CUSTOMISED TEACHER'S TRAINING PACKAGE FOR KGBV TEACHERS

MATHEMATICS

TEACHING OF ALGEBRA, RATIO AND PROPORTION, MENSURATION, DATA HANDLING AND ANNEXURE

BOOK 3

विद्यया ऽमृतमश्नुते
एन सी ई आर टी NCERT
Department of Women's Studies
राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

ISBN 978-93-5007-267-7

First Edition
September 2014 Bhadrapada 1936

PD 1T MJ

₹ ??.00

Printed on 80 GSM paper with NCERT watermark

Published at the Publication Division by the Secretary, National Council of Educational Research and Training, Sri Aurobindo Marg, New Delhi 110 016 and printed at .....
.....

OFFICES OF THE PUBLICATION DIVISION, NCERT

NCERT Campus
Sri Aurobindo Marg
New Delhi 110 016 **Phone : 011-26562708**

108, 100 Feet Road
Hosdakere Halli Extension
Banashankari III Stage
Bangaluru 560 085 **Phone : 080-26725740**

Navjivan Trust Building
P.O. Navjivan
Ahmedabad 380 014 **Phone : 079-27541446**

CWC Campus
Opp. Dhankal Bus Stop
Panihati
Kolkata 700 114 **Phone : 033-25530454**

CWC Complex
Maligaon
Guwahati 781 021 **Phone : 0361-2674869**

Publication Team

Head, Publication Division : *N.K. Gupta*

Chief Production Officer : *Kalyan Banerjee*

Chief Editor : *Shveta Uppal*

Chief Business Manager : *Gautam Ganguly*

Editorial Assistant : *Mathew John*

Assistant Production : .....

Cover and Layout
Digital Expressions

Foreword

National Curriculum Framework–2005 states that a critical function of education for equality is to enable all learners to claim their rights as well as to contribute to society and the polity. We need to recognise that rights and choices in themselves cannot be exercised until central human capabilities are fulfilled. Thus, in order to make it possible for all learners from different socio-economic backgrounds, especially girls, to claim their rights as well as play an active role in shaping collective life, education must empower them to overcome the disadvantages of unequal socialisation and enable them to develop their capabilities of becoming equal citizens.

Reaching out to the girl child has been central to the efforts of Universalising Elementary Education (UEE). The Sarva Shiksha Abhiyan (SSA), a national flagship programme for UEE, recognises the need for special efforts to bring girls, especially from disadvantaged groups, to schools, and to bridge gender disparities in education at the elementary level. In this regard the Ministry of Human Resource Development instituted the Kasturba Gandhi Balika Vidyalaya (KGBV) scheme, an innovative and promising initiative that attempts to address the social, cultural and economic deprivation faced by girls from deprived and disadvantaged sections of rural society. Introduced as a scheme in 2004, it became a part of SSA in 2007. Currently it is operational in twenty-four states and one Union Territory.

A National Consultation on KGBV was organised by NCERT from 11-12 August 2008 to share experiences generated by the KGBV scheme over the last few years. This consultation brought together scholars in the field. The consultation strongly recommended development of Bridge Course for girls entering in KGBV and Customised Teacher Training Package for upgrading the skills of KGBV teachers. Under this backdrop Department of Women's Studies took initiative for developing Bridge Course and Teacher Training Package based on NCF-2005, in collaboration with other Curricular Departments of NIE, RIEs, University Departments, DIETs of Delhi, NGOs and practising school teachers including teachers of KGBV. This

material has been developed in the content areas of science, maths, history, geography, social and political life, languages—English and Hindi—and art education and is based on NCERT textbooks at elementary level.

The success of this effort largely depends on the multiple contextual steps that KGBV school principals and teachers would undertake to encourage girls to reflect on their own learning and to pursue imaginative activities and questions. We must recognise that, given space, time and freedom, girls generate new knowledge by engaging with the information passed onto them by adults. Treating the prescribed textbooks as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. Inculcating creativity and initiative is possible if we perceive and treat girls as participants in learning and not as mere receivers of a fixed body of knowledge. The teacher should encourage girls to build on their own acquired and perceived knowledge and link it with their lived realities.

The present training material attempts to upgrade the skills of teachers during their in-service training in their subject areas of science, maths, history, geography, social and political life, languages—English and Hindi—and art education. Different participatory pedagogical methods have been adopted in all the subject areas to encourage activity based teaching and learning. This material developed by the NCERT can be treated as an initial material that can be supplemented later. It is not exhaustive in nature and it can be adopted or adapted according to the contextual needs of KGBV teachers.

The Department of Women's Studies (DWS) could not have gone ahead with this endeavour without the direction and guidance of Professor Krishna Kumar, former Director, NCERT. He had rightly envisioned the importance of the present Teacher Training Package in meeting the academic challenges of teachers of KGBV scheme.

We also gratefully acknowledge contributions of the Review Committee chaired by Dr. Sharda Jain, Director, Sandhan, Jaipur; and other members – Sister Sabina, former State Project Director, Mahila Samakhya Society, Patna, Bihar; Ms. Seema Bhaskaran, State Project Director, Mahila Samakhya Society, Kerala; Ms. Amukta Mahapatra, Director, School Scape, Chennai for their expert reviews and suggestions. We are thankful to the members of Evaluation Team constituted by MHRD – Ms. Sarita

Mittal, Director EE8; Ms. Kiran Dogra, Consultant Gender, Ed.CIL; and Ms. Dipta Bhog, Director, Nirantar for their inputs and suggestions.

As an organisation committed to systemic reform and continuous improvement in the quality of its products, NCERT welcomes comments and suggestions which will enable us to undertake further revision and refinement.

Director
National Council of Educational
Research and Training

New Delhi
September, 2011

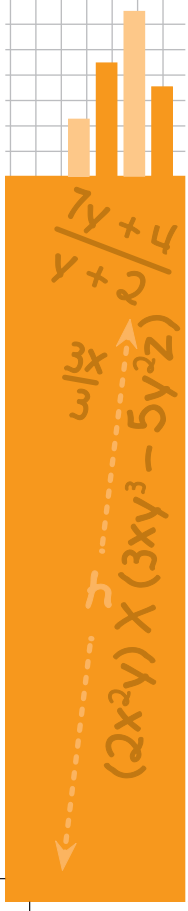
Gandhiji's Talisman

I will give you a talisman. Whenever you are in doubt or when the self becomes too much with you, apply the following test:

Recall the face of the poorest and the weakest man whom you may have seen and ask yourself if the step you contemplate is going to be of any use to him. Will he gain anything by it? Will it restore him to a control over his own life and destiny? In other words, will it lead to Swaraj for the hungry and spiritually starving millions?

Then you will find your doubts and your self melting away.

M.K. Gandhi



Preface

The training materials for the Kasturba Gandhi Balika Vidyalaya (KGBV) teachers have been developed keeping in view the principles of the National Curriculum Framework–2005 of the NCERT. These materials developed in different subject areas, viz. English, Hindi, History, Geography, Social and Political Life, Arts Education, Science, and Mathematics are based on the NCERT upper primary textbooks. All these areas will contribute to the upgradation of professional skills of the KGBV teachers. These materials provide ample avenue to the KGBV teachers for their growth in pedagogy, methodology and approach in dealing with their subject areas. There is a considerable scope for exploration and creativity in the classroom. The use of bilingual technique in English will take teachers ahead in their thinking skills. The flexibility in the approach and suggested activities such as taking the help of worksheets, teacher demonstration, anecdotes, reciting poems, crossword puzzles, experimenting, hands on skills, oral traditions and reading material across various subjects are the highlights of the manual.

Each subject area has picked up key concepts across the upper primary textbooks. Each concept has been dealt through a different kind of activity without bringing any definition and the content for rote learning. The concept or the idea has been floated through activities for the learners to catch and analyse. It is hoped that this material will be of use as a resource and also as reference material. The activities are suggestive. Any alternate activity can also be carried out based on the local-specific contexts. Each activity has the scope of creating similar other local-specific activities not making it necessary to stick to the materials given in this package. Its scope will get enhanced if this creates a space for more such activities.

The motivating material on Legendary Women of Science makes the training package of Science even more interesting and gives an edge for making it very gender sensitive. The women of India and the world, who have achieved heights in this area, will always encourage girls of the KGBVs to even explore these areas which have a masculine image. Kalpana Chawla will motivate

girls to reach the heights of space while Florence Nightingale, The Lady of Lamp, will encourage them to think in numbers. Marie Curie, the only woman to receive the Nobel Prize twice, will make them feel proud for being the women themselves. The KGBV teachers thus oriented to take up such challenges will certainly become guides and agents of social change.

In mathematics there has been a conscious effort of demystifying the masculine image of mathematics. The processes underlying everyday mathematics done by women both within the home and outside have been highlighted. In several concepts there is emphasis on the use of mathematic kit by teachers to make the learning of mathematics more concrete and useful for girls.

Women have always been the backbone of several historical and contemporary movements. Recognising their important role in social reform and national movement the teacher training package in history highlights the contributions made by women like Rani Gaidinliu, Pandita Ramabai, Sarojini Naidu, Aruna Asif Ali and many more. Their trials and successes will continue to inspire girls to meet multifaceted challenges in life.

Similarly the motivating material on legendary women like Rani Jhansi and Ila Sachani has been included in languages. The training package for social and political life attempts to make teachers sensitive towards unconventional roles and responsibilities. Examples like Laxmi Lakra and Fatima Bi will touch hearts of common people. Geography equips teachers in spatial phenomenon. It instills human values and appreciations for regional inter dependence and resource conservation. Arts and aesthetics attempt to inspire diversity in expression of art. It is through performing art that all girls through various art forms can become communicative, creative and expressive.

