly, form different quadrilaterals by moving strips and measure the angles of each quadrilateral so formed in the table given below:

S. No.	∠1	∠2	∠3	∠4	∠1+ ∠2+ ∠3+ ∠4
1.					
2.					
3.					

Inference : ....

Properties of a Parallelogram

1. Form different parallelograms by moving the strips as shown in Fig. 2 and for each parallelogram measure the angles and the lengths of the sides.

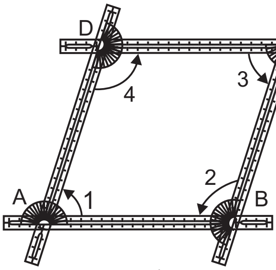


Fig.2

Now complete the following table:

S. No.	Lengths of the sides				Measures of angles				∠1 + ∠2	∠1 + ∠4	∠3 + ∠4	∠2 + ∠3
	AB	BC	DC	AD	∠1	∠2	∠3	∠4				
1.												
2.												
3.												

Inference : 1. Opposite angles are ....
2. Opposite sides are ....
3. Adjacent angles are ....

2. Take two strips and fix them as diagonals AC and BD as shown in Fig. 3.

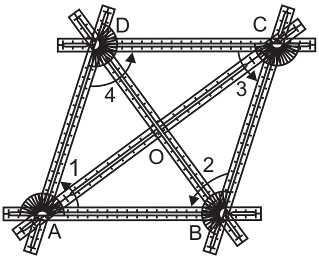


Fig. 3

3. Similarly, form different parallelograms with diagonals and note down the distance of vertices of each parallelogram from the point of intersection *O* in the following table:

S. No.	Measure of angle				Length along the diagonal					
	∠1	∠2	∠3	∠4	AC	AO	OC	BD	BO	OD
1.										
2.										
3.										

Inference : 1. Diagonals ....
2. Point of intersection of diagonals ....

Properties of a Rhombus

1. Make different rhombuses by moving the strips and measure the angles, lengths of sides of each rhombus so formed as shown in Fig. 4.

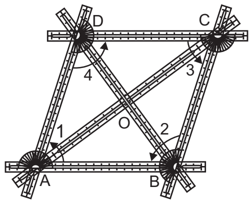


Fig. 4

Now complete the following table:

S. No.	∠1	∠2	∠3	∠4	AB	BC	CD	DA
1.								
2.								
3.								

Inference : 1. Opposite angles ....
2. Opposite sides ....

2. Take two strips and fix them as diagonals AC and BD as shown in Fig. 4. Now, measure the angles formed by the

diagonals and the distance of the vertices from the point of the intersection *O* of the diagonals.

S. No.	**Length of diagonal**		**Distance from vertices to center**				**Measure of angle**			
	BD	AC	AO	OC	BO	OD	∠ BOA	∠BOC	∠DOC	∠AOD
1. 2. 3.										

Inference : 1. Diagonals ....

2. Point of intersection of diagonals ....

3. Angles between the diagonals ....

Properties of a Rectangle

1. Make different rectangles by moving the strips and fix two strips as diagonals AC and BD of the rectangle so formed as shown in Fig. 5.

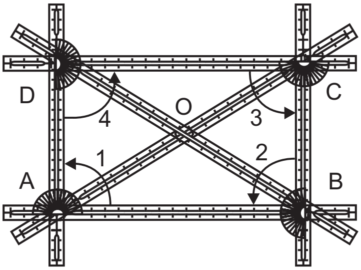


Fig.5

2. Now measure the angles, sides and diagonals of the rectangle so formed. Also, measure the distances of vertices from the point of intersection *O* and the angles formed by diagonals of each rectangle so formed and complete the following table:

S. No.	Lengths of the side				Measures of angle				Length along the diagonal					
	AB	BC	CD	AD	∠1	∠2	∠3	∠4	AC	AO	OC	BD	BO	OD
1.														
2.														
3.														

Inference : 1. Opposite sides ....

2. Angles ....

3. Point of intersection of diagonals ....

4. Lengths of diagonals ....

Properties of a Square

Make different sqaures by moving the strips (Fig. 6) and measure its angles, sides and different line segments. Complete the following table:

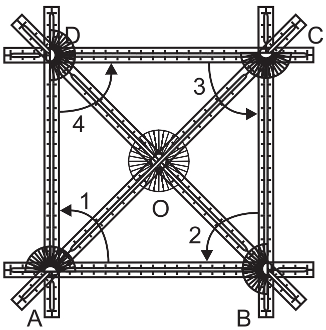


Fig.6

S. No.	Measure of angle				Distance along diagonal						Measure of angles			
	∠1	∠2	∠3	∠4	AC	AO	OC	BD	BO	OD	∠BOA	∠BOC	∠DOC	∠AOD
1.														
2.														
3.														

.**Inference :** 1. Sides ....

2. Point of intersection of diagonals ....

3. Angles..

4. Angles between diagnonals............................

5. Lengths of diagonals ....

Activity 9

Quadrilateral Formed by Mid-points of Sides of a Given Quadrilateral

Objective

To verify that a quadrilateral formed by joining the mid-points of the sides of a quadrilateral, taken in order, is a parallelogram.

Material required

Eight plastic strips, two half protractors, and fly screws.

How to proceed?

1. Fix four plastic strips using fly screws to form a quadrilateral ABCD as shown in Fig.1.

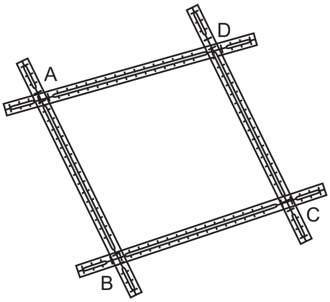


Fig. 1

2. Fix the remaining four strips at the mid-points E, F, G and H of the sides of quadrilateral in such a manner that they form a quadrilateral as shown in Fig. 2.

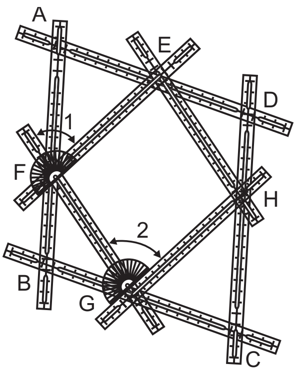


Fig.2

3. Now measure ∠1, ∠2 and length of sides HE and GF.
4. Repeat the above activity by forming different types of quadrilaterals and complete the following table :

S. No.	∠1	∠2	Is ∠1 = ∠2?	HE	GF	Is HE = GF?
1.						
2.						
3.						

Inference: Since ∠1 = ∠2, so HE ∥ ....
Also, HE and GF are.....................................
So, quadrilateral EFGH is a...........................

Activity 10

Exploring Area using Geoboard

Objective

To form different shapes on a geoboard and explore their areas.

Material required

Geoboard, rubber bands, and geoboard pins.

How to proceed?

Area of a Rectangle

1. Form shapes of different rectangles using geoboard pins and rubber bands on the geoboard as shown in Fig. 1.

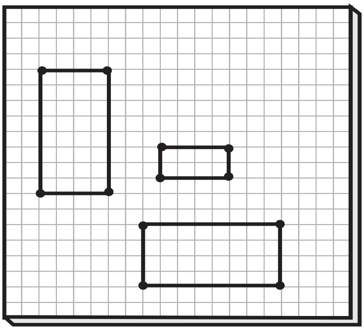


Fig. 1

2. Now count the number of unit squares enclosed in each rectangle and complete the following table:

S. No.	Total number of unit squares in a rectangle	Length of the rectangle	Breadth of the rectangle	Length × Breadth
1.				
2.				
3.				

Inference: Area of a rectangle is ....

Area of a Square

1. Form the shapes of different squares using geoboard pins and rubber bands on a geoboard as shown in Fig. 2.

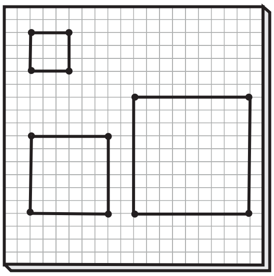


Fig.2

2. Count the number of unit squares enclosed in each of the three squares and complete the following table:

S. No.	Total number of unit squares in the square	Side length of the square	Side × Side
1.			
2.			
3.			

Inference : Area of a square is ....

Area of a Right Triangle

1. Form shapes of different right angled triangles with the help of geoboard pins and rubber bands on a geoboard as shown in Fig. 3.

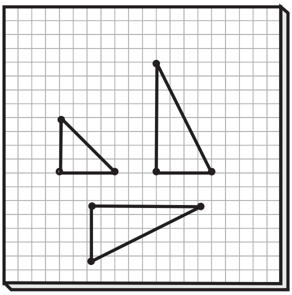


Fig.3

2. Now count the number of unit squares enclosed in each right angled triangle using the criterion for finding the area of any irregular figure by counting the number of unit squares enclosed in it as given below and complete the following table:

S. No.	**Total number of unit squares in the right triangle**	**Height (h)**	**Base (b)**	$\frac{1}{2}\times(b\times h)$
1.				
2.				
3.				

Inference: Area of a right angled triangle is ....

Area of Irregular Figures

1. Form an irregular figure on the Geoboard with the help of geoboard pins and rubber bands as shown in Fig.4.

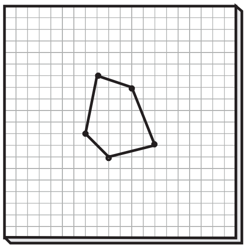


Fig.4

2. Find out the area of this figure by counting the number of unit squares enclosed in it in the following manner:
   (a) Count one complete unit square enclosed by the figure as 1 and take its area as 1 square unit.
   (b) Count the unit square which is more than half enclosed by the figure as 1 and take its area as 1 square unit.
   (c) Count the unit square which is half enclosed by the figure as ½ and take its area as ½ square unit.
   (d) Neglect the unit squares which are less than half enclosed by the figure.
3. The above Fig.4 enclosed 17 complete unit squares, 4 more than half unit squares and 5 half unit squares.
4. Make more irregular figures on the geoboard and try to

find out their areas.

Inference: The area of the above figure is ....

Activity 11

AREAS OF SIMILAR TRIANGLES

OBJECTIVE

To verify that the ratio of areas of two similar triangles is equal to the ratio of squares of their corresponding sides.

MATERIAL REQUIRED

Geoboard, rubber bands, and geoboard pins.

HOW TO PROCEED?

1. Make two similar triangles ABC and DEF by fixing 6 geoboard pins at suitable places on a geoboard and rubber bands as shown in Fig.1.

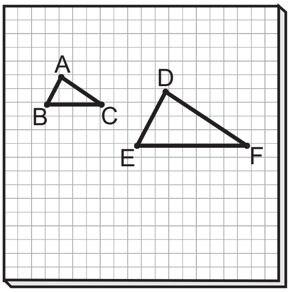


Fig. 1

2. Using rubber bands and geoboard pins make squares on sides BC and EF as shown in Fig. 2.

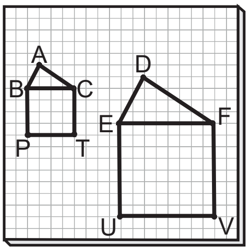


Fig. 2

3. Find out the area of the two triangles and the two squares formed above by counting the number of unit squares enclosed in it as discussed earlier in Activity No. 10.
4. Also, find out the lengths of sides BC and EF.
5. Repeat the above activity by forming different pairs of similar triangles by suitably changing the positions of the geoboard pins and complete the following table:

S. No.	$\frac{ar(\text{D}ABC)}{ar(\text{D}DEF)}$	**Area of square on BC $(BC)^2$**	**Area of square on EF $(EF)^2$**	$\frac{ar(\text{D}ABC)}{ar(\text{D}DEF)} = \frac{(BC)^2}{(EF)^2}$
1.				
2.				
3.				

Inference: ....

Activity 12

Median and Area of a Triangle

Objective

To verify that median of a triangle divides it into two triangles of equal area.

Material required

Geoboard, rubber bands, and geoboard pins.

How to proceed?

1. Fix three pins on a geoboard and use a rubber band to form the ΔABC such that the length of base BC has even number of units.