Overall, keeping these variations in mind, the pedagogical approaches needed in the KGBVs will be multilevel and diverse for meeting the needs of KGBVs in different socio-cultural contexts.

New Delhi
September, 2011

Gouri Srivastava
Chief Coordinator
Professor and *Head*
Department of Women's Studies
NCERT, New Delhi

Development Team

Members

Anup Rajput, *Associate Professor*, NCERT, New Delhi

Ashutosh K. Wazalwar, *Professor*, NCERT, New Delhi

G.P. Dikshit, *Professor* (Retd.), Lucknow University

Hukum Singh, *Professor*, NCERT, New Delhi

Mahendra Shankar, *Lecturer* (SG) (Retd.), NCERT, New Delhi

P.N. Yadav, *Lecturer*, DIET, Kushinagar,

Ram Avtar, *Professor* (Retd.), NCERT, New Delhi

Roohi Fatima, *Assistant Professor*, Jamia Millia Islamia, New Delhi

S.K.S. Gautam, *Professor* (Retd.), NCERT, New Delhi

Academic Coordinator

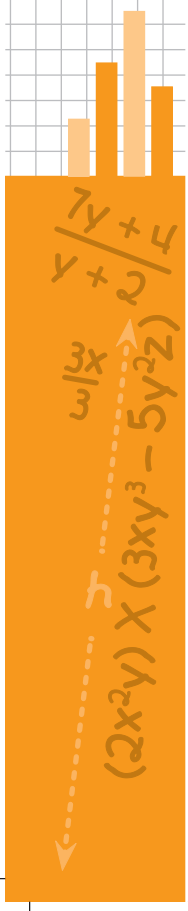
P. K. Chaurasia, *Associate Professor*, RIE, Ajmer

Coordinator

Mona Yadav, *Associate Professor*, DGS, NCERT, New Delhi

Acknowledgement

The National Council of Educational Research and Training gratefully acknowledges and appreciates the hard work done by the development team responsible for this book. Special thanks to Professor Gouri Srivastava, *Head*, Department of Women's Studies, NCERT, the chief coordinator of this book for her constant support and guidance at every step. We are grateful to Professor Sushma Jaireth, Department of Women's Studies, NCERT for providing us the motivating material on women mathematician to be included in the book. We are thankful to the following participants for the review of this training package: Jyoti Tyagi, *TGT* Mathematics, G.S. Joshi, *Lecturer*, DIET Almorah, Uttarakhand; Babulal Mali, *TGT* Mathematics, DM School RIE, Ajmer; Rahul Kumar, *TGT* Mathematics, Sanatan Dharma Inter College, Muzaffernagar; T.P. Sarma, *Assistant Professor*, NCERT, New Delhi; V.P. Singh, *Associate Professor*, NCERT, New Delhi; Nidhika Malhotra, ITL Public School, Dwarka, New Delhi; Rajeev Kumar, *TGT* Mathematics, Kendriya Vidyalaya, JNU, New Delhi; Sulkeha Ram, *Lecturer*, BD Institute of Technology, Sultanpur, New Delhi and Reemu Verma, *TGT* Mathematics, KV Masjid Moth, New Delhi. We also thank the Publication Department, NCERT, for their support. We gratefully acknowledge the services of Pawan Kumar Barriar, Narender Kumar Verma and Monika Rajput, *DTP Operators* and Pradeep Kumar, *Mathematics Laboratory Assistant*.



Contents

Data Handling

Annexures

Block

Teaching of Algebra

Students are expected to have opportunities to develop their understanding of patterns, to represent and analyse mathematical situations, etc. in their day-to-day life. By helping students move from specific numeric situations to develop general rules that model all situations of that type, teachers, in fact, begin to address learning of algebra. Generalising numeric situations can create strong connections between the mathematical content of numbers, number operations and their properties. In addition, these generalising activities built on what students already know about numbers and number operations can help students to develop a deeper understanding of formal algebraic symbols.

Algebra is a part of the reasoning process, a problem-solving strategy, a key to thinking mathematically and to communicating with mathematics. The key prerequisites for success in algebra are:

(i) understanding the concept of variables;
(ii) understanding the technical language of algebra; and
(iii) understanding the concepts of relations and functions.

Variable is the basic building block of Algebra. Due care and enough practice need to be given while introducing the

concept of a variable. Using matchstick patterns we can have a transition from arithmetic to algebra as follows. For example, look at the pattern of letter T.

T, T T, T T T, T T T T, T T T T T. . . . . .

No. of Ts	No. of matchsticks
1	2
2	4=2×2
3	6=2×3
4	8=2×4
5	10=2×5

While doing this activity, a certain pattern can be seen for the number of matchsticks required. This pattern continues further for any number of Ts to be formed. We can see that number of matchsticks required= 2× number of Ts. So if the number of Ts formed is 50, we instantly get number of matchsticks required as 2×50=100. Observe that the 'number of Ts formed' is a 'number'. But this number varies as per our choice. At one time 'the number of Ts formed' is 100 and at some other it could be 200. Every time it is not convenient for us to write the entire phrase as 'the number of Ts formed' so we choose a 'letter' say 'n' as 'the number of Ts formed'. So, we have number of matchsticks required = $2\times n$ or $2n$. This is a short and convenient notation where 'n' is a variable which represents a number.

A variable is a quantity which varies. We may choose any letter in the above case to represent the 'number of Ts formed'. It could be 'm' or 'a' or 'b'. In that case, number of matchsticks required would be $2m$ or $2a$ or $2b$. We have thus generated a rule to find the number of matchsticks required to form any number of Ts.

Students should learn to use symbols as a language in which they can express their own ideas, before they are introduced to formal algebra. Then algebra will not appear to them as just a meaningless collection of rules and procedures. Most of the laws in arithmetic are taught explicitly in algebra.

Idea of a variable that leads to different concepts in algebra is given below. This also shows the importance of algebra and the importance of interpreting things in symbolic form.

(i) A variable can be used in formula as shown below:

(a) Perimeter of a square = $4 \times l = 4l$, l is the length of its side

(b) Perimeter of a rectangle= $2(l+b)$, l = length, b = breadth

(c) $I = \dfrac{P \times R \times T}{100}$; I = simple interest, P = Principal, R = Rate, T = Time

In a formula, we use letters to represent numbers, such letters are known as literal numbers or variables.

(ii) Variable (x) is used as an unknown in $17+x = 35$. Here it is used to show a relationship among two quantities.

(iii) Variable is a very useful and powerful tool to express properties in arithmetic, geometry and other branches, in general form. Using the variables a and b for two numbers, we can express the commutativity of addition/multiplication of two numbers as $a+b = b+a$ and $a.b = b.a$. Other rules like associativity and distributivity are also expressed using variables.

Each of these understandings regarding variables requires appropriate language experience and students

must be required to translate and generalise it by using symbols as a language, to express their descriptive and numerical explanations.

Students should be required to explain some of the typical conflicts between the language of arithmetic, with which they are familiar, and the more technical language of algebra, which they will need to master. In algebra, it is often necessary to indicate operations because we do not always know the numbers these letters represent.

For example

Statement		Result
7+5	=	12
3 Pens + 2 Pens	=	5 pens
3 Pens + 2 Pencils	=	.....

[Either the students do not give answer or give wrong answer]

$x+y$ = ...

Thus we can add 7+5 but we cannot add x and y, so we indicate addition by $x+y$. Similarly, in algebra we indicate multiplication of x and y by xy, but in arithmetic we do not write 3×5 as 35. In arithmetic, we find that $7+\frac{1}{2}=7\frac{1}{2}$ and 4+0.5=4.5 but in algebra $3a+b$ does not mean $3ab$. It is necessary to familiarise the students with the language of algebra by comparing it with that of arithmetic.

In arithmetic, we come across expressions like (3×4)+7 or (2×5)+(3×7), etc. Similarly variables being numbers, we can perform operations of addition, subtraction, multiplication and division on them, like $2x+3$, $4-3x$, etc. This leads us to the idea of algebraic expressions.

Some of the algebraic expressions are : $2+3x$, $\frac{5}{x}$, $2x-7$, etc.

Familiarity with vocabulary is an important aspect at this stage. The students can be made aware that the same fact can be stated in different ways. Sufficient practice of this kind will be helpful in forming equations in different situations— $x + 7$ can be read as: x plus 7, sum of x and 7, 7 more than x, x increased by 7.

The exercise should be done both ways, i.e. given an expression write it in a statement form and vice versa.

The next step could be forming expressions in daily life situations. Some simple situations are given as:

(i) In a four-storeyed building, height of each storey is h metres. So, height of building is $4h$ metres.

(ii) Rita covers 'm' cm in one step. She walks 7 steps. Distance covered by her will be '$7m$' cm.

Students can be encouraged to form such questions or statements in different ways. This will help them to understand the process.

After giving enough practice of such expressions we can move on to expressions like $x^2 - 5$, $2mn + 8$, x^2+3y, etc.

An algebraic expression is made up of terms which in turn are made up of factors. The terms are added together to form an algebraic expression. In the expression $8y + 3x^2$, the terms are $8y$ and $3x^2$, whereas in the expression $4p^2q - 3pq^2 - 7$, the terms are $4p^2q$, $-3pq^2$ and -7, since $4p^2q - 3pq^2 - 7 = 4p^2q + (-3pq^2) + (-7)$

Factors of $4p^2q$ are $4, p, p, q, 4p^2$, etc.

Factors of $-3pq^2$ are $-3, p, q, q^2$, etc.

The factors p, p, q in $4p^2q$ and p, q, q in $-3pq^2$ are called the **algebraic factors**. Since they are different, we say that $4p^2q$ and $-3pq^2$ are unlike terms. The terms in which the algebraic factors are same are called like terms. Thus, the terms $-7mn$ and $-2mn$ are like terms.