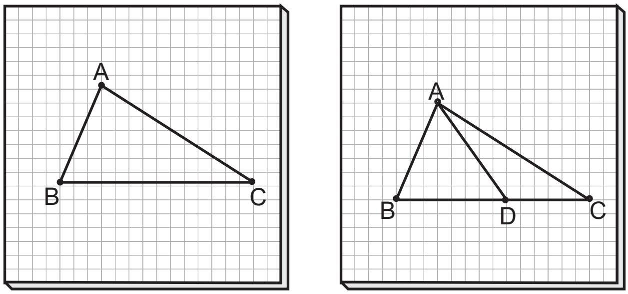


Fig. 1.

2. From the mid-point of BC say D, join A using a rubber band to form the median AD.
3. Find out the area of triangle ADC and triangle ADB by counting the number of unit squares enclosed in it as discussed earlier in Activity No. 10.
4. Repeat the above activity by forming different triangles by changing the positions of geoboard pins and complete the following table :

S. No.	ar($\triangle$ABD)	ar($\triangle$ACD)	Is ar($\triangle$ABD) = ar($\triangle$ACD)?
1.			
2.			
3.			

Inference : The median of a triangle divides it in two triangles having ....

Activity 13

Figures on the Same Base and between the Same Parallel Lines

Objective

To form different figures on a Geoboard satisfying the following conditions:

(a) Lying on the same base.

(b) Lying between the same parallel lines but not on the same base.

(c) Lying on the same base and between the same parallel lines.

Material required

Geoboard, rubber bands, and geoboard pins.

How to proceed?

Figures Lying on the Same Base

1. Fix two geoboard pins on a geoboard at two different points say A and B and place three different coloured rubber bands between the two geoboard pins as shown in Fig.1.

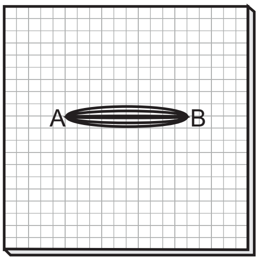


Fig.1

2. Now with these three rubber bands make three different figures say a triangle, rectangle and trapezium by fixing geoboard pins at suitable places as shown in Fig.2.

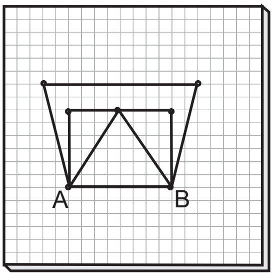


Fig.2

All these figures formed above in Fig. 2 are lying on the same base AB

Figures Lying Between the Same Parallels but not on the Same Base

1. Fix four geoboard pins suitably in a line. Now fix two geoboard pins on a line parallel to the previous line on the geoboard.
2. Take three rubber bands and make three different figures say a square, a triangle and a trapezium such that they lie between the same parallel lines but **not** on the same base as shown in the Fig. 3 given below:

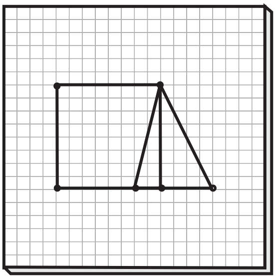


Fig.3

Figures Lying on the Same Base and Between the Same Parallel Lines

1. Fix two geoboard pins on the geoboard at two points and place three different coloured rubber bands between these two pins.
2. Now with these rubber bands make three different figures say a triangle, rectangle and trapezium in such a manner

that they lie on the same base and between the same parallel lines as shown in Fig. 4:

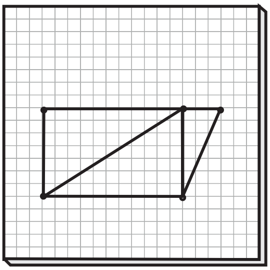


Fig.4

3. Repeat the above activities by making different shapes using geoboard pins and rubber bands satisfying any of the above three conditions.

Activity 14

Triangles on the Same Base and between the Same Parallels

Objective

To verify that triangles on the same base and between the same parallels are equal in area.

Material required

Geoboard, rubber bands, and geoboard pins.

How to proceed?

1. Fix geoboard pins on the geoboard to make ΔABC using rubber band as shown in Fig. 1.

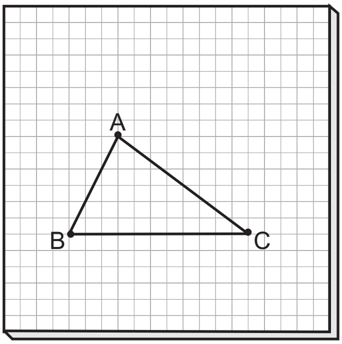


Fig. 1

2. Fix more geoboard pins on the geoboard at suitable places to form another ΔDBC (use different coloured rubber band) such that both the triangles are on the same base and between the same parallel lines as shown in Fig. 2.

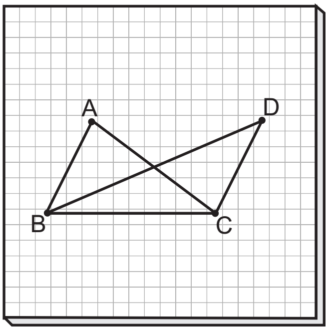


Fig. 2

3. Find out the areas of the two triangles by counting the number of unit squares enclosed in it as discussed earlier in Activity No.10.
4. Repeat the above activity by forming different pairs of triangles lying on the same base and between the same parallel lines by suitably changing the positions of geoboard pins and complete the following table :

S.No.	*ar* (ΔABC)	*ar* (ΔDBC)	Is *ar* (ΔABC) = *ar* (ΔDBC)?
1.			
2.			
3.			

Inference : Triangles on the same base and between same parallels are ....

Activity 15

PARALLELOGRAMS ON THE SAME BASE AND BETWEEN SAME PARALLEL LINES

OBJECTIVE

To verify that parallelograms on the same base and between the same parallels are equal in area.

MATERIAL REQUIRED

Geoboard, rubber bands, and geoboard pins.

HOW TO PROCEED?

1. Fix geoboard pins on a geoboard to make a parallelogram ABCD using rubber band as shown in Fig. 1.

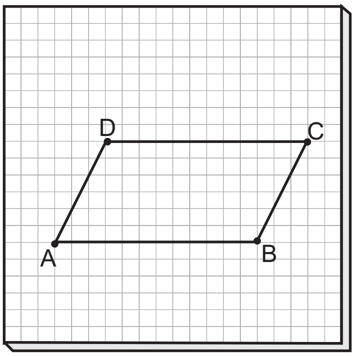


Fig. 1

2. Fix more geoboard pins on the geoboard at suitable places to form another parallelogram ABEF (use different coloured rubber band) such that both the parallelograms are on the same base and between the same parallel lines as shown in Fig. 2.

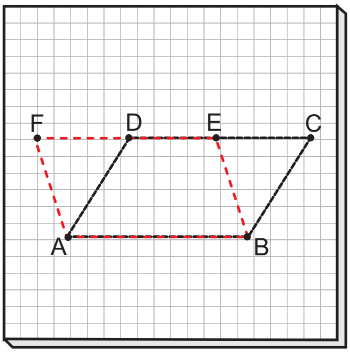


Fig. 2

3. Find the area of the two parallelograms by counting the number of unit squares enclosed as done earlier in Activity No.10.
4 . Repeat the above activity by forming different pairs of parallelograms by suitably changing the positions of geoboard pins and complete the following table :

S. No.	**Area of ||gm ABCD**	**Area of ||gm ABEF**	**Is ar (||gm ABCD) = ar (||gm ABEF)?**
1.			
2.			
3.			

Inference : Parallelograms on the same base and between same parallel lines are ....

Activity 16

Triangle and Parallelogram on the Same Base and between Same Parallel Lines

Objective

To verify that for a triangle and a parallelogram on the same base and between the same parallels, the area of the triangle is half the area of the parallelogram.

Material required

Geoboard, rubber bands, and geoboard pins.

How to proceed?

1. Fix geoboard pins on a geoboard to make a parallelogram ABCD using rubber band as shown in Fig. 1.

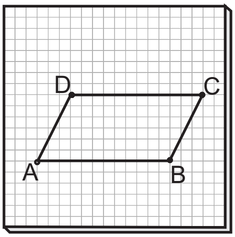


Fig. 1

2. Fix more geoboard pins on the geoboard at suitable places to form a triangle ABE (use different colours of rubber bands) such that the triangle and the parallelogram lie on the same base and between the same parallel lines as shown in Fig. 2.

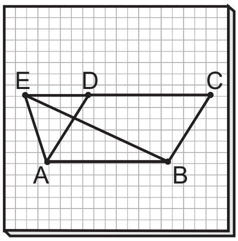


Fig. 2.

3. Find the area of parallelogram ABCD and ΔABE by counting the number of unit squares enclosed as done earlier in Activity No.10.
4. Repeat the above activity by forming different pairs of triangles and parallelograms by suitably changing the positions of geoboard pins and complete the following table :

S. No.	**Area of ||gm ABCD**	**Area of (ΔABE)**	**Is *ar* (ΔABE)=$\frac{1}{2}$ *ar* (||gm ABCD)?**
1.			
2.			
3.			

Inference : If a triangle and a parallelogram are on the same base and between the same parallel lines, then area of triangle is .... of the area of ....

Activity 17

Areas of Triangle, Parallelogram and Trapezium

Objective

To explore area of a triangle, parallelogram and trapezium.

Material required

Cut-outs of different shapes.

How to proceed?

Area of Parallelogram

1. Put cut-outs of a triangular piece 'A' and a trapezium piece 'B' together to form a parallelogram as shown in Fig. 1.

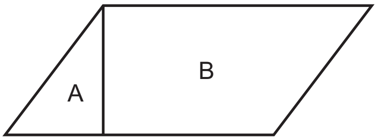


Fig. 1

2. Remove cut-out A of triangular piece and attach it to the other side of cut out B of trapezium piece as shown in Fig. 2 given below. It will form a rectangle.

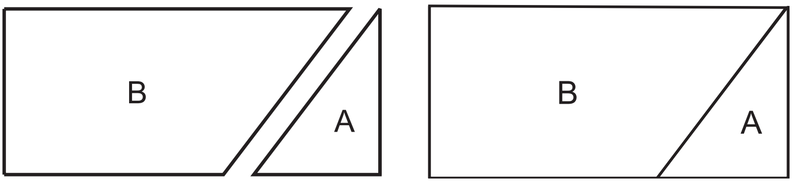


Fig. 2

Inference:

Area of the Parallelogram = Area of ....

= Length × .... of rectangle.

= Base × .... of parallelogram.

Area of Triangle

1. Put the cut-outs of the two congruent triangles T_1 and T_2 together to form a parallelogram as shown in Fig. 3.

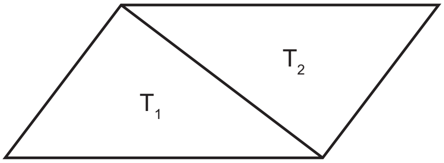


Fig.3

Inference :

Area of the triangle (T_1 or T_2) = $\frac{1}{2}$ × Area of ....

2. Take a parallelogram and three triangular pieces A, B and C which exactly cover the parallelogram as shown in Fig. 4 (I).

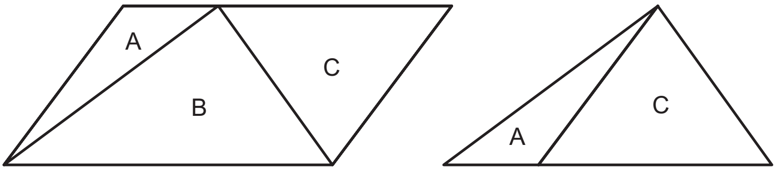


Fig.4

(I) (II)

3. Put the triangular pieces A and C together. They will cover the triangular piece B completely as shown in Fig.4 (II).

Inference : Area of piece B = Area of piece A + Area of piece C

Area of the parallelogram = 2 × Area of piece B

Area of piece B = ...