This block contains the following four units:

Unit 1: Introduction to Algebraic Expressions
Unit 2: Linear Equations
Unit 3: Multiplication of Algebraic Expressions
Unit 4: Factorisation and Division of Algebraic Expressions

Unit 1 deals with the basic concepts related to algebra, namely variable, constant, algebraic expressions and their terms, like and unlike terms, coefficient of a term, addition and subtraction of algebraic expressions (polynomials). It also deals with finding the value of algebraic expressions for some given values of the variable(s).

Unit 2 deals with the concept of an equation and an identity. Different methods of solving linear equations in one variable along with their applications have also been discussed.

Unit 3 starts with the multiplication of algebraic expressions in a graded manner, namely, multiplication of two or more monomials, multiplication of a polynomial by a monomial, by a binomial, and so on. Four standard identities have been introduced in this unit through the process of multiplication of polynomials. The facts of these identities have also been reinforced through four separate activities. Applications of these identities in simplification and evaluations of some expressions have also been dealt with.

Finally, **Unit 4** deals with different methods of factorisation of algebraic expressions and their use in division of algebraic expressions.

UNIT 1

Introduction to Algebraic Expressions

Structure

- Introduction
- Main Concepts and Sub-concepts
- Objectives
- Teaching Points
  1. Concept of a Variable and a Constant
  2. Algebraic Expressions: How are they formed?
  3. Monomial, Binomial, Trinomial and Polynomial
  4. Addition and Subtraction of Algebraic Expressions
  5. Value of an Expression
- Common Errors
- Exercise

Introduction

In the block on the Number System, the students have studied about natural numbers, whole numbers, integers, fractions and rational numbers. They have also studied the basic operations of addition, subtraction, multiplication and division on these numbers. Now instead of known numbers, usually called constants, we shall consider unknown numbers, called variables. We shall also discuss various operations on variables and constants. The branch of mathematics in which such type of study is done is called **algebra** which is one branch of mathematics. Here, only few concepts will be discussed.

Main Concepts and Sub-concepts

- Variables and constants
- Algebraic expressions

  Term of an expression, Factors of a term, Coefficients, Like and unlike terms
- Monomial, Binomial, Trinomial and Polynomial
- Addition and subtraction of algebraic expressions
- Value of an expression

Objectives

After learning this unit, students can

- distinguish between a variable and a constant;
- understand algebraic expressions, their terms and coefficients of terms;
- understand the meaning of a monomial, binomial, trinomial and polynomial;
- add two algebraic expressions and subtract one expression from the other; and
- find the value of an expression for a given value of the variables.

Teaching Points

1. Concept of a Variable and a Constant

The teacher may introduce the concept of a variable and a constant through the following examples:

Suppose the age of Sarla is 3 years more than the age of Salma. What will be the age of Sarla when Salma's age is 10 years, 11 years, 12 years and n years? Look at the following table:

Salma's Age (in years)	10	11	12	n
Sarla's Age (in years)	10+3=13	11+3=14	12+3=15	n+3

In the above pattern, in 2nd row teacher should tell her students to observe that each time the number 3 is fixed,

i.e. $10 + \boxed{3} = 13$ $\quad 11 + \boxed{3} = 14$ $\quad 12 + \boxed{3} = 15$

But the other number is continuously varying, so **variable** should be assigned for that number which is **varying.**

(Teacher should explain the similarity between words **varying** and **variable.**)

Age of Sarla is expressed as $(n + 3)$ years when the age of Salma is n years. The number n has no fixed value. By taking $n = 10, 11, 12$, you can find $n + 3$, as 13, 14, 15, respectively. Here, n can take different values and so it is called a **variable** while in $n + 3$, 3 is a **constant**. Instead of the symbol n, you can use any other English alphabet e.g., $m, p, q, r, s, t, x, y, z$ etc. to denote a variable.

Another situation may be as follows:

Suppose each student of a class has to receive a set of 7 books from the Government. How many books will be required for a class of 20 students, 25 students, 30 students and n students? Look at the following table:

Number of students	20	25	30	n
Number of books required	20×7=140	25×7=175	30×7=210	n×7=7n

Again $7n$ has no fixed value. Its value depends on the value of n. For n = 20, 25, 30, the values of $7n$ are 140, 175, 210, respectively. In $7n$, 7 is a **constant** and n is a **variable**.

2. EXPRESSION

Algebraic Expressions: How are they formed?

Teacher should introduce the term "expression" by giving few examples of number expression and algebraic expression.

Students are already familiar with number expression e.g. 2 × 3 + 7, 7 × 3, etc.

So, they can be told that an expression is formed by relating numbers with basic operations. Now, since students have been introduced to variables, by examples like n + 3, n × 7, teacher can emphasise that an algebraic expression is formed by the combination of variables and numbers.

Thus, the expressions, like n + 3, $7n$, given above, are called **algebraic expressions**. More examples of algebraic expressions are:

$6n + 25,\ 2n - 7,\ 3m + 2n + 1,\ 5x - 3y + 2,\ 2a - b,\ n + \frac{1}{n}$

The symbols n, m, x, y, a, b (letters or literal numbers) used in algebraic expressions are **variables** while the fixed numbers 6, 25, 2, –7, etc. are **constants**.

An algebraic expression $6n$ + 25 is formed by first multiplying n by 6 and then adding 25 to it. Similarly an algebraic expression $2n$ – 7 is formed by first multiplying n by 2 and then subtracting 7 from it. Thus, **an algebraic**

expression can be formed by applying basic operations of addition, subtraction, multiplication and division on variables and constants. Teacher may point out that $2x^2 + 3x + 2$, $x^2 y^2 + 3$ etc. are also algebraic expressions.

Here, x^2 is nothing but $x \times x$ and is pronounced as x squared or square of x or x raised to the power 2. More algebraic expressions can be constructed using different powers of the variable or variables and basic operations. For example, using numbers, variables and sign of basic operations, an expression for the statement 7 added to y and to sum of 5 times x is written as $y + 5x + 7$.

Terms of an Expression: Teacher may explain the idea of terms of an algebraic expression by taking some examples such as $2x^2 - 3x + 2$, $4x^2 - 3xy + 2y^2 - 6x - 2y - 3$ etc. $2x^2 - 3x + 2$ can be written as $2x^2 + (-3x) + 2$. Each of $2x^2$, $-3x$, 2 is a term of $2x^2 - 3x + 2$. Similarly, $4x^2$, $-3xy$, $2y^2$, $-6x$, $-2y$, -3 are terms of the expression $4x^2 - 3xy + 2y^2 - 6x - 2y - 3$. Teacher may emphasise here that in the algebraic expression $2x^2 - 3x + 2$, $-3x$ is a term but $3x$ is not.

"Thus, the parts of an algebraic expression separated by the operation of addition are called terms."

Teacher may emphasise that in an algebraic expression, terms are separated by the operations + and – only and not by × and ÷.

Example: $2x + 3 \div y + 7$.

Here $2x$ is a single term, $3 \div y$ is a single term. So the above algebraic expression has 3 terms, namely $2x$, $\frac{3}{y}$, 7 only, not 5 terms as 2, x, 3, y and 7.

Factors in a Term: Students already know that 3 and 4 are factors of 12. Similarly, 3 and n are the factors of the term $3n$. In the term $-3x^2y^3$; -3, x^2, x, y, y^3, $(-3x^2)$, etc. are all its factors. x and y are called **algebraic factors** and -3 is called a **numerical factor.**

Teacher may point out that 1 is always a factor of any term.

Coefficient of a term: Consider the term $-3x^2y^3$. Here, it may be pointed out that the coefficient of x^2 y^3 is (–3), coefficient of y^3 is $(-3x^2)$ and the coefficient of x^2 is $(-3y^3)$. Usually when we speak about the coefficient of a term, we mean the **numerical coefficient** of that term. For example –3 is the coefficient of the term $-3x^2y^3$.

Teacher can discuss about coefficient by providing them an idea of hiding that part of the term whose coefficient has been asked. For example, in $-3x^2y^3$ to find coefficient of x^2, we can hide x^2 and left out is the answer.

Like and Unlike Terms: When terms have the same algebraic factors, they are called **like terms**. For example, $2x$ and $3x$ are like terms, so are $3x^2$ y, $-5x^2y$. However the terms $2x$, $2x^2$ are not like terms. These are called **unlike terms**. Here it may also be pointed out that terms –7 and 5 are like terms. Teacher may further reinforce the idea by taking more examples.

Next consider the following expressions:

$2x^2y - 3y^2x + 2x - 3y + 6$ and $-xy^2 + 3x^2y - 4y + 2$. Ask the students to look carefully the terms of these expressions. They will find that $-3y^2x$, a term of first expression which is the same as $-3xy^2$ and the term '$-xy^2$' of second expression are like terms. Similarly, the terms '$2x^2y$' and $3x^2y$ are like terms, $-3y$ and $-4y$ are also like terms, etc. The term $2x$ of first expression has no corresponding like term in the second expression. The teacher is expected to construct some pairs of expressions and ask the students to identify like and unlike terms.

Teacher may help the students to solve questions in Exercise 11.1 to 11.4 of Class VI, Mathematics, NCERT.

3. MONOMIAL, BINOMIAL, TRINOMIAL AND POLYNOMIAL

The algebraic expressions can be classified according to their number of terms. An algebraic expression having only one term is called a **'monomial'**, having two unlike

terms is called a **'binomial'**, having three unlike terms is called a **'trinomial'** and having any number of unlike terms is called a **'polynomial'**. Here, teacher has to point out that the word 'polynomial' is used in a restricted sense in which exponents of the variable(s) are **non negative** integers. For example, $2x^2$ is a monomial, $3x^2 + 2$ is a binomial, $2x^5y + 3x + y$ is a trinomial and $2x^5 + 3x^3 - 2x - 4$ is a polynomial (in one variable x). In fact, $2x^2$, $3x + 2$, $2x^5y + 3x + y$ are also referred to as polynomials. The last one is a polynomial in two variables x and y.