Area of Trapezium

1. Take cut-outs of two congruent trapeziums 'C' and 'D' having height 'h' and parallel sides 'a' and 'b'.
2. Put the cut-outs together to form a parallelogram as shown in Fig. 5 given below:

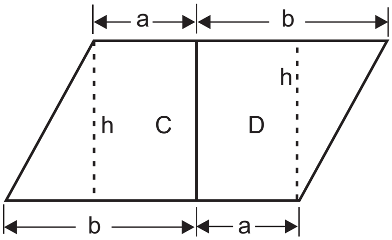


Fig.5

Inference :

Area of trapezium C = Area of trapezium D.

Area of parallelogram = Area of .... + Area of ....

$\therefore$ Area of trapezium = $\frac{1}{2}$ Area of ....

$= \frac{1}{2}(a+b) \times$ ....

> ***Note***
>
> *This activity may be repeated by using suitable cut-outs of triangles, trapeziums and parallelograms of different sizes and students may be encouraged to make more cut-outs of parallelograms, trapeziums and triangles to explore the inter-relationship of the areas of various shapes.*

Activity 18

Pythagoras Theorem

Objective

To verify Pythagoras theorem, i.e., "In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides".

Material required

Cut-out of a right angled triangle with sides *a*, *b,* and *c*, cut-outs of squares of sides *a*, *b,* and *c*.

How to proceed?

1. Arrange the cut-outs of three squares of sides *a*, *b* and *c* and right angled triangle as shown in Fig. 1.

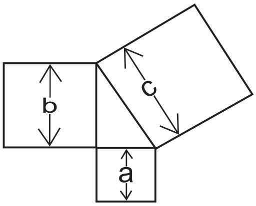


Fig. 1

2. Put the squares of sides a and b together as shown in Fig.2.

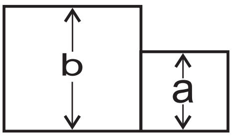


Fig.2

3. Three cut-outs from squares of side b and 2 cut-outs from squares of side a (given in the kit) are prepared by marking

the two right angled triangles with sides a, b and c in Fig. 2 and then cutting it along the lines as shown in Fig. 3.

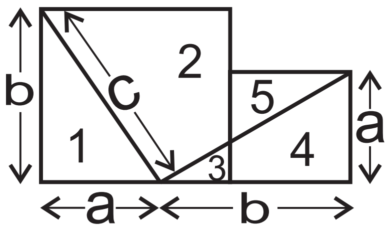


Fig.3

4. Now rearrange these 5 pieces on the square of side c as shown in Fig.4. The square of side c is exactly covered by the five cut-out pieces of squares of sides a and b respectively.

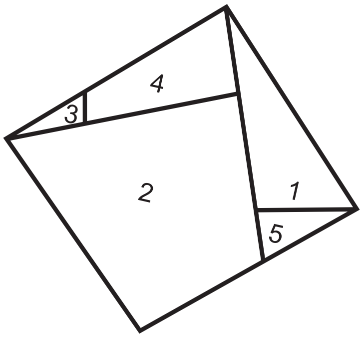


Fig.4

5. From step 4, we have $a^2 + b^2 = c^2$.
6. Repeat this activity by taking different right angled triangles.

Inference : In a right angled triangle with sides a, b and c, where c is hypotenuse.

$a^2 + b^2 =$ ....

In a right angled triangle, square of hypotenuse is ....

Activity 19

ALGEBRAIC IDENTITIES

OBJECTIVE

To verify the algebraic identities:

(i) $(a+b)^2 = a^2 + 2ab + b^2$

(ii) $(a-b)^2 = a^2 - 2ab + b^2$

MATERIAL REQUIRED

Cut-outs of squares of sides a and b units, two rectangular cut-outs of length a units, and breadth b units.

HOW TO PROCEED?

$$(a+b)^2 = a^2 + 2ab + b^2$$

1. Arrange the four cut-outs on a table as shown in Fig. 1.

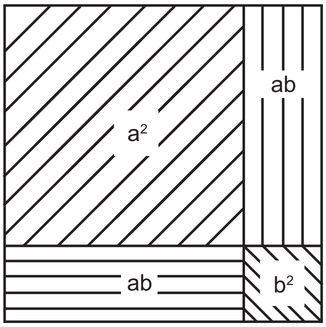


Fig. 1.

2. Look at the shape obtained in Fig. 1. It is a square of side (a+b) units.
3. Find the area of the shape so formed in Fig. 1, using the formula for area of square, i.e., side × side.
   $(a + b) \times (a + b) = (a + b)^2$
4. Also, find the area of the shape so formed by adding the areas of the four pieces as:

$$a^2 + ab + ab + b^2 = a^2 + 2ab + b^2$$

5. Repeat the activity by making squares and rectangles with different values of a and b using chart paper etc., and complete the following table:

S. No.	a	b	a^2	b^2	ab	2ab	$a^2+2ab+b^2$	(a+b)	$(a+b)^2$
1.									
2.									
3.									

Inference : $(a + b)^2 = a^2 + 2ab + b^2$, for all values of a and b

$(a - b)^2 = a^2 - 2ab + b^2$

1. Take cut-outs of two squares of sides a and b units. Arrange the two square cut-outs on the table as shown in Fig. 2.

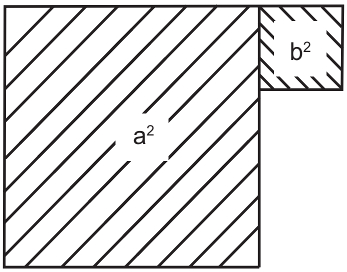


Fig.2

2. Now put one cut-out of rectangle of length a units and breadth b units on the shape which is obtained in Fig. 2 as shown in Fig. 3.

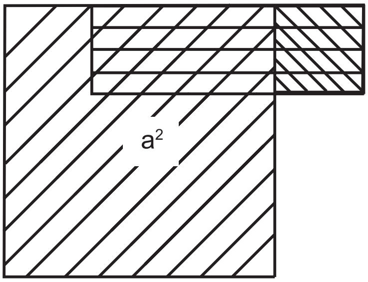


Fig.3

3. Put another rectangular piece of same dimensions on the shape in Fig.3 as shown in Fig.4.

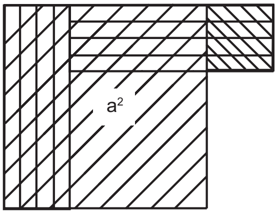


Fig.4

4. Look at uncovered portion in Fig. 4. Is it a square?
5. Find the length of its side. It is (a – b) units.
6. Find the area of the shape given in Fig. 2. It is $(a^2 + b^2)$ sq. units.
7. Find the area of shaded portion given in Fig. 4. It is 2ab sq. units .
8. Find the area of uncovered portion given in Fig. 4 from step 2 and step 3. It is $(a^2 + b^2 - 2ab)$ sq. units.
9. Now compute the area of uncovered portion in Fig. 4 using formula for area of square. It is $(a - b) \times (a - b) = (a - b)^2$ sq. units.
10. Repeat the above activity by taking squares and rectangles with different values of a and b and complete the following table :

S. No.	a	b	a^2	b^2	ab	2ab	$a^2-2ab+b^2$	(a–b)	$(a-b)^2$
1.									
2.									
3.									

Inference : $(a - b)^2 = a^2 - 2ab + b^2$, for all values of a and b.

Extension : Try verifying the identity $(a + b)^2 - 4ab = (a - b)^2$

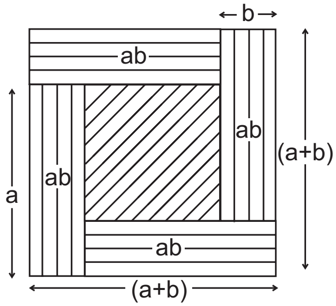


Fig.5

Activity 20

ALGEBRAIC IDENTITY

OBJECTIVE

To verify the algebraic identity:
$a^2 – b^2 = (a + b) (a – b)$

MATERIAL REQUIRED

Cut-outs of squares of side a and b units, two cut-outs of congruent trapeziums having parallel sides of length a, and b units.

HOW TO PROCEED?

1. Arrange the two cut-outs of squares on a table as shown in Fig.1.

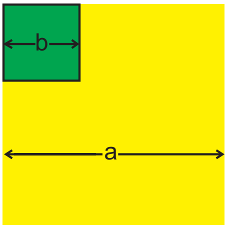


Fig. 1

2. Arrange the cut-outs of two trapeziums on uncovered portion of square of side a units as shown in Fig. 2 .They will cover the remaining portion of the square completely.

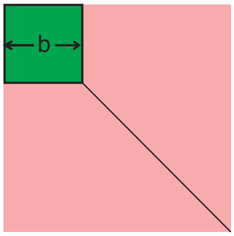


Fig. 2

3. Now take out the cut-outs of trapeziums and arrange them. They will form a rectangle of length (a+b) units and breadth (a–b) units as shown in Fig. 3.

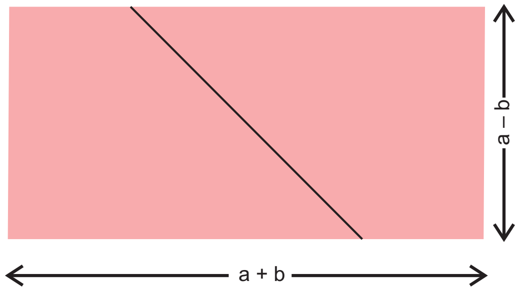


Fig. 3

4. Look at Fig. 1 and find the area of uncovered portion. It is $(a^2 - b^2)$ sq. units
5. Find the area of two trapezium pieces in Fig. 2. It is $(a^2 - b^2)$ sq. units
6. Look at Fig. 3. It is a rectangle with sides (a + b) and (a – b) units. Its area is (a + b) (a – b) sq. units.
7. Repeat the above activity by taking square and trapezium with different values of a and b and hence complete the following table:

S. No.	a	b	a^2	b^2	a + b	a – b	$a^2 - b^2$	(a+b)(a–b)	Is $a^2 - b^2$ = (a+b)(a–b)
1.									
2.									
3.									

Inference : $a^2 - b^2 = (a + b)(a - b)$, for all values of a and b.

Activity 21

Factorisation of a Quadratic Polynomial

Objective

To factorise expressions of the type $Ax^2 + Bx + C$, say

- $x^2 + 5x + 6$
- $x^2 - x - 6$
- $2x^2 - 7x + 6$

Material required

Blue, and red coloured algebraic tiles.

How to proceed?

To factorise $x^2 + 5x + 6$

1. Take one x^2 tile, five x tiles and six unit tiles. Arrange them to form a rectangle as shown in Fig.1.

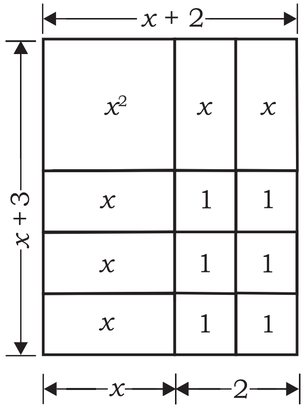


Fig. 1

2. The rectangle obtained in Fig.1 has sides of length $(x + 3)$ and $(x + 2)$. So, the area of this rectangle is $(x + 2)(x + 3)$.
3. Also, by adding the area of all the tiles enclosed by rectangle, we get
   $x^2 + x + x + x + x + x + 1 + 1 + 1 + 1 + 1 + 1 = x^2 + 5x + 6$

Inference : This shows that :

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

To factorise $x^2 - x - 6$

1. Take one x^2 tile, one '$(-x)$' tile and six '(-1)' tiles. Try to arrange these tiles in the form of a rectangle.
2. In this case, the tiles will not form a rectangle. Now, we will take one x^2 tile, three '$(-x)$' tiles, two x tiles and six '(-1)' tiles to form a rectangle as shown in given Fig. 2.

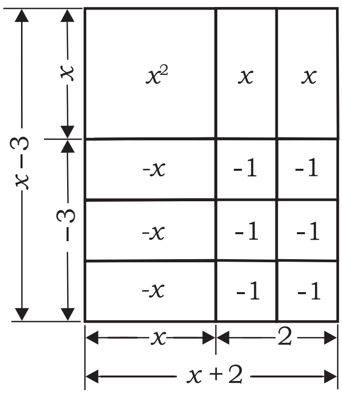


Fig.2

3. The rectangle obtained in Fig.2 has sides of lengths $(x + 2)$ and $(x - 3)$.
   So, the area of this rectangle is $(x + 2)(x - 3)$.
4. Also, by adding the area of all the tiles enclosed by the rectangle given in Fig.2, we get :
   $x^2 + x + x + (-x) + (-x) + (-x) + (-1) + (-1) + (-1) + (-1) + (-1) + (-1) = x^2 - x - 6$

Inference : This shows that :

$$x^2 - x - 6 = (x+2)(x-3)$$

To factorise $2x^2 - 7x + 6$

1. Take two x^2 tiles, seven $-x$ tiles and six unit tiles. Arrange these tiles in the form of a rectangle as shown in Fig. 3.

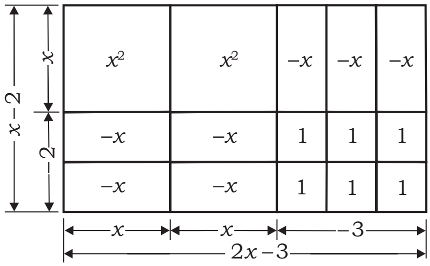


Fig. 3

2. The rectangle obtained in Fig.3 has sides of lengths $(2x - 3)$ and $(x - 2)$. So, the area of this rectangle is $(2x - 3)(x - 2)$.
3. Also, by adding the areas of all the tiles enclosed by the rectangle given in Fig.3, we get :

$$x^2 + x^2 + (-x) + (-x) + (-x) + (-x) + (-x) + (-x) + (-x) + 1 + 1 + 1 + 1 + 1 + 1$$
$$= 2x^2 - 7x + 6$$

Inference : This shows that :

$$2x^2 - 7x + 6 = (2x - 3)(x - 2)$$

Now, using the similar process of factorisation by algebraic tiles, complete the following table:

No. of tiles required for polynomials	x^2	x	$-x$	+1	–1	1st factor	2nd factor
$x^2 + 7x + 10$							
$-2x^2 - 3x + 5$							
$2x^2 + 10x$							
$x^2 - 7x + 12$							

Activity 22

Area of a Circle

Objective

To explore the area of a circle.

Material required

Cut-outs of circles.

How to proceed?

1. Take a circular cut-out which is divided into 4 cut-outs of equal sectors, half of which, i.e., 2 are labelled as 'A' as shown in Fig. 1.

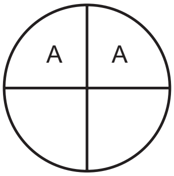


Fig. 1

2. Now arrange these sectors to form a figure as shown below in Fig. 2.

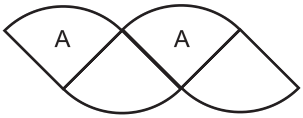


Fig. 2

3. Take a circular cut-out which is divided into 6 cut-outs of equal sectors, half of which, i.e., 3 are labelled as 'B' as shown in Fig. 3.

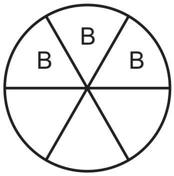


Fig. 3

4. Arrange these sectors to form a shape as shown below in Fig. 4.

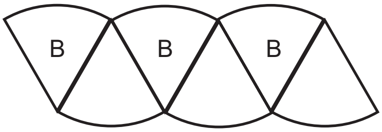


Fig. 4

5. Take a circular cut-out which is divided into 8 cut-outs of equal sectors, half of which, i.e., 4 are labelled as 'C' as shown in Fig. 5.

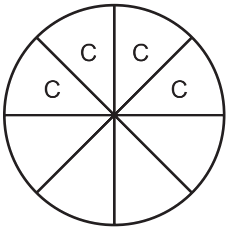


Fig. 5

6. Arrange these sectors to form a shape as shown below in Fig. 6.

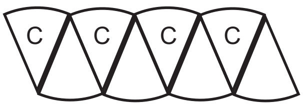


Fig. 6

7. Take a circular cut-out which is divided into 12 cut-outs of equal sectors, half of which, i.e., 6 are labelled as 'D' as shown in Fig. 7.

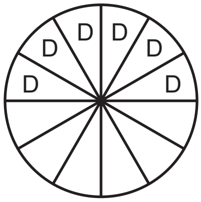


Fig. 7

8. Arrange these sectors to form a shape as shown below in Fig. 8.

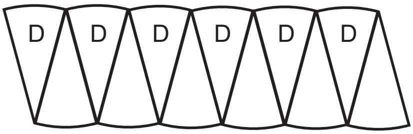


Fig. 8

9. Take a circular cut-out which is divided into 16 cut-outs of equal sectors, half of which, i.e., 8 are labelled as 'E' as shown in Fig. 9.

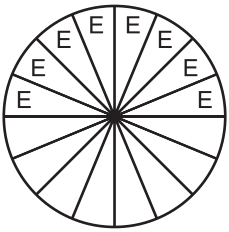


Fig. 9

10. Arrange these sectors to form a shape as shown below in Fig. 10.

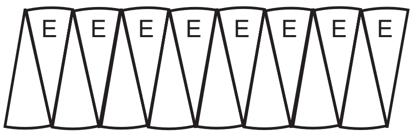


Fig. 10

11. What do you observe in all the figures above?

Inference : The figures which is so formed above looks like a parallelogram having length equal to half of circumference of respective circular cut-out and height equal to radius of circular cut-out. Hence, we get

Area of circle = Area of Parallelogram

= Base of parallelogram × Corresponding height of parallelogram

= πr^2

Extension: This activity may be repeated by taking 32 or 64 equal cut-outs of sectors of the circle.

Activity 23

Angle Subtended at the Centre of a Circle by Unequal Chords

Objective

To verify that a longer chord subtends greater angle at the centre of a circle.

Material required

Circular board, two plastic strips, rubber bands, connectors for circular board, and full protractor.

How to proceed?

1. Take two plastic strips and fix them on the circular board with the help of connectors representing two chords AB and CD of the circle of different lengths as shown in Fig.1.

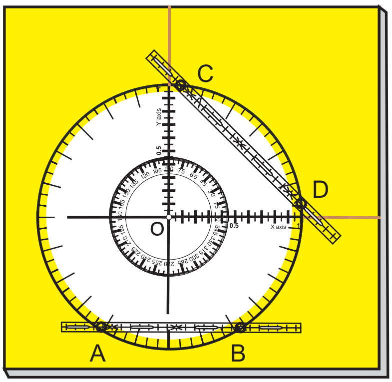


Fig. 1

2. With the help of rubber bands of different colours make two angles at the centre *O* subtended by the two chords as shown in Fig. 2.

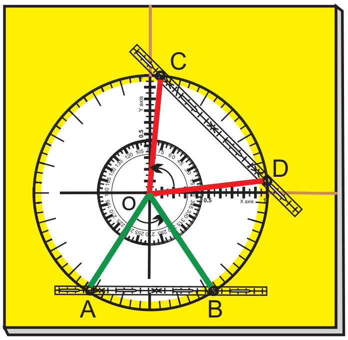


Fig.2

3. Measure ∠AOB and ∠COD at the centre of circle with the help of markings on the circular board or a full protractor.
4. Repeat the above activity by taking different lengths of the strips and complete the following table:

S. No.	AB	CD	∠AOB	∠COD	Longer chord	Greater angle
1.						
2.						
3.						

Inference : Angle at the centre of a circle made by longer chord is ....

Activity 24

Angles Subtended at the Centre of a Circle by Equal Chords

Objective

To verify that equal chords subtend equal angles at the centre of the circle.

Material required

Circular board, two plastic strips, rubber bands, connectors for circular board, and full protractor.

How to proceed?

1. Take two plastic strips and fix them on the circular board with the help of connectors in such a manner that they will represent two equal chords AB and CD of the circle as shown in Fig. 1.

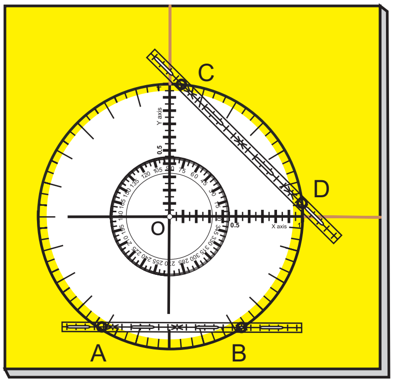


Fig. 1

2. With the help of different coloured rubber bands make angles at the centre *O* of given circle subtended by the two chords as shown in Fig. 2.

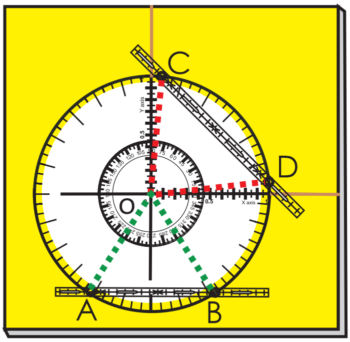


Fig.2

3. Measure ∠AOB and ∠COD with the help of markings on the circular board or a full protractor.
4. Repeat the above activity by taking different chords of equal lengths and complete the following table:

S. No.	AB	CD	∠AOB	∠COD	Is ∠AOB = ∠COD?
1.					
2.					
3.					

Inference : If AB .... CD, then
∠AOB .... ∠COD
i.e., equal chords of a circle subtend ....

Activity 25

Chords Subtending Equal Angles at the Centre of a Circle

Objective

To verify that the chords subtending equal angles at the centre of a circle are equal.

Material required

Circular board, two plastic strips, rubber bands, and connectors for circular board.

How to proceed?

1. Fix two plastic strips at suitable places with the help of connectors representing two chords AB and CD on the circular board such that the angles subtended by them at the centre of given circle are equal as shown in the Fig.1.

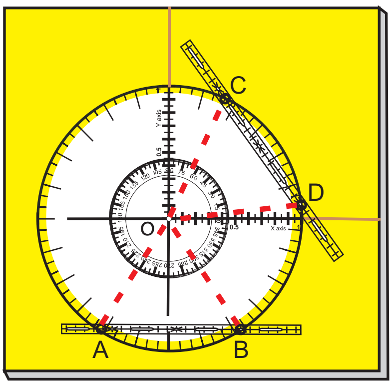


Fig. 1

2. Measure the length of each strip between the two connectors representing the two chords AB and CD.

3. Repeat the above activity by taking other pairs of chords subtending equal angles at the centre of circle on circular board and complete the following table:

S. No.	∠AOB	∠COD	AB	CD	Is AB = CD?
1.					
2.					
3.					

Inference : Chords subtending equal angles at the centre are ....

Activity 26

PERPENDICULAR FROM THE CENTRE TO A CHORD IN A CIRCLE

OBJECTIVE

To verify that the perpendicular drawn from the centre of a circle to a chord bisects the chord.

MATERIAL REQUIRED

Circular board, set squares, plastic strip, and connectors for circular board.

HOW TO PROCEED?

1. Fix a plastic strip on the circular board representing chord AB with the help of connectors.
2. Take a set square and place it on the circular board such that its one of the edges (other than the hypotenuse) coincides with the edge of the plastic strip as shown in Fig.1.

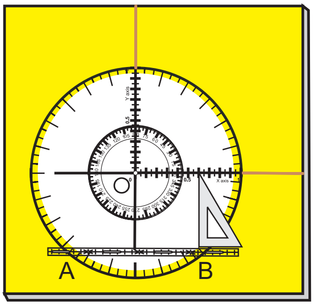


Fig. 1

3. Now slide the set square along the strip so that the other edge of the set square passes through the centre '*O*' of the

circle on the circular board and mark the meeting point of set square and plastic strip as '*M*' as shown in Fig. 2.

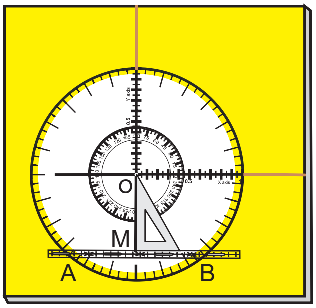


Fig. 2

4. Now measure the lengths of AM and BM of chord AB.
5. Repeat the above activity by forming different chords of different lengths by using plastic strips and complete the following table:

S. No.	**AM**	**BM**	**Is AM = BM?**
1.			
2.			
3.			

Inference : If OM ⊥ AB, then AM = ...