Teacher may give examples to make students understand that a monomial, binomial, trinomial can be algebraic expression.

	Algebraic Expression	Polynomial
Monomial	$3/x$	x^2
Binomial	$x^2 + 2x^{-7}$	$4x^2 + 3x^3$
Trinomial	$x^{-3} + x^2 + 7$	$x + 2 + x^{100}$

Students may be helped to do Exercise 12.1 on Page 234 of Class VII, Mathematics, NCERT.

4. Addition and Subtraction of Expressions

The students know about like terms. The teacher should explain the addition of two (or more) expressions by collecting the like terms together and then performing the addition. This process can be explained in the following ways:

Add: $2x^2 + 3x - 5$ and $-x^2 + 2x - 8$

First Method:

$(2x^2 + 3x - 5) + (-x^2 + 2x - 8) = 2x^2 + 3x - 5 - x^2 + 2x - 8$

$= (2x^2 - x^2) + (3x + 2x) + (-5 - 8)$ ⟶ writing like terms together

$= x^2 + 5x - 13$

Second Method:

Write the expression one below the other in such a way that like terms are arranged in the same column, i.e.

$$\begin{array}{ll} & 2x^2 + 3x - 5 \\ \text{Adding} & -x^2 + 2x - 8 \\ \hline \text{We get} & x^2 + 5x - 13 \end{array}$$

Teacher may also take few examples in which there are unlike terms; e.g. $2x^2 + 3xy - 5$ and $-x^2 + 2x - 8$.

Horizontal Method:

$$(2x^2 + 3xy - 5) + (-x^2 + 2x - 8)$$

$$= (2x^2 - x^2) + (-5 - 8) + 3xy + 2x$$

To be kept in the end as they are not like terms

$$= x^2 - 13 + 3xy + 2x$$

Vertical Method:

$$\begin{array}{llll} 2x^2 - & 5 + 3xy & & \\ -x^2 - & 8 & + 2x & \\ \hline x^2 & - 13 + 3xy + 2x & & \end{array}$$

Unlike terms written separately

Three or more expressions can be added in the same way.

Subtraction, as you know, is an inverse process of addition. That is, if you want to subtract a number b from another number a, simply add a and $-b$. The same process can be used in case of expressions also. Suppose you want to subtract $2x^2 - 3x + 4$ from $x^2 + x - 1$. This means to find $(x^2 + x - 1) - (2x^2 - 3x + 4)$. Teacher may emphasise that if there is a minus sign before a bracket, the sign of each term inside that bracket will change from + to – and – to + on removing the bracket. We can also say that if there is a

minus sign before a bracket, then each term of the polynomial inside the bracket is to be multiplied by (–1) or the sign of each term of the polynomial inside the bracket is to be changed. For example, $-(2x^2 + 3x - 5) = -2x^2 - 3x + 5$.

Thus, $(x^2 + x - 1) - (2x^2 - 3x + 4)$

$= x^2 + x - 1 - 2x^2 + 3x - 4$

$= (x^2 - 2x^2) + (x + 3x) + (-1 - 4)$

$= -x^2 + 4x - 5.$

The other way of doing this is as follows:

Write the expression to be subtracted below the other expression with like terms occurring in the same column, change the signs of the terms of lower expression and then add, i.e.,

$$\begin{array}{r} x^2 + x - 1 \\ +\,2x^2 - 3x + 4 \\ - \quad + \quad - \\ \hline -\,x^2 + 4x - 5 \\ \hline \end{array}$$

Another Example:
Subtract $3x^2 - 2x + 7$ from $7x^3 + 1$

Write $7x^3 + 1$ as: $7x^3 + 0x^2 + 0x + 1$

and $3x^2 - 2x + 7$ as: $0x^3 + 3x^2 - 2x + 7$

Then change the signs as: $- \quad - \quad + \quad -$

$7x^3 - 3x^2 + 2x - 6$

The teacher may give some more exercises to students for solving by using both ways of addition and subtraction of expressions as discussed above.

5. VALUE OF AN EXPRESSION

An expression may contain one variable or more than one variable. Therefore, the expression will have different values for different values of the variable or variables. The teacher may discuss with students, as follows:

Suppose the given expression is $2x^2 - x - 2$. For $x = 1$, its value will be $2 \times 1^2 - 1 - 2 = 2 - 1 - 2 = -1$. For $x = 3$, its value will be $2 \times 3 \times 3 - 3 - 2 = 13$. For $x = 0$, its value will be $0 - 0 - 2 = -2$ and for $x = -2$, its value will be $2 (-2) (-2) - (-2) - 2 = 8 + 2 - 2 = 8$.

This can be given in the form of a table as:

$x =$	-2	0	1	3
$2x^2 - x - 2 =$	8	-2	-1	13

Similarly to find the value of the expression $2x^2y^2 - 3xy \times 4x - 3y - 1$, when $x = 1$, $y = -1$, substitute these values of x and y in the expression to get

$2 \times 1^2 \times (-1)^2 - 3 \times 1 \times (-1) + 4 \times 1 - 3 \times (-1) - 1$

$= 2 + 3 + 4 + 3 - 1$

$= 11$

If the value(s) of (all) variable(s) in a given expression is known, then on replacing these variables by their numerical values, we get an arithmetic expression which can be evaluated. This process of replacing variables by their numerical values is called **substitution**.

The teacher may now give more exercises for finding the values of expressions for given values of variables to the students. They may be helped to do Exercise 12.2 on page 239 of Class VII, Mathematics, NCERT and Exercise 9.1 of Class VIII, Mathematics, NCERT.

Common Errors

1. In the expression $-x^2 - 3x + 2$ the terms are taken as x^2, $3x$, 2 in place of $-x^2$, $-3x$, 2.
2. Coefficient of xy in the term $-6xy$ is taken as 6 in place of -6. Also the coefficient of x in $-2xy$ is taken as -2 in place of $-2y$.

3. In problems of addition and subtraction, like and unlike terms are sometimes combined together in place of combining only like terms. e.g., $(3x + 5) + (x^2 + 1)$ is written as $4x + 6$ in place of $x^2 + 3x + 6$.

   You may evaluate the students through the following exercise.

Exercise

1. A rectangular box has height h cm. Its length is 5 times the height and breadth is 10 cm less than the length. Express the length and breadth of the box in terms of the height.
2. Identify terms and their two factors in the expressions:

   (i) $xy + 2x^2y^2$ (ii) $1.2ab - 2.4b + 3.6a$
3. Classify the following into monomials, binomials and trinomials:

   (i) $4y - 7z$ (ii) y^2 (iii) $x + y - xy$

   (iv) 100 (v) $z^2 - 3z + 8$ (vi) $5 - 3t$
4. Identify the like terms in the following and make their groups:

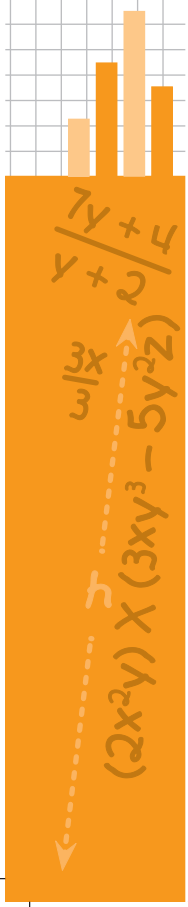
   $-xy^2$, $-4xy^2$, $8x^2$, $2xy^2$, $7y$, $-11x^2$, $-100x$, $-11yx$, $-20x^2y$, $-6x^2$, y, $2xy$, $3x$.
5. Add: $14x + 10y - 12xy - 13$; $18 + 7x - 10y + 8xy$ and $4xy$.
6. Subtract $4pq - 5q^2 - 3p^2$ from $5p^3 + 3q^2 - pq$.
7. If $p = -2$, find the value of $-p^3 - 3p^2 + 4p + 7$.
8. If $a = -1$, $b = -2$, find the value of $2a - 2b - 4ab - 5 + a$.
9. If $a = 5$, $b = -3$, find the value of $2(a^2 + ab) + 3 - ab$.

2 UNIT

Linear Equations

Structure

- Introduction
- Main Concepts and Sub-concepts
- Objectives
- Teaching Points
  1. Meaning of an Equation
  2. Solution of a Linear Equation
  3. Difference between an Equation and an Identity
  4. Applications
- Common Errors
- Exercise



Introduction

The students are already familiar with the concepts of variable, constant and formation of algebraic expressions. They have also learnt how to find the value of an expression for different values of the variables. In this unit, the concept and meaning of an equation alongwith the solution of a linear equation will be discussed. The difference between an equation and an identity will be explained. The solution of equations is very useful in solving many daily life problems. We shall also explain the formulation of some daily life situations into linear equations and solve them.

Main Concepts and Sub-concepts

- Equation
- LHS and RHS of an equation
- Formation of an equation
- Solution of a linear equation

Objectives

After teaching this unit, the students can

- form an equation for a given situation;
- solve the equation; and
- solve daily life problems using equations.

Teaching Points

1. Meaning of an Equation

An equation consists of two expressions with an equality sign between them. The expression to the left of the equality sign is called left hand side (LHS) and the expression to the right of the equality sign is called right hand side (RHS) of the equation.

Teacher may point out that of the two expressions, at least one of them must contain a variable.
For example, $x + 1 = 0$, $0 = -1 + x$, $2x + 3y = 5$, $-x + 8 = -\frac{1}{2}$, $x^2 = 1$ are equations, but $x + 3$, $x^2 - 1 + x$, $2x < 3$, $x^2 - 1$ are not equations, because they do not contain an equality sign.