Think and Discuss!

Is AM = BM, when OM is not perpendicular to AB?

Activity 27

LINE THROUGH THE CENTRE OF A CIRCLE BISECTING A CHORD

OBJECTIVE

To verify that a line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.

MATERIAL REQUIRED

Circular board, 1 plastic strip, connectors for circular board, connectors for strips, rubber band, and one half protractor

HOW TO PROCEED?

1. Fix a plastic strip on the circular board at suitable points with the help of connectors to represent chord AB.
2. Fix a connector at the centre *O* of the circle and a connector (for strips) at the mid-point M of the chord. Now tie a rubber band on the two connectors to represent the line drawn through the centre of circle to bisect the chord AB as shown in Fig. 1.

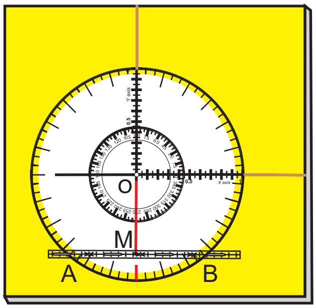


Fig. 1

3. Place a protractor such that its centre coincides with the mid-point M of the chord AB as shown in Fig.2.

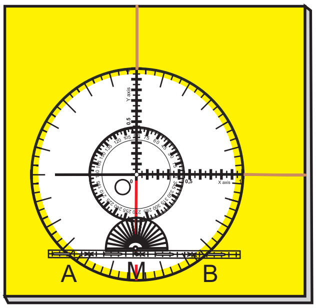


Fig. 2

4. Now measure ∠OMA and ∠OMB.
5. Repeat the above activity by forming chords of different lengths by fixing strips at different positions and complete the following table :

S. No.	∠OMA	∠OMB
1.		
2.		
3.		

Inference : ∠OMA is a .... angle. So, ...

Activity 28

Equal Chords of a Circle

Objective

To verify that equal chords of a circle are equidistant from the centre of the circle.

Material required

Circular board, 2 plastic strips, one set square, and connectors (for circular board).

How to proceed?

1. Take 2 plastic strips and suitably fix them with the help of connectors so that they represent two equal chords AB and CD of the circle on circular board with centre *O* as shown in Fig. 1.

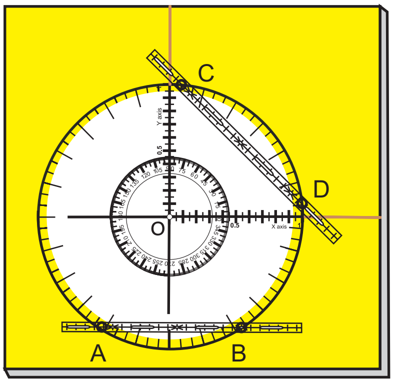


Fig. 1

2. Take a set square and slide it along the strip AB so that the other edge of the set square passes through centre *O* of circle on the circular board and is perpendicular to AB.

Mark the meeting point of set square and plastic strip as M as shown in Fig. 2.

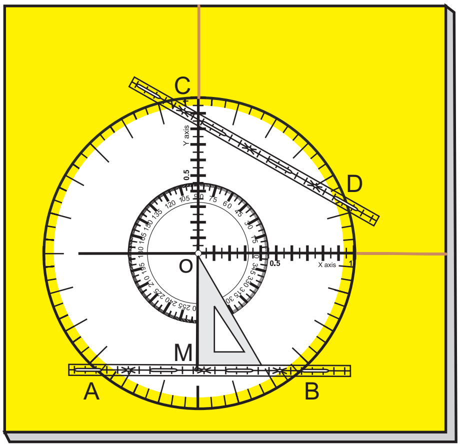


Fig.2

3. Measure the length OM. Now repeat step 2 for chord CD and measure the length of the perpendicular ON from centre *O* to chord CD.

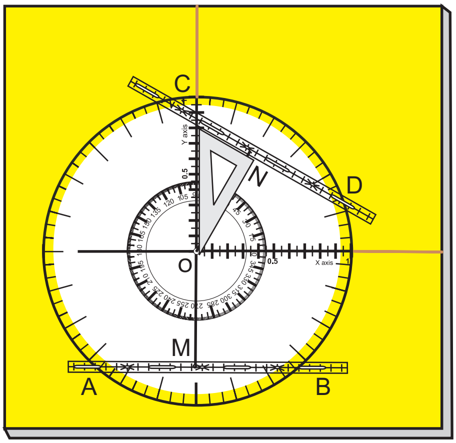


Fig.3

4. Repeat the same activity by varying lengths of equal chords AB and CD by fixing plastic strips at different positions and complete the following table:

S. No.	AB	CD	OM	ON	Is OM = ON?
1.					
2.					
3.					

Inference : Equal chords of a circle are ....

Think and Discuss!

(i) Is OM = ON, if AB ≠ CD ?

(ii) If AB > CD, is OM > ON or OM< ON ?

Activity 29

Chords Equidistant from the Centre of a Circle

Objective

To verify that the chords equidistant from the centre of a circle are equal.

Material required

Circular board, 2 plastic strips, one set square, and connectors for circular board.

How to proceed?

1. Take a plastic strip and fix it at a convenient distance OM (say 5 cm) from centre *O* of circle on the circular board with the help of set square to represent a chord AB as shown in Fig. 1.

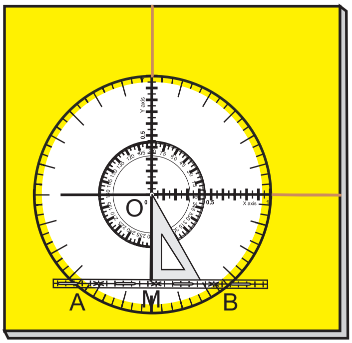


Fig. 1

2. Now again with the help of set square place another plastic strip at the same distance ON (5 cm) to represent another chord CD as shown in Fig. 2.

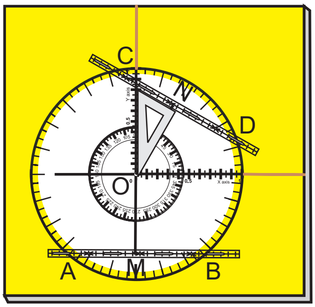


Fig.2

3. Now measure lengths of chords AB and CD using markings on plastic strips.
4. Repeat the above activity by fixing the plastic strips at different positions to represent other pairs of chords which are equidistant from the centre *O* of circle on circular board and complete the following table :

S. No.	OM	ON	AB	CD	Is AB = CD?
1.					
2.					
3.					

Inference : Chords equidistant from the centre of a circle are.........................

Think and Discuss!

(i) Is AB = CD, if ON ≠ OM?
(ii) If ON > OM, is AB > CD or AB < CD ?

Activity 30

Equal Arcs of a Circle

Objective

To verify that equal arcs of a circle subtend equal angles at the centre.

Material required

Circular board, connectors for circular board, rubber bands, and full protractor.

How to proceed?

1. Fix four connectors at suitable positions on the boundary of circle on the circular board to represent two equal arcs AB and CD as shown in Fig.1.

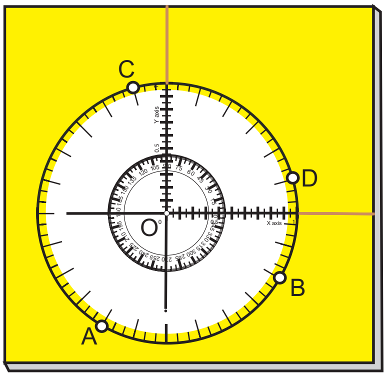


Fig. 1

2. Fix a connector at the centre *O* of the circle on circular board and with the help of 2 different coloured rubber bands make ∠AOB and ∠COD subtended by these two arcs at the centre O as shown in the Fig. 2.

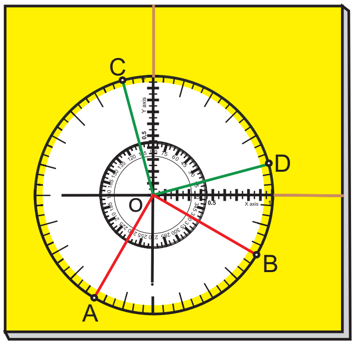


Fig. 2

3. Now measure ∠AOB and ∠COD with the help of degree markings at centre of circle or by fixing a full protractor at the centre of the circle.
4. Repeat the above activity by taking different pairs of equal arcs on the circular board and complete the following table :

S. No.	∠AOB	∠COD	Is ∠AOB = ∠COD?
1.			
2.			
3.			

Inference : Equal arcs of a circle subtend...........................

Think and discuss!

(i) Is ∠AOB = ∠COD when arc AB and arc CD are not equal?
(ii) If arc AB > arc CD, is ∠AOB > ∠COD?

Activity 31

Angle Subtended at the Centre and at any Point on the Circle

Objective

To verify that an angle subtended by an arc of a circle at the centre is double the angle subtended by it on any point on the remaining part of the circle.

Material required

Circular board, connectors for circular board, rubber bands, and one half protractor.

How to proceed?

1. Fix two connectors at two convenient places on the boundary of circle on the circular board to represent an arc AB as shown in Fig. 1.

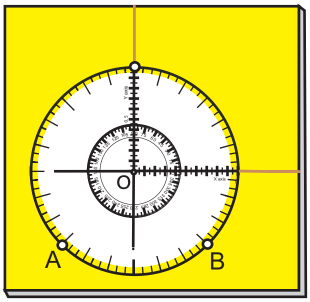


Fig. 1

2. Fix a connector at the centre *O* of circle and make ∠AOB subtended by this arc at the centre by using rubber band.
3. Now fix a connector at any point P on the remaining part of the boundary of circle on the circular board and show

∠APB subtended by the same arc AB at point P with the help of another rubber band as shown in Fig. 2.

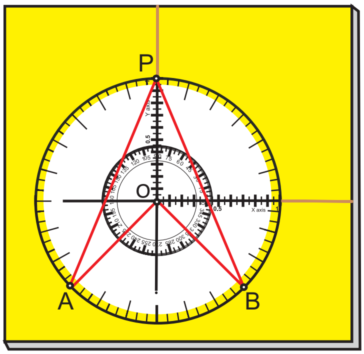


Fig.2

4. Measure ∠AOB with the help of degree marking at the centre of circle on the circular board and measure ∠APB by fixing a half protractor at P as shown in Fig. 3.

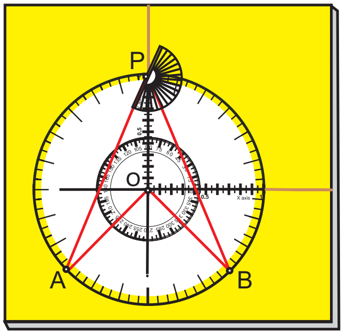


Fig.3

5. Repeat the activity by taking point P at different positions on the boundary of circle on circular board.
6. Also, repeat the same activity by varying length of arc AB with the help of connectors and complete the following table:

S. No.	∠AOB	∠APB	Is ∠AOB = 2∠APB?
1.			
2.			
3.			

Inference: Angle subtended by an arc at the centre is ....

Think and Discuss!

(i) If point P lies outside the circle, is ∠AOB = 2 ∠APB ?

(ii) If point P lies within the circle, is ∠AOB = 2 ∠APB ?

Activity 32

ANGLES IN THE SAME SEGMENT OF A CIRCLE

OBJECTIVE

To verify that the angles in the same segment of a circle are equal.

MATERIAL REQUIRED

Circular board, connectors for circular board, rubber bands, one plastic strip, and one half protractor.

HOW TO PROCEED?

1. Fix a plastic strip with the help of connectors to represent a chord AB of the circle on circular board.
2. Fix two pins at two different points P and Q on the same side of AB on the boundary of circle on the circular board.
3. Use two rubber bands of different colours to represent ∠APB and ∠AQB in the same segment of the circle as shown in Fig.1.