Teacher may ask the students to identify LHS and RHS of the given equations. Teacher may explain the formation of an equation through different situations as discussed in pages 236–237 of Class VI, Mathematics, NCERT.

Linear equation: The equation where the power of the variable involved is only 1 is called linear equation.

$x+1= 0$, $0 = -1 + y$, $-x + 8 = \frac{x}{2}$, $2x+3y = 5$ and $x- y = 2x+5$ are linear equations. Of these equations, the equations $x+1= 0$, $0 = -1 + y$, $-x + 8 = \frac{x}{2}$, are **linear equations in one variable** while the equations $2x + 3y = 5$, $x - y = 2x + 5$ are **linear equations in two variables**.

Thus an equation involving only a linear polynomial is called a linear equation.

Equations of the type $x^2 - 1 = 0$, $2x^2 + x - 3 = 0$ are not linear equations. Why they are not linear, has again to be explained. At this stage, we will restrict ourselves to linear equations in one variable only.

2. SOLUTION OF A LINEAR EQUATION

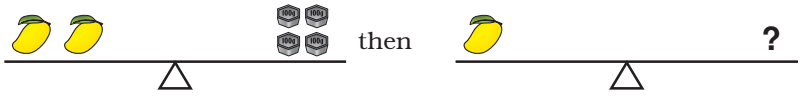
Before coming to the solution of an equation, you may ask the students to form a table as given below for an equation, say $3x - 1 = 8$:

Equation	Value of the variable	Value of the LHS	Value of the RHS	Is LHS = RHS? (Yes/No)
$3x - 1 = 8$	–1	–4	8	No
	0	–1	8	No
	1	2	8	No
	2	5	8	No
	3	8	8	Yes
	4	11	8	No

Let the students observe from the table that for $x = 3$, the LHS and RHS of the given equation both become 8, i.e. they are equal. Here, the teacher may explain that the value of x for which LHS = RHS is called a **solution of the equation.** So, $x = 3$ is a solution of the equation. The other values of x, i.e. –1, 0, 1, 2 and 4 are not solutions of the equation. Teacher may highlight the fact using different examples that there is only one solution of a linear equation. The method of finding solution in this way is called the **trial and error** method. The teacher may give a number of equations to the students to find the solutions by this method. Let the students themselves feel that this method is not always convenient. This may motivate them to learn a systematic method.

An equation can be compared with a weighing balance with equal weights on its LHS and RHS pans and the beam of the balance being exactly horizontal. If we add the same weight or remove (subtract) the same weight from both the pans, still it remains horizontal. Also if we double or triple the weights or make them half or one third on both the pans, the beam still remains horizontal. If we interchange the weights of LHS and RHS pans, the

beam again remains horizontal. To explain the concept of balance, teacher can take up some visuals as follows:

1. 

2. 

This gives a clue for solving an equation:

1. You can add or subtract the same non-zero number to LHS and RHS of the equation.
2. You can multiply or divide by the same non-zero number to LHS and RHS of the equation.

   (Teacher may explain why non-zero number)
3. You can interchange LHS and RHS of an equation.

You may give some equations to the students and they can see how these facts help in solving a linear equation. Let the linear equation be $3x - 1 = 8$.

Step I: Add 1 on both sides to get

$$3x - 1 + 1 = 8 + 1$$
$$3x = 9$$

Step II: Divide both sides by 3 to get

$$\frac{3x}{3} = \frac{9}{3}$$
$$x = 3$$

In the beginning, advise the students to check whether the solution they have found satisfies the given equation or not. Infact this is a good practice if the students do this to be sure whether they got correct solution or not.

It may be noted that adding or subtracting the same term to LHS and RHS of an equation is equivalent to

transposing the term from one side of the equation to the other side by changing the sign of the terms, for example,

$3x - 1 = 8$ gives $3x = 8 - (-1) = 8 + 1 = 9$ (transposing (-1) from LHS to RHS) or $x = 3$

Take another example, say $5x + 2 = 7x - 10$

or, $5x - 7x = -10 - 2$ by transposing the terms containing x on one side and constants on the other side.

i.e. $-2x = -12$ or $x = \frac{12}{2}$ or $x = 6$

Sometimes, you come across some equations which are not linear but can be converted into a linear equation by doing the above mentioned operations. For example, the equation

$$\frac{3y + 4}{2} = \frac{-2}{5}$$

is not a linear equation in this form, but when it is rewritten as

$$5(3y + 4) = -2(2 - 6y) \ldots\ldots \text{(i)},$$

it becomes a linear equation.

i.e. $15y + 20 = -4 + 12y$

i.e. $15y - 12y = -4 - 20$

i.e. $3y = -24$

i.e. $y = -24 \div 3 = -8$.

Note: Teacher may also explain the method of obtaining equation in (i), as follows:

$\frac{3y + 4}{2}(2 - 6y) = \frac{-2}{5}(2 - 6y)$ [Multiply both sides by $(2-6y)$]

or $5(3y + 4) = \frac{2}{5}(2 \quad 6y)5$ [Multiply both sides by 5]

or $5\ (3y + 4) = -2\ (2 - 6y)$

It may be emphasised that this equation can directly be obtained by cross multiplying, as

$$\frac{3y+4}{2-6y} = \frac{-2}{5}$$

This method is called **cross multiplication** method.

3. EQUATIONS AND IDENTITIES

You have seen above how to solve a linear equation in one variable and also have observed that only one value of the variable satisfies the equation. Sometimes, you may find a situation, like the equation $2(x + 3) = 2x + 6$.

By definition, it appears to be a linear equation in one variable. Ask the students to find the solution of this equation by the trial and error method, as follows:

Equation	Value of the variable	Value of the L.H.S.	Value of the R.H.S.	Is LHS = RHS? (Yes/No)
$2(x+3) = 2x+6$	-1	4	4	Yes
	0	6	6	Yes
	1	8	8	Yes
	2	10	10	Yes
	3	12	12	Yes
	4	14	14	Yes

You may observe from the table that the given equation is true for all values of the variable x. Similarly, if you consider the equation,

$(x + 1)(x + 1) = x^2 + 2x + 1$, you can find that the above equation is also true for all the values of x. Such equations are named as **identities.** Now, a number of equations and identities may be written on the blackboard and let the

students identify which is an equation and which is an identity.

4. APPLICATIONS

Many real life problems can be solved by using the concept of linear equations. Students can now translate word statements into equations. Some real life problems can be converted into linear equations. The process is to be explained through some examples, as follows:

Example: The length of a rectangle is 5 cm and its perimeter is 18 cm. Find its breadth.

The unknown quantity is breadth of the rectangle. Let it be x cm.

It is given that perimeter = 18 cm

i.e. $18 = 2(5 + x)$ [Formulation of the problem]

i.e. $\frac{18}{2} = 5 + x$

i.e. $9 = 5 + x$

i.e. $5 + x = 9$

i.e. $x = 9 - 5 = 4$

Therefore, breadth = 4 cm

Check: Perimeter $= 2\ (5 + 4)$cm

$= 2 \times 9$ cm

$= 18$ cm

Note: The teacher should advise the students to check the final answer with the situation given in the question.

Example: The age (in years) of mother is 5 times the age of her daughter. After 21 years the age of mother will become twice the age of the daughter. Find their present ages.

Let the age of the daughter be x years. Then, age of mother will be $5x$ years.

Teacher may encourage students for tabulation, as follows:
Thus, the given condition expresses that

$5x + 21 = 2\ (x + 21)$

or $5x + 21 = 2x + 42$

or $5x - 2x = 42 - 21$

or $3x = 21$

or $x = \frac{21}{3} = 7$

Therefore, age of daughter = 7 years and age of mother = 5 × 7 = 35 years.

CHECK: After 21 years,

age of daughter will be 7 + 21 = 28 years

age of mother will be 35 + 21 = 56 years

and 56 = 2 × 28

Teacher may help the students in Exercise 11.5 on page 240 of Class VI Mathematics, NCERT; Exercises 4.1 to 4.4 of Class VII Mathematics, NCERT; and Exercises 2.1 to 2.6 in Class VIII Mathematics, NCERT.

Common Errors

(i) In transposing a term from LHS to RHS or vice versa, the students sometimes do not change the sign of the term and this leads to wrong answer.

(ii) In the solution of an equation, the students sometimes forget to write equality sign at proper place.

(iii) Sometimes in solving equations like $\frac{x\ \ 1}{2} = \frac{2x\ \ 3}{3}$, the students write this as $\frac{3(x\ \ 1) = 2(2x\ \ 3)}{6}$ in place of $3\ (x - 1) = 2\ (2x - 3)$. Sometimes, they write it as $3(x - 1) = 2 \times 2x - 3$.

(iv) Sometimes, solution comes out to be a rational number of the form say $\frac{1}{3}$. Some students write it as x = 0.33 which is not correct. The reason is that $x=\frac{1}{3}$ will satisfy the equation but x = 0.33 will not.

You may evaluate the students through the following exercise:

Exercise

1. Set up an equation for the following situation: Lakshmi's father is 49 years old. He is 4 years older than three times her age.
2. Solve the equations:

   (i) $\frac{x}{5}=\frac{7}{15}$ (ii) $3n-2=46$ (iii) $4\,(2-x) = 8$

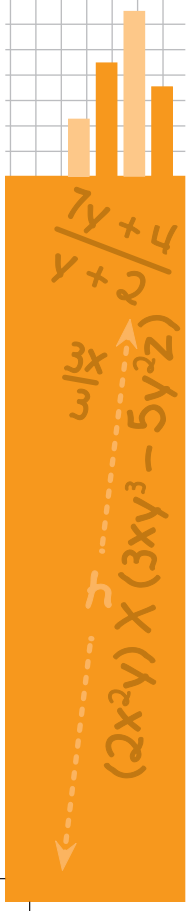
   (iv) $3\,(n-5)= -21$ (v) $5t-3=3t-5$ (vi) $\frac{2x}{3}+1=\frac{7x}{15}+3$
3. Sum of digits of a two digit number is 9. When we interchange the digits, it is found that the new number is greater than the original number by 27. Find the number.
4. Solve the equation $\frac{7y+4}{y+2}=\frac{4}{3}$
5. The denominator of a fractional number is greater than its numerator by 8. If the numerator is increased by 17 and denominator is decreased by 1, the number obtained is $\frac{3}{2}$. Find the original fractional number.
6. The length of a rectangle is 7 cm and its area is 35 cm^2. Find its breadth.