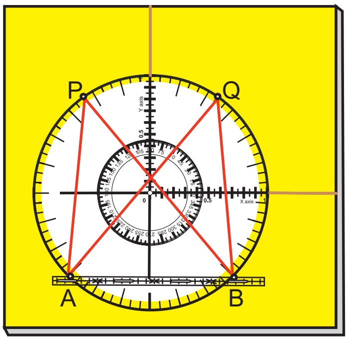


Fig. 1

4. Measure ∠APB and ∠AQB using half protractor.

5. Repeat the activity by making different pairs of angles in the same segment of the circle and complete the following table :

S. No.	$\angle$APB	$\angle$AQB	Is $\angle$APB = $\angle$AQB?
1.			
2.			
3.			

Inference: Angles in the same segment of a circle are ....

Think and Discuss!

Is $\angle APB = \angle AQB$ if one of the points P and Q is not on the boundary of the circular board?

Activity 33

Angle in a Semicircle

Objective

To verify that an angle in a semicircle is a right angle.

Material required

Circular board, connectors for circular board, rubber bands, one plastic strip, and one half protractor.

How to proceed?

1. Fix a plastic strip at a suitable place on the circular board with the help of connectors to represent diameter AB of the circle.
2. Fix a connector at any point P on the boundary of circle on the circular board.
3. Using rubber band make ∠APB in the semicircle as shown in Fig.1.

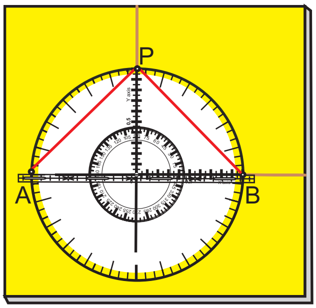


Fig. 1

4. Measure ∠APB using a half protractor.

5. Repeat the activity by taking different positions of point P on the semicircle and complete the following table :

S.No.	∠APB
1.	
2.	
3.	

Inference : Angle in a semicircle is ....

Think and Discuss!

1. *If AB is not a diameter, then what will be the measure of ∠APB?*
2. *What can you say about an angle (i) in a major arc (ii) in a minor arc?*

Activity 34

Pair of Opposite Angles of a Cyclic Quadrilateral

Objective

To verify that the sum of either pair of opposite angles of a cyclic quadrilateral is 180°.

Material required

Circular board, connectors for circular board, rubber bands, and two half protractors.

How to proceed?

1. Fix 4 connectors at suitable points A, B, C and D on the boundary of circle on the circular board and form a cyclic quadrilateral ABCD with the help of the rubber bands as shown in the Fig.1.

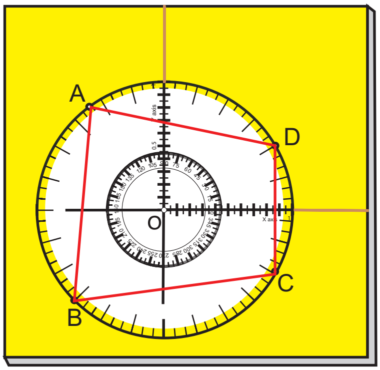


Fig.1

2. Measure ∠ABC and ∠ADC with the help of half protractors and find their sum.

3. Also, measure $\angle$BAD and $\angle$BCD with the help of half protractors and find their sum.
4. Repeat the activity by forming other cyclic quadrilaterals by changing the positions of connectors on the circular board and complete the following table :

S. No.	$\angle$ABC	$\angle$ADC	$\angle$ABC+ $\angle$ADC	$\angle$BCD	$\angle$BAD	$\angle$BCD+$\angle$BAD
1.						
2.						
3.						

Inference : Sum of either pair of opposite angles of a cyclic quadrilateral is ....

Think and Discuss!

If one of the points A, B, C and D is not on the boundary of circle on circular board, will $\angle ABC + \angle ADC = 180°$ or $\angle BAD + \angle BCD = 180°$?

Activity 35

Pair of Opposite Angles of a Non-cyclic Quadrilateral

Objective

To verify that the sum of either pair of opposite angles of a non-cyclic quadrilateral is not equal to180°.

Material required

Circular board, plastic strips, connectors for circular board, connectors for strip, rubber bands, and two half protractors.

How to proceed?

1. Fix 4 connectors at points A, B, C and D on the circular board such that 3 connectors are on the boundary of circle and the fourth connector is outside the boundary of circle.
2. Form a quadrilateral ABCD, using rubber band as shown in Fig. 1.

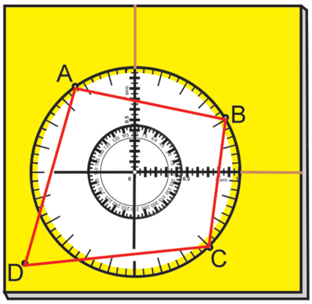


Fig. 1

3. Measure ∠ADC and ∠ABC with the help of half protractors and find their sum.

4. Similarly, measure ∠DAB and ∠DCB with the help of half protractors and find their sum. Repeat the activity by taking different positions of points and complete the following table :

S. No.	∠ADC	∠ABC	∠ADC+∠ABC	∠DAB	∠DCB	∠DAB+∠DCB
1.						
2.						
3.						

Inference: Is ∠ADC + ∠ABC = 180°?
Is ∠DAB + ∠DCB = 180°?

5. Now again fix four connectors on the circular board such that 3 connectors are on the boundary and the fourth connector is inside the boundary as shown in Fig.2.

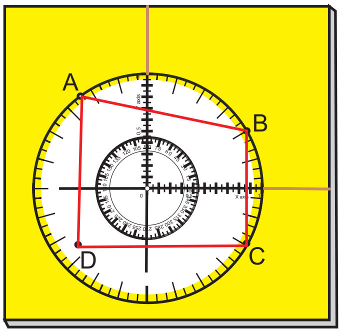


Fig.2

6. Measure ∠ADC and ∠ABC using half protractors and find their sum.
7. Similarly, measure ∠DAB and ∠DCB by half protractors and find their sum. Repeat the activity by varying positions of points and complete the following table:

S. No.	∠ADC	∠ABC	∠ADC+∠ABC	∠DAB	∠DCB	∠DAB+∠DCB
1.						
2.						
3.						

Inference : The sum of either pair of opposite angles of a non cyclic quadrilateral is ....

Think and Discuss!

In Fig.1 and Fig.2, try to move point D towards the boundary of the circular board and find the sum of each pair of opposite angles accordingly.

Is the sum coming closer and closer to 180°?

Activity 36

Tangent at a Point on a Circle

Objective

To verify that a tangent at any point of circle is perpendicular to the radius through the point of contact.

Material required

Circular board, 1 plastic strip (B-type), one half protractor, rubber band, and connector for circular board.

How to proceed?

1. Fix a plastic strip at a point say P on the boundary of circle on the circular board such that the strip touches the circle representing a tangent APB .
2. Fix a connector at the centre '*O*' of circular board and join it with point P using a rubber band representing the radius OP of the circle as shown in the Fig. 1.

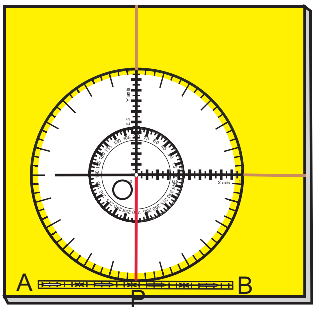


Fig. 1

3. Measure ∠OPA and ∠OPB with the help of half protractor as shown in the Fig. 2.

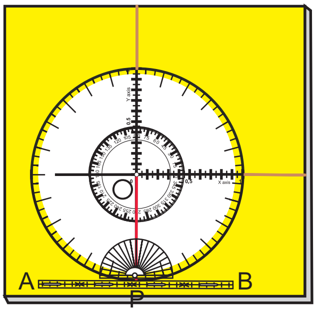


Fig.2

4. Repeat the activity by placing the strip at different points on the boundary of circle on the circular board and complete the following table :

S. No.	∠OPA	∠OPB
1.		
2.		
3.		

Inference : The tangent at any point of a circle is...............

Think and Discuss!

If Q is any point other than P on the strip. Is OQ > OP or OQ = OP or OQ < OP?

Activity 37

TANGENTS TO A CIRCLE FROM AN EXTERNAL POINT

OBJECTIVE

To verify that the lengths of two tangents drawn from an external point to a circle are equal.

MATERIAL REQUIRED

Circular board, 2 plastic strips (B-type), and connectors for circular board.

HOW TO PROCEED?

1. Choose a convenient point P outside the boundary of circle on the circular board and fix 2 plastic strips at that point.
2. Now adjust the two strips in such a way that the two strips touch the boundary of circle on the circular board at two different points say 'A' and 'B' as shown in Fig. 1.

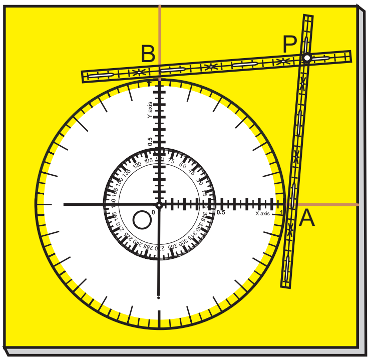


Fig. 1

3. Measure the lengths of PA and PB using markings on the

plastic strips.

4. Repeat the activity by fixing the two strips at different positions outside the boundary of circle on the circular board and complete the following table:

S.No.	PA	PB	Is PA = PB?
1.			
2.			
3.			

Inference : Two tangents drawn from an external point to a circle are ....

Think and Discuss!

What happens when P lies on the circle?

Activity 38

TRIGONOMETRIC RATIOS-I

OBJECTIVE

To understand the meaning of different trigonometric ratios using a circular board.

MATERIAL REQUIRED

Circular board, wire needle, set square, plastic strips (B type), and connectors for circular board.

HOW TO PROCEED?

a) sinθ and cosθ

1. Fix the wire needle at the centre O of the circular board to make an angle POX = θ with the horizontal line marked on the circular board as shown in Fig. 1.

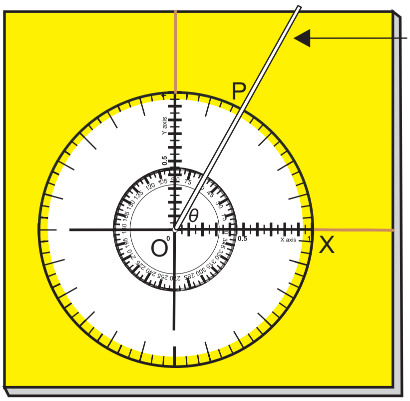


Fig. 1

2. Place the set square such that one edge of the set square (other than the hypotenuse) coincides with the horizontal line.

3. Now slide the set square along the horizontal line so that other edge of the set square forming the right angle passes through the point P (if necessary you may remove the needle temporarily to get the exact perpendicular) as shown in Fig. 2.

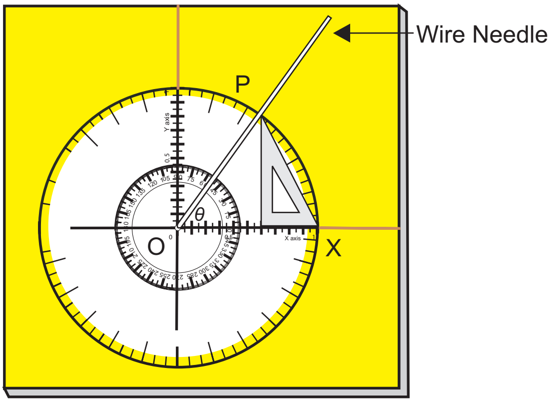


Fig.2

4. Place a plastic strip (B type) along this edge of the set square to represent a perpendicular PM on horizontal line OX as shown in Fig. 3.