3 UNIT

Multiplication of Algebraic Expressions

Structure

- Introduction
- Main Concepts and Sub-concepts
- Objectives
- Teaching Points
  1. Multiplication of Two or More Monomials
  2. Multiplication of Two Polynomials
  3. Standard Identities
  4. Application of Identities
- Common Errors
- Exercise



Introduction

In the unit 'Introduction to Algebraic Expression', operations of addition and subtraction on algebraic expression have been discussed. In this unit, multiplication of algebraic expressions will be discussed.

Main Concepts and Sub-concepts

- Multiplication of two or more monomials
- Multiplication of a polynomial by a monomial
- Multiplication of two polynomials
- Application

Objectives

After teaching of this unit, students can

- multiply two or more monomials;
- multiply a binomial or a trinomial by a monomial;
- multiply a polynomial by a binomial or a trinomial; and
- state and use some standard identities

Teaching Points

1. Multiplication of Two or More Monomials

Students are already aware of the multiplication of numbers, their properties and the laws of exponents. Teacher may recall these concepts through examples before starting this unit. They also know that any monomial can be written as a product of its factors. For example, $5x^2y^3z^4$ can be written as $(5) \times (x^2) \times (y^3) \times (z^4)$, where 5 is a constant and x, y and z are variables. To multiply two monomials, you may proceed as follows:

Step I: Collect all the numerical coefficients and the same variables of the monomials together in

separate brackets with multiplication sign in between. You may point out that while collecting the coefficients and variables together, the commutative and associative properties of multiplication are being used.

Step II: Use the laws of exponents to find the product in each of the brackets. You may explain the process by taking some examples:

Example: Multiply $5x^3$ and $7x^2$

Solution:
$$5x^3 \times 7x^2 = (5 \times x^3) \times (7 \times x^2)$$
$$= (5 \times 7) \times (x^3 \times x^2)$$
$$= 35 \times (x^{3+2})$$
$$= 35x^5$$

Thus, product of $5x^3$ and $7x^2$ is $35x^5$

Example: Find the product of $8x^2y^3$ and $6xy^5z$

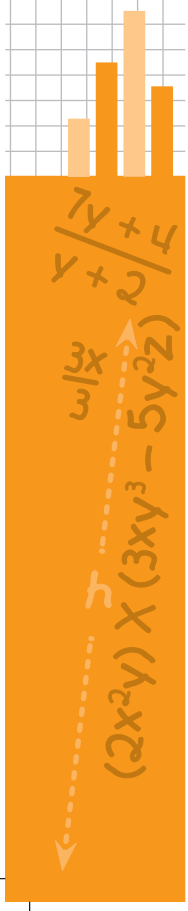
Solution:
$$8x^2y^3 \times 6xy^5z = (8 \times x^2 \times y^3) \times (6 \times x \times y^5 \times z)$$
$$= (8 \times 6) \times (x^2 \times x) \times (y^3 \times y^5) \times (z)$$
$$= 48 \times (x^{2+1}) \times (y^{3+5}) \times (z)$$
$$= 48x^3y^8z$$

Thus, product of $8x^2y^3$ and $6xy^5z$ is $48x^3y^8z$

Example: Find the product of the monomials $3x^2y$, $5x^3y^2z$ and $-7xyz^3$

Solution:
$$(3x^2y) \times (5x^3y^2z) \times (-7xyz^3)$$
$$= (3 \times x^2 \times y) \times (5 \times x^3 \times y^2 \times z) \times [(-7) \times x \times y \times z^3]$$
$$= [3 \times 5 \times (-7)] \times (x^2 \times x^3 \times x) \times (y \times y^2 \times y) \times (z \times z^3)$$
$$= (-105) \times (x^6) \times (y^4) \times (z^4)$$
$$= -105x^6y^4z^4$$

The teacher may give more questions for practice and understanding of the concept of multiplication of monomials.



2. MULTIPLICATION OF TWO POLYNOMIALS

Using the idea of multiplication of two or more monomials, you may now explain multiplication of two polynomials, using distributive property, by taking some examples.

Example: Find the product of $2x^2y$ and $3xy^3 - 5y^2z$

Solution: $(2x^2y) \times (3xy^3 - 5y^2z)$

$=(2x^2y) \times (3xy^3) - (2x^2y) \times (5y^2z)$ [by distributive property]

$=(2 \times 3) \times (x^2 \times x) \times (y \times y^3) - (2 \times 5) \times (x^2) \times (y \times y^2) \times (z)$

$=6x^3y^4 - 10x^2y^3z$

Thus, the product of $2x^2y$ and $3xy^3 - 5y^2z$ is $6x^3y^4 - 10x^2y^3z$.

Example: Find the product of $(2a - 3b)$ and $(a + 5b)$

Solution: $(2a - 3b) \times (a + 5b)$

$=2a \times (a+5b) - 3b \times (a+5b)$

$=(2a) \times (a) + (2a) \times (5b) - (3b) \times (a) - (3b) \times (5b)$

$=2a^2 + 10ab - 3ba - 15b^2$

$=2a^2 + 7ab - 15b^2$ [Since $ab = ba$]

Example: Find the product of $(2x^2 - 3y)$ and $(x^2+3xy^3-2y^2)$

Solution: $(2x^2 - 3y) \times (x^2 + 3xy^3 - 2y^2)$

$= (2x^2) \times (x^2 + 3xy^3 - 2y^2) - 3y \times (x^2 + 3xy^3 - 2y^2)$

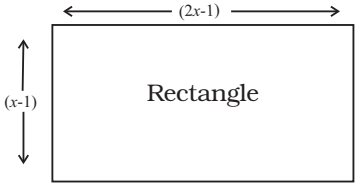
$= (2x^2) \times (x^2) + (2x^2) \times (3xy^3) + (2x^2) \times (-2y^2) - (3y) \times (x^2) - (3y) \times (3xy^3) - (3y) \times (-2y^2)$

$= 2x^4 + 6x^3y^3 - 4x^2y^2 - 3yx^2 - 9xy^4 + 6y^3$

Teacher may also take few application based questions as:

Question: Find the area of the following:

(i)



(ii) Circle with radius $7x$

You may help the students to solve questions in Exercise 9.2 on page 143, Exercise 9.3 on page146, Exercise 9.4 on page 148 of Class VIII Mathematics, NCERT.

3. STANDARD IDENTITIES

In the unit on linear equations, identities and the difference between an equation and an identity have been discussed. Now, we will discuss some of the identities which may be explained using the idea of multiplication of polynomials.

(i) $(a+b)^2 = a^2 + 2ab + b^2$

$(a+b)^2 = (a+b)\,(a+b)$

$= a\,(a+b) + b\,(a+b)$

$= a^2 + ab + ba + b^2$

$= a^2 + 2ab + b^2$ [Since $ab = ba$]

Thus, $(a+b)^2 = a^2 + 2ab + b^2$

(ii) $(a - b)^2 = a^2 - 2ab + b^2$

$(a - b)^2 = (a - b)\,(a - b)$

$= a\,(a - b) - b\,(a - b)$

$= a^2 - ab - ba + b^2$

$= a^2 - 2ab + b^2$

Thus, $(a-b)^2 = a^2 - 2ab + b^2$

(iii) $(a + b)\,(a - b) = a^2 - b^2$

$(a + b)\,(a - b) = a(a - b) + b\,(a - b)$

$= a^2 - ab + ba - b^2$

$= a^2 - b^2$

Thus, $(a+b)\,(a-b) = a^2 - b^2$

(iv) $(x + a)\,(x + b) = x^2 + (a+b)x + ab$

$(x + a)\,(x + b) = x\,(x+b) + a(x+b)$,

$$= x^2 + xb + ax + ab$$

$$= x^2 + ax + bx + ab \quad [\text{Since } xb = bx]$$

$$= x^2 + (a+b)x + ab$$

Thus, $(x+a)(x+b) = x^2 + (a+b)x + ab$

You may also explain these identities through Activities 1 to 4 given at the end of this unit.

4. APPLICATION OF IDENTITIES

You may now explain the use of these identities in simplification of some expressions and also in finding some specific values of the expressions through some examples.

Example: Solve $(3x + 4y)^2$ using a suitable identity.

Solution: Using the identity $(a + b)^2 = a^2 + 2ab + b^2$

$$(3x + 4y)^2 = (3x)^2 + 2 \times (3x) \times (4y) + (4y)^2$$

$$= 9x^2 + 24xy + 16y^2$$

Example: Evaluate $(102)^2$, using a suitable algebraic identity.

Solution: $(102)^2 = (100 + 2)^2$ (Highlight this step)

$$= (100)^2 + 2 \times (100) \times (2) + (2)^2$$

(Using $(a + b)^2 = a^2 + 2ab + b^2$)

$$= 10000 + 400 + 4$$

$$= 10404$$

Thus, $(102)^2 = 10404$

Example: Using the identity $(a - b)^2 = a^2 - 2ab + b^2$, find (i) $(5p - 4q)^2$ and (ii) $(9.9)^2$.