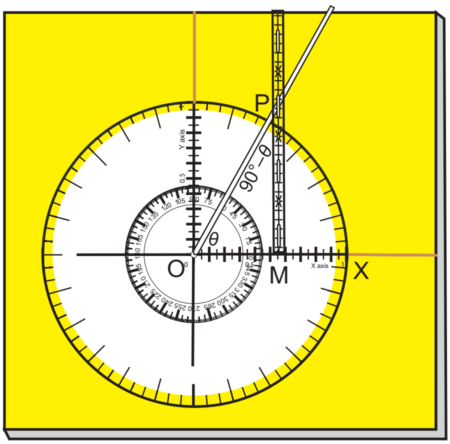


Fig.3

5. Now measure lengths of PM and OM.

6. $\sin q = \dfrac{Perpendicular}{Hypotenuse} = \dfrac{PM}{OP} = \dfrac{PM}{1} = PM$

   and $\cos q = \dfrac{Base}{Hypotenuse} = \dfrac{OM}{OP} = \dfrac{OM}{1} = OM$

7. Repeat the activity by making different angles by varying the position of the needle and complete the following table:

S. No.	θ	**PM**	**OM**	**sin**θ	**cos**θ
1.					
2.					
3.					

sine and cosine of complementary angles

8. In Fig. 3 above, $\angle OPM = 90° - \theta$

$$\sin(90° - \theta) = \frac{Perpendicular}{Hypotenuse} = \frac{OM}{OP} = \frac{OM}{1} = OM = \cos\theta$$

$$\cos(90° - \theta) = \frac{Base}{Hypotenuse} = \frac{PM}{OP} = \frac{PM}{1} = PM = \sin\theta$$

   Thus, we have:
   $\sin(90° - \theta) = \cos\theta$ and $\cos(90° - \theta) = \sin\theta$

b) tan θ**, sec** θ**, cot** θ **and cosec** θ

1. Fix the wire needle at the centre O of circle on the circular board to make an angle POX = θ, with the horizontal line as shown in Fig. 1.
2. Fix a plastic strip tangential to the circular board at point P and place another plastic strip along the horizontal line OX to meet first strip at point A as shown in Fig. 4.

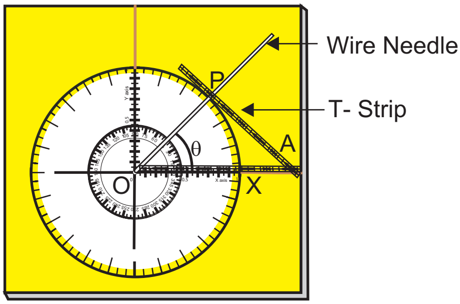


Fig.4

3. Place another plastic strip along the vertical line passing through centre O to meet the first strip at point B as shown in Fig.5.

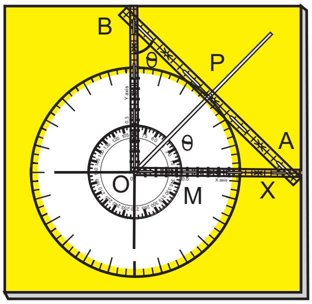


Fig.5

4. From right triangle POA,

$$\tan\theta = \frac{Perpendicular}{Base} = \frac{AP}{OP} = \frac{AP}{1} = AP$$

$$\text{and } \sec\theta = \frac{Hypotenuse}{Base} = \frac{OA}{OP} = \frac{OA}{1} = OA$$

5. From right ΔPOB, $\angle PBO = \theta$,

$$\cot\theta = \frac{Base}{Perpendicular} = \frac{BP}{OP} = \frac{BP}{1} = BP$$

$$\text{and } \operatorname{cosec}\theta = \frac{Hypotenuse}{Perpendicular} = \frac{OB}{OP} = \frac{OB}{1} = OB$$

6. Repeat the activity by making different angles by varying the position of needle and complete the following table:

S. No.	θ	tan θ	sec θ	cot θ	cosec θ
1.					
2.					
3.					

tan, sec, cot and cosec of complementary angles.

7. From triangle OAP given in Fig 5, ∠OAP = 90° – θ

$$\cot(90-\theta) = \frac{Base}{Perpendicular} = \frac{AP}{OP} = \frac{AP}{1} = AP = \tan\theta$$

$$\text{cosec}(90-\theta) = \frac{Hypotenuse}{Perpendicular} = \frac{OA}{OP} = \frac{OA}{1} = OA = \sec\theta$$

Now, from Δ POB given in Fig 5, ∠BOP = $(90^\circ - \theta)$

$$\tan(90-\theta) = \frac{Perpendicular}{Base} = \frac{BP}{OP} = \frac{BP}{1} = BP = \cot\theta$$

$$\sec(90-\theta) = \frac{Hypotenuse}{Base} = \frac{OB}{OP} = \frac{OB}{1} = OB = \text{cosec}\,\theta$$

Thus, we have: $\cot(90^\circ - \theta) = \tan\theta$

$\text{cosec}\,(90^\circ - \theta) = \sec\theta$

$\tan(90^\circ - \theta) = \cot\theta$

$\sec(90^\circ - \theta) = \text{cosec}\,\theta$

Think and Discuss!

Can the trigonometric ratios $\tan\theta$, $\cot\theta$, $\sec\theta$ and $\text{cosec}\,\theta$ be obtained from Fig. 3? Why are we using Fig.5?

Activity 39

Trigonometric Ratios-II

Objective

To find the trigonometric ratios of some special angles such as 0°, 30°, 45°, 60° and 90°.

Material required

Circular board, plastic strips (B type), connectors for circular board, wire needle, and set square.

How to proceed?

For sine and cosine of 30°, 45° and 60°

1. Fix a wire needle at the centre O of circle on the circular board such that it makes an angle ∠POX = 30° with the horizontal line OX as shown in Fig. 1.

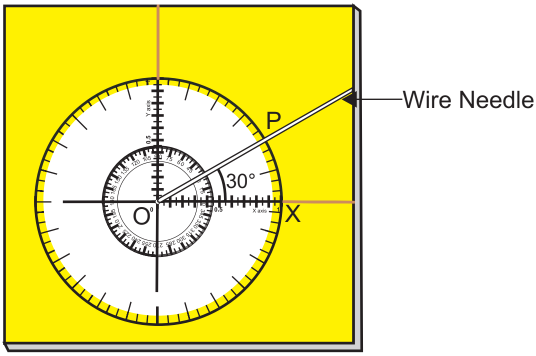


Fig. 1

2. Now place a plastic strip passing through point P to represent a perpendicular PM on horizontal line OX using a set square (as done earlier in Activity No. 38) as shown in Fig. 2.

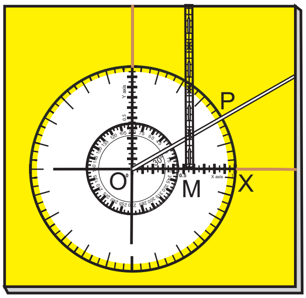


Fig.2

3. Now measure lengths of PM and OM.
4. $\sin 30° = \frac{PM}{OP} = \frac{PM}{1} = PM = 0.5$

   $\cos 30° = \frac{OM}{OP} = \frac{OM}{1} = OM = 0.9$ (approx).
5. The same activity can be repeated by taking ∠POX as 45° and 60°.

6.1 For sin 0° and cos0°

(i) Fix the wire needle to make any angle say ∠POX = θ.

(ii) Fix a plastic strip to represent perpendicular PM on horizontal line OX using set square as explained earlier.

(iii) Now rotate the wire needle in clockwise direction and accordingly change the position of the strip representing perpendicular PM.

(iv) When the wire needle coincides with OX, i.e., when θ = 0°, point P and M coincides.Then OM = 1 unit and length PM becomes 0 as shown in Fig. 3.

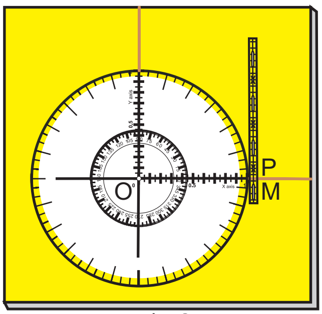


Fig.3

So, sin0°= PM = 0 and cos0°= OM = 1.

6.2 For sin90° and cos90°

(i) Fix a plastic strip to represent perpendicular PM on horizontal line OX as explained earlier in 6.1.

(ii) Now rotate the wire needle in anticlockwise direction and accordingly change the position of strip representing PM.

(iii) When the wire needle coincides with vertical line OY, i.e., when $\theta = 90°$, then point O and M coincides. Then PM=1 and OM= 0 as shown in Fig. 4.

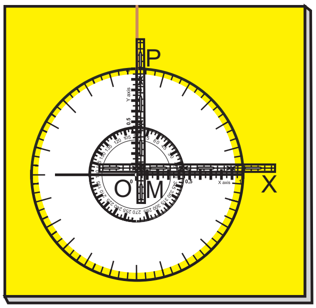


Fig.4

So, sin90° = PM = 1 and cos 90° = OM = 0.

6.3 For tan, sec, cot and cosec of 30°, 45° and 60°

(i) Fix the wire needle at the centre O of circle on the circular board to make an ∠POX = 30° with horizontal line OX (as explained earlier).

(ii) Fix a plastic strip tangential to the circular board at point P and place another strip along OX to meet the first strip at point A as shown in Fig. 5.

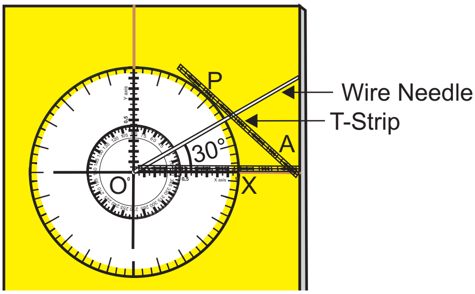


Fig.5

(iii) Place another plastic strip along the vertical line passing through the centre O to meet the first strip at point B as shown in Fig.6.

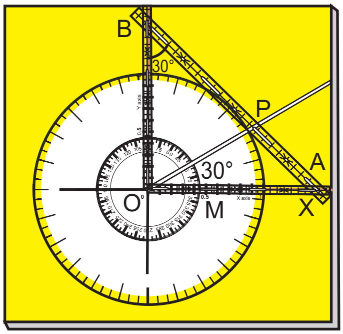


Fig.6

(iv) Now measure AP, OA, PB and OB.

(v) $\tan 30° = \frac{AP}{OP} = \frac{AP}{1} = AP$

$\sec 30° = \frac{OA}{OP} = \frac{OA}{1} = OA$

$\cot 30° = \frac{BP}{OP} = \frac{BP}{1} = BP$

$\text{cosec } 30° = \frac{OB}{OP} = \frac{OB}{1} = OB$

(vi) Same activity can be repeated by taking ∠POX as 45° and 60°.

7.1 For tan, sec, cot and cosec of 0°

(i) Fix the wire needle to make any angle say

$\angle POX = \theta$.

(ii) Fix a plastic strip to represent perpendicular PM on horizontal line OX using set square as explained earlier. Rotate the wire needle in clockwise direction and accordingly change the position of plastic strip, such that when wire needle coincides with OX, i.e., when $\theta = 0°$, AP becomes 0 as P and A coincide as shown in Fig. 7.

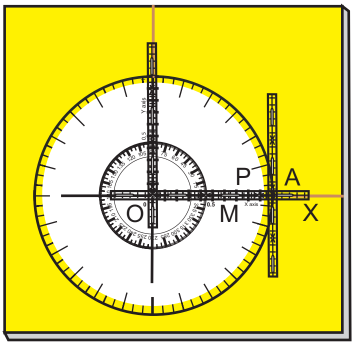


Fig. 7

(iii) Now, $\tan 0° = \frac{AP}{1} = \frac{0}{1} = 0$.

(iv) Also, cot 0° = PB where B is the point of intersection of the 2 plastic strips. Since the two strips are parallel, so PB is not defined. Thus, cot 0° is not defined.

(v) cosec 0° = OB where B is the point of intersection of the two plastic strips. Since the two strips are parallel so, OB is not defined. Thus, cosec 0° is not defined.

7.2 For tan, sec, cot and cosec of 90°

(i) Arrange the strips as explained earlier in subsection 7.1. Now rotate the wire needle anticlockwise and accordingly change the position of plastic strip such that when needle coincides with vertical line OY, i.e., when $\theta = 90°$, PB is nearly 0 and points P and B coincide as shown in Fig. 8.

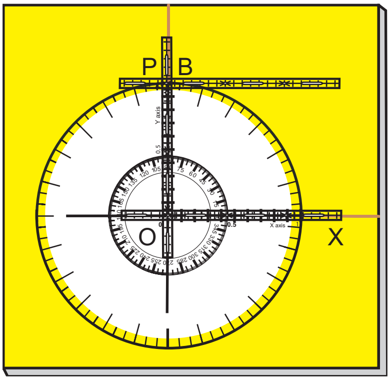


Fig.8

(ii) Now tan 90° = $\frac{AP}{1}$, where A is the intersection point of the two plastic strips. Since two strips are parallel, so AP is not defined. Thus, tan 90° is not defined.

(iii) sec 90° = OA where A is the point of intersection of the two strips. But the two strips are parallel. So, OA is not defined. Thus, sec 90° is not defined.

(iv) Also, cot 90° = $\frac{PB}{1} = \frac{0}{1} = 0$,

and cosec 90° = $\frac{OB}{1} = \frac{1}{1} = 1$.

Think and Discuss!

(i) Is sin30° = cos 60°?
(ii) Is tan0° = cot 90°?
(iii) Is sin45° = cos 45°?
(iv) Is sin30° = cosec 30°?
(v) Is cos60° = sec 60°?

Activity 40

Trigonometric Ratios and Sides of Right Triangle

Objective

To verify that the values of trigonometric ratios of an angle do not vary with the lengths of the sides of the triangle.

Material required

Geoboard, rubber bands, ruler, and geoboard pins.

How to proceed?

1. Fix 5 geoboard pins on the geoboard at suitable points A, B, C, D and E and join them with 2 rubber bands of different colours to represent two similar right triangles as shown in the Fig.1.

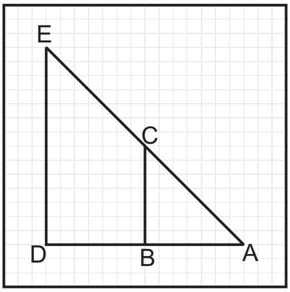


Fig. 1

2. Measure lengths of AB, BC, AD, DE, AC and AE using a ruler.
3. Find $\frac{BC}{AC}, \frac{AB}{AC}, \frac{BC}{AB}, \frac{DE}{AE}, \frac{AD}{AE}, \frac{DE}{AD}$

4. Repeat the activity by making other pairs of similar right triangles and complete the following table:

S. No.	AB	BC	AC	$\frac{BC}{AC}$	$\frac{AB}{AC}$	$\frac{BC}{AB}$	AD	DE	AE	$\frac{DE}{AE}$	$\frac{AD}{AE}$	$\frac{DE}{AD}$
1.												
2.												
3.												

Inference: (i) Since $\frac{BC}{AC} = \frac{DE}{AE}$,

so, sin A does not vary with the change of the lengths of the sides of the triangle.

(ii) $\frac{AB}{AC}$ = ... , So, cos A = ...

(iii) $\frac{BC}{AB}$ = ... , So, tan A = ...

Activity 41

TRIGONOMETRIC IDENTITIES

OBJECTIVE

To verify standard trigonometric identities.

MATERIAL REQUIRED

Circular board, wire needle, set square, T-strips (B-type), and connectors (for circular board).

HOW TO PROCEED?

(A) For identity $\sin^2\theta + \cos^2\theta = 1$

1. Fix the wire needle at the centre *O* of the circular board to make an angle POX as θ with the horizontal line as shown in Fig. 1.

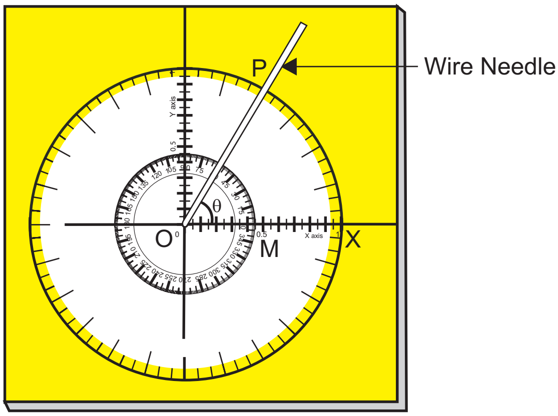


Fig. 1

2. Place a T-strip to represent a perpendicular PM on OX using set square (as done in earlier activity) as shown in Fig. 2.

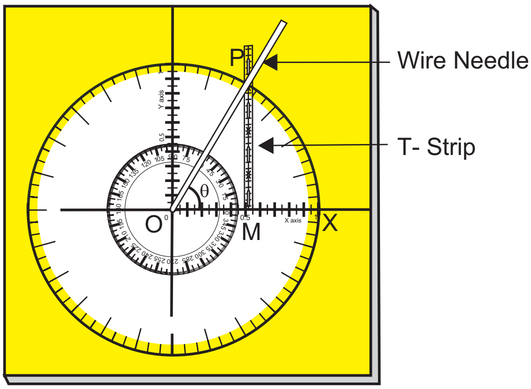


Fig.2

3. Now $\sin\theta = \dfrac{Perpendicular}{Hypotenuse} = \dfrac{PM}{1} = PM$

   and $\cos\theta = \dfrac{Base}{Hypotenuse} = \dfrac{OM}{1} = OM$

   Now, $\sin^2\theta + \cos^2\theta = (PM)^2 + (OM)^2$

   $= (OP)^2$ (Pythagoras theorem)

   or $\sin^2\theta + \cos^2\theta = (1)^2 = 1.$

(B) For identities $1 + \tan^2\theta = \sec^2\theta$ and $1 + \cot^2\theta = \text{cosec}^2\theta$

1. Fix the wire needle at the centre *O* of the circular board to make an angle POX as θ with horizontal line OX as shown in Fig. 1 (done earlier).
2. Fix a T-strip tangential to circular board at point P and place another T-strip along horizontal line OX to meet first strip at point A as done earlier.
3. Now place another T-strip along the vertical line, passing through centre *O* to meet the T-strip at point B as shown in the Fig. 3.

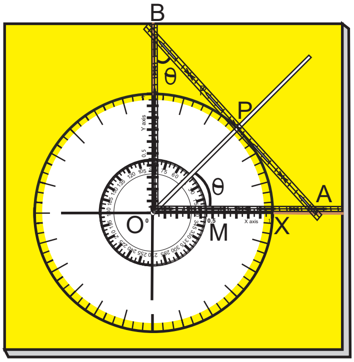


Fig.3

4. From right ΔPOA,

$$\tan\theta = \frac{Perpendicular}{Base} = \frac{AP}{1} = AP$$

and $\sec\theta = \frac{Hypotenuse}{Base} = \frac{OA}{1} = OA$

So, $1 + \tan^2\theta = 1 + (AP)^2$

$= (OP)^2 + (AP)^2$

$= (OA)^2$ (Pythagoras theorem)

$= \sec^2\theta$

5. From right ΔPOB,

$$\cot\theta = \frac{Base}{Perpendicular} = \frac{BP}{1} = BP$$

and $\operatorname{cosec}\theta = \frac{Hypotenuse}{Perpendicular} = \frac{OB}{1} = OB$

So, $1 + \cot^2\theta = 1 + (BP)^2$

$= (OP)^2 + (BP)^2$

$= (OB)^2$ (Pythagoras theorem)

$= \operatorname{cosec}^2\theta$

Activity 42

Surface Area and Volume of Solids

Objective

(i) To understand the concepts of surface area and volume of solids.

(ii) To verify the fact that increase/decrease in the volume of a solid may not result the same change in its surface area.

Material required

One solid wooden cube, cut outs of a cuboid, right circular cylinder, right circular cone, and hemisphere embedded in cube.

How to proceed?

1. Take solid wooden cube (say of side a) along with all the cut-outs fitted in it as shown in Fig.1.

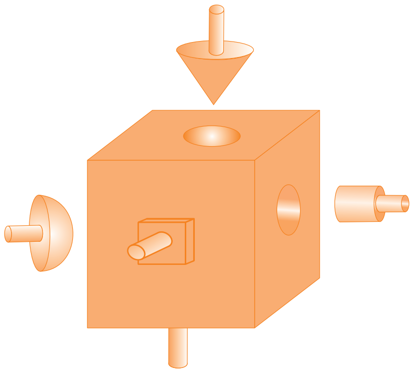


Fig.1

2. Take out the cut-out of cuboid say of dimensions l, b and h from cube as shown in Fig. 2.

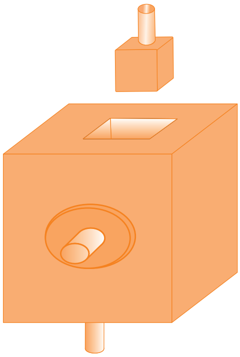


Fig.2

3. Find the volume and surface area of the cut-out of cuboid.
4. Find the volume and surface area of the remaining solid i.e., solid left after taking out cuboid from the given cube.
5. Take different values of a, l, b and h and find the change in the volume and surface area of the original cube and the left out solid. Now complete the following table:

S. No.	**Cube**		**Cuboid**		**Left Out Solid**	
	Volume a^3	TSA $6a^2$	Volume lbh	TSA 2(lb+bh+lh)	Volume a^3-lbh	TSA $6a^2+2h(l+b)$
1.						
2.						
3.						

6. Repeat the same activity by taking out cut-outs of a cylinder, a cone and a hemisphere as shown in following figures:

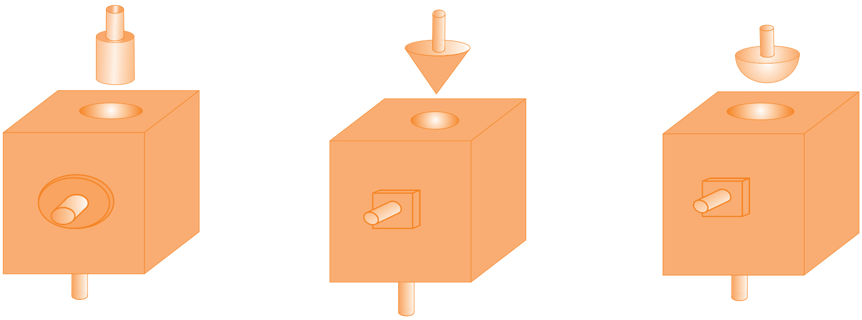


Fig. 3

Cylinder

Let r be the base radius and h be the height of the cylinder.

Take different values of r and h to complete the following table :

S. No.	Cube		Cylinder		Left Out Solid	
	Volume a^3	TSA $6a^2$	Volume $\pi r^2 h$	TSA $2\pi r(r+h)$	Volume $a^3 - \pi r^2 h$	TSA $6a^2 + 2\pi rh$
1.						
2.						
3.						

Cone

Let r be the base radius, h be the height and l be the slant height of given cut-out of cone. Take different values of r, l and h to complete the following table:

S. No.	Cube		Cone		Left Out Solid	
	Volume a^3	TSA $6a^2$	Volume $\frac{1}{3}\pi r^2 h$	TSA $\pi r(l+r)$	Volume $a^3 - \frac{1}{3}\pi r^2 h$	TSA $6a^2 + \pi r(l-r)$
1.						
2.						
3.						

Hemisphere

Let r be the radius of given cut-out of hemisphere. Take different values of r to complete the following table:

S. No.	Cube		Hemisphere		Left Out Solid	
	Volume a^3	TSA $6a^2$	Volume $\frac{2}{3}\pi r^3$	TSA $3\pi r^2$	Volume $a^3 - \frac{2}{3}\pi r^3$	TSA $6a^2 + \pi r^2$
1.						
2.						
3.						

Inference : (i) Volume of left out solid decreases by the amount of volume of the cut out solid.

(ii) Surface area of the left out solid increases in each case as compared to the original surface area of the solid.

(iii) From the above observations, we conclude that decrease (increase) in the volume of a solid need not result in decrease (increase) in the surface area of the resulting solid.

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