Solution: (i) $(5p - 4q)^2 = (5p)^2 - 2 \times (5p) \times (4q) + (4q)^2$

$$= 25p^2 - 40pq + 16q^2$$

(ii) $(9.9)^2 = (10 - 0.1)^2$ (Highlight this step)

$$= (10)^2 - 2 \times 10 \times (0.1) + (0.1)^2$$

$= 100 - 2 + 0.01$

$= 98 + 0.01$

$= 98.01$

Hence, $(9.9)^2 = 98.01$

Example: Using a suitable identity, find (i) $(3m + 4)(3m - 4)$ and (ii) 1002×998

Solution: (i) $(3m + 4)(3m - 4) = (3m)^2 - (4)^2$

$= 9m^2 - 16$

(ii) 1002×998 can be written as $(1000 + 2) \times (1000 - 2)$

$(1000 + 2) \times (1000 - 2) = (1000)^2 - (2)^2$

$= 1000000 - 4$

$= 999996$

Example: Using the identity $(x + a)(x+b) = x^2 + (a + b)x + ab$ find 103×107.

Solution: $103 \times 107 = (100 + 3)(100 + 7)$

Highlight this step

$= (100)^2 + (3 + 7) \times 100 + 3 \times 7$

$= 10000 + 1000 + 21$

$= 11021$

Thus, $103 \times 107 = 11021$

Ask the students to solve Exercise 9.5 on page 151 of Class VIII Mathematics, NCERT.

Activity 1: For $(a+b)^2 = a^2 + 2ab + b^2$

1. Cut out a square of length a units from a drawing sheet/ cardboard and name it as square ABCD [See Fig. 1].
2. Cut out another square of length b units from a drawing sheet/cardboard and name it as square CHGF [See Fig. 2].

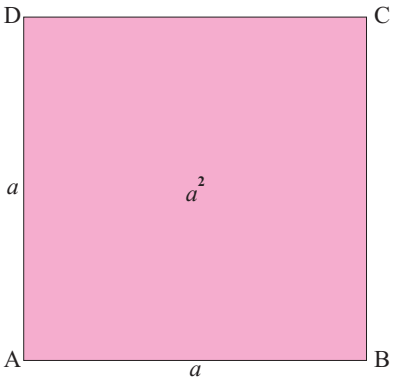


Fig. 1

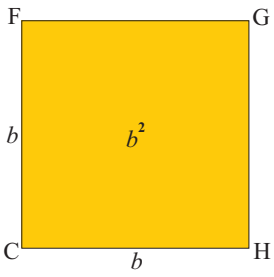


Fig. 2

3. Cut out a rectangle of length a units and breadth b units from a drawing sheet/cardboard and name it as rectangle DCFE [See Fig. 3].

4. Cut out another rectangle of length b units and breadth a units from a drawing sheet/cardboard and name it as rectangle BIHC [See Fig. 4].

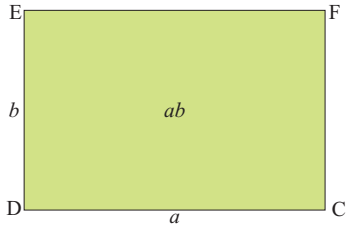


Fig. 3

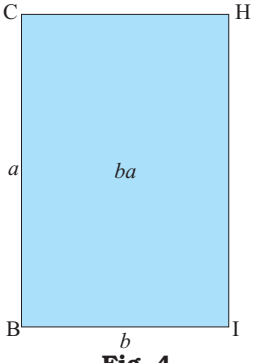


Fig. 4

5. Total area of these four cut-out figures

   = Area of square ABCD + Area of square CHGF + Area of rectangle DCFE + Area of rectangle BIHC

   = $a^2 + b^2 + ab + ba = a^2 + 2ab + b^2$

6. Join the four cut-outs using sellotape as shown in Fig. 5.

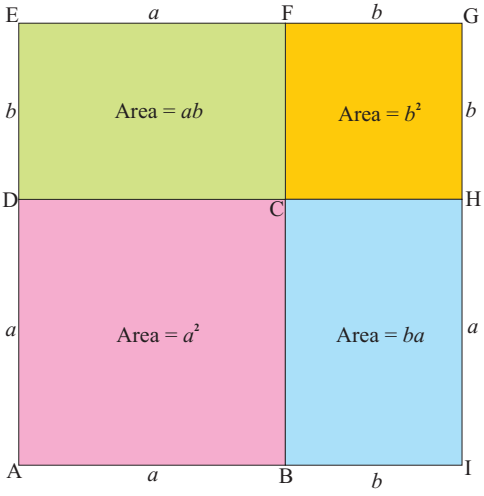


Fig. 5

Clearly, AIGE is a square of side $(a + b)$. Therefore, its area is $(a + b)^2$. The combined area of all four parts $= a^2 + 2ab + b^2$.

Hence, the algebraic identity $(a+b)^2 = a^2 + 2ab + b^2$ is true.

Here, area is in square units.

Activity 2 : For $(a - b)^2 = a^2 - 2ab + b^2$

1. Cut out a square ABCD of side a units from a drawing sheet/cardboard [see Fig. 6].
2. Cut out a square EBHI of side b units $(b < a)$ from a drawing sheet/cardboard [see Fig. 7].
3. Cut out a rectangle GDCJ of length a units and breadth b units from a drawing sheet/cardboard [see Fig. 8].
4. Cut out a rectangle IFJH of length a units and breadth b units from a drawing sheet/cardboard [see Fig. 9].
5. Arrange these cut-outs as shown in Fig. 10.

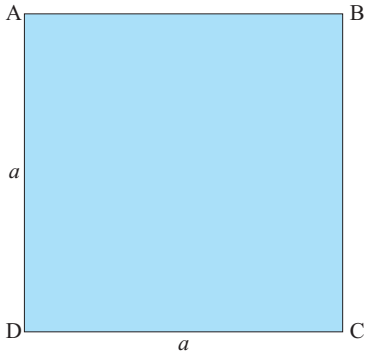


Fig. 6

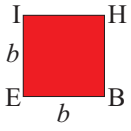


Fig. 7

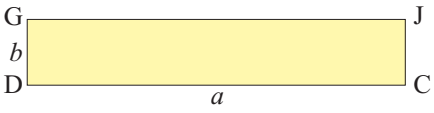


Fig. 8

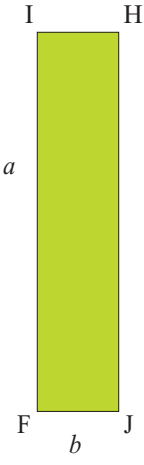


Fig. 9

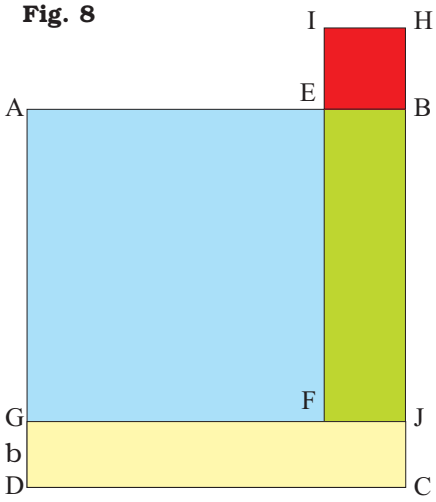


Fig. 10

Area of square ABCD = a^2, Area of square EBHI = b^2

Area of rectangle GDCJ = ab, Area of rectangle IFJH = ab

From Fig.10, area of square AGFE = AG × GF = $(a - b)(a - b)$ = $(a - b)^2$

Now, area of square AGFE = Area of square ABCD + Area of square EBHI

– Area of rectangle IFJH – Area of rectangle GDCJ

$= a^2 + b^2 - ab - ab$

$= a^2 - 2ab + b^2$

Thus, $(a - b)^2 = a^2 - 2ab + b^2$

Another Activity:

For $(a-b)^2 = a^2 - 2ab + b^2$

1) Cut-out a square ABCD of side a units from drawing sheet.

2) Cut-out a square MNOP of side b units from drawing sheet.

3) Arrange them as shown below (Fig. 11):

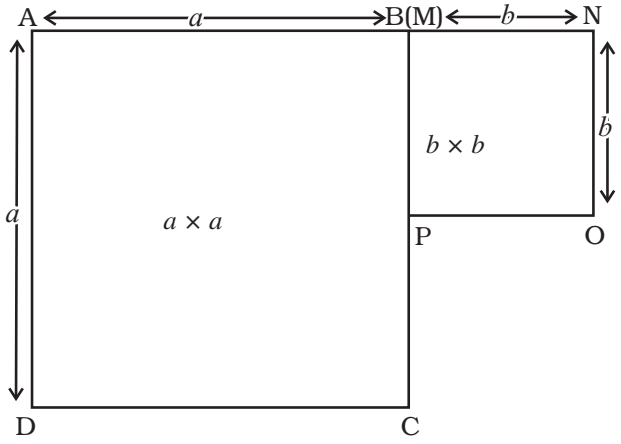


Fig. 11

4) Draw a line ST in the bigger square to divide it into 2 rectangles of dimensions $a \times b$ and $a \times (a-b)$ as shown in Fig. 12.

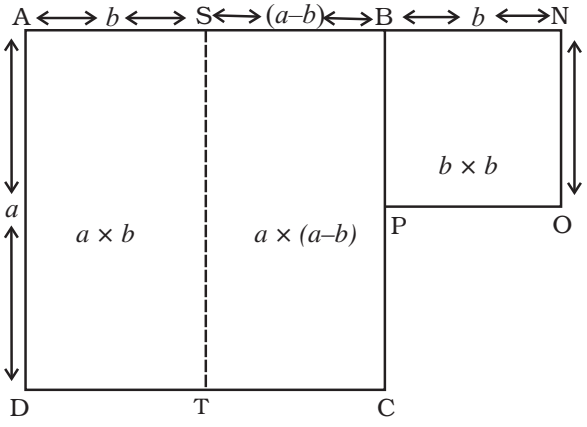


Fig. 12

5) Extend OP to meet ST at X.

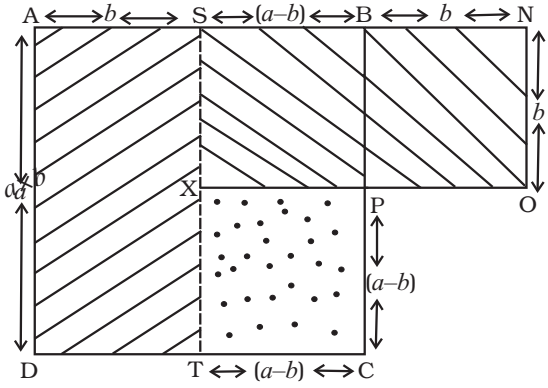


Fig. 13

6) $a^2 + b^2$ = Area of rectangle ASTD + Area of Rectangle SNOX + Area of square PCTX (Fig. 13)

$a^2 + b^2 = a \times b + a \times b + (a-b)\ (a-b)$

$a^2 + b^2 = 2ab + (a-b)^2$

$$a^2 + b^2 - 2ab = (a-b)^2$$

Activity 3 : For $a^2 - b^2 = (a+b)(a-b)$

1. Cut out a square ABCD of side a units from a drawing sheet [see Fig. 14].
2. Cut out one square AEFG of side b units ($b < a$) from another drawing sheet [see Fig. 15].
3. Arrange these squares as shown in Fig. 16.
4. Join F to C using sketch pen. Cut out trapeziums congruent to EBCF and GFCD using a transparent sheet and name them as EBCF and GFCD, respectively [see Fig. 17 and Fig. 18]
5. Arrange the cut-outs of trapeziums as shown in Fig. 19.

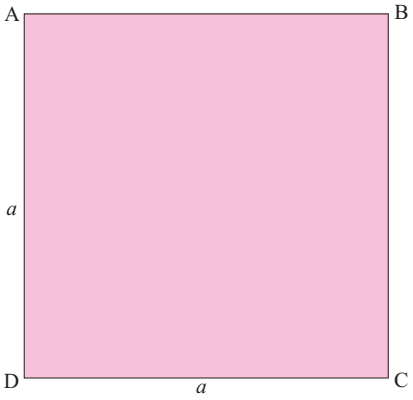


Fig. 14

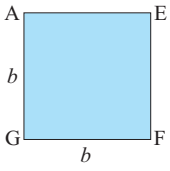


Fig. 15

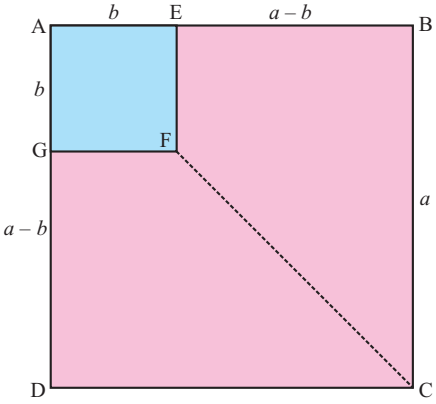


Fig. 16

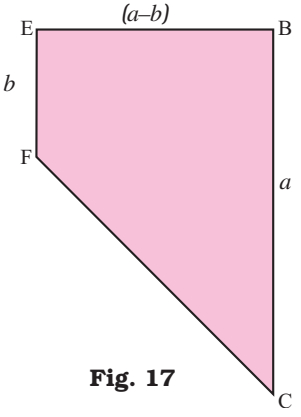


Fig. 17

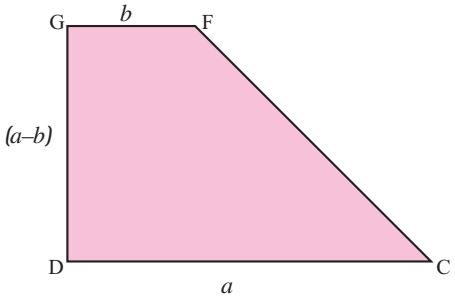


Fig. 18

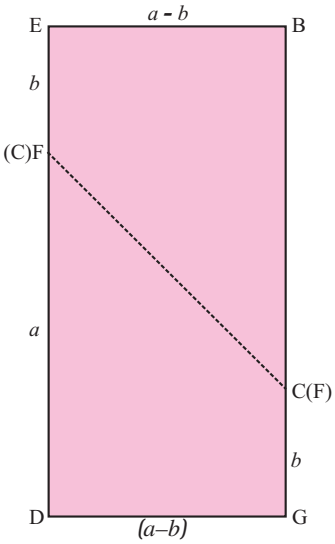


Fig. 19

Area of square ABCD = a^2, Area of square AEFG = b^2

In Fig. 16,

Area of square ABCD – Area of square AEFG

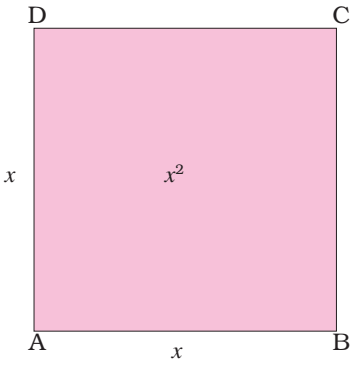
= Area of trapezium EBCF + Area of trapezium GFCD

= Area of rectangle EBGD [Fig. 19]

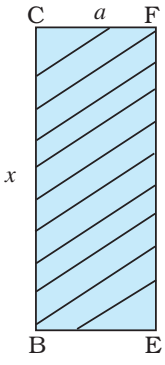
= ED × DG

Thus, $a^2 - b^2 = (a + b)(a - b)$

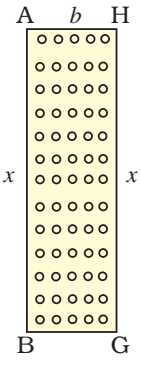
Activity 4 : For $(x+a)(x+b)=x^2+(a+b)x+ab$



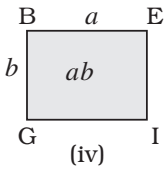
(i)



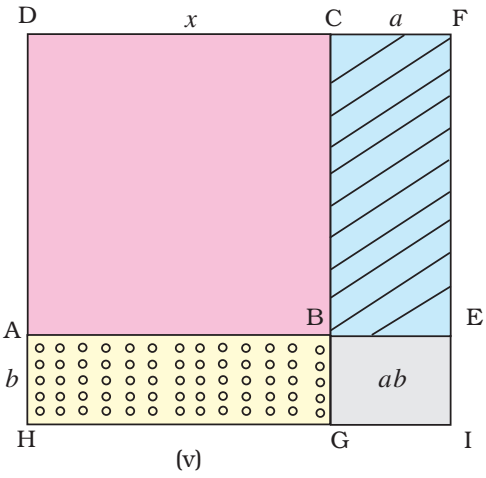
(ii)



(iii)



(iv)



(v)

Fig. 20

1. Cut out a square of length x units from a drawing sheet/cardboard and name it as square ABCD [See Fig. 20 (i)].
2. Cut out a rectangle of length x units and breadth a units from a drawing sheet/cardboard and name it as rectangle BEFC [See Fig. 20 (ii)].
3. Cut out another rectangle of length x units and breadth b units from a drawing sheet/cardboard and name it as rectangle BGHA [See Fig. 20 (iii)].
4. Cut out third rectangle of length a units and breadth b units from a drawing sheet/cardboard and name it as rectangle GIEB [See Fig. 20 (iv)].
5. Join the four quadrilaterals using a sello tape as shown in Fig. 20 (v)
6. Total area of these four cut out figures = Area of square ABCD + Area of rectangle BEFC + Area of rectangle ABGH + Area of rectangle GIEB = $x^2 + ax + bx + ab$

Clearly HIFD is a rectangle with sides $(x + a)$ and $(x + b)$, so its area = $(x+a)\ (x+b)$

Hence,

$$(x + a)\ (x + b) = x^2 + ax + bx + ab$$
$$= x^2 + x\ (a+b) + ab$$

Common Errors

1. $3x^2(2x - x^3) = 6x^3 - x^3$, i.e. some students multiply $3x^2$ with only first term in the bracket.
2. $-2y\,(x - 3y) = -2yx - 6y^2$ in place of $-2yx + 6y^2$
3. $(2x)^2 + 4(2x) + 7 = 2x^2 + 8x + 7$ in place of $4x^2 + 8x + 7$
4. $(2a + 3b)\,(a - b) = 2a^2 - 3b^2$ in place of using the identity $(x+a)(x+b) = x^2 + (a+b) + ab$
5. $(a - 4)\,(a - 2) = a^2 - 8$
6. $(a + b)^2 = a^2 + b^2$
7. $(a - b)^2 = a^2 - b^2$

You may evaluate learning of the concepts through the following exercise.

Exercise

1. Find the product of

   (i) m, $-mn$, and mnp (ii) $\frac{10}{3}pq^3$ and $\frac{-9}{10}p^3q$

   (iii) $3x^2$ and $(ax - by)$ (iv) $-2y$ and $-x^2 + 3x - 2y^2$

2. Find the product of:

   (i) $(2pq + 3q^2)$ and $(3pq - 2q^2)$

   (ii) $(x + y)\,(x^2 - xy + y^2)$

   (iii) $(a + b + c)\,(a + b - c)$

3. Using the appropriate identities, simplify:

   (i) $(2y + 5)\,(2y + 5)$

   (ii) $\left(3a - \frac{1}{2}\right)\left(3a - \frac{1}{2}\right)$

   (iii) $(2a + 5)\,(2a + 9)$

   (iv) $(7a - 9b)\,(7a + 9b)$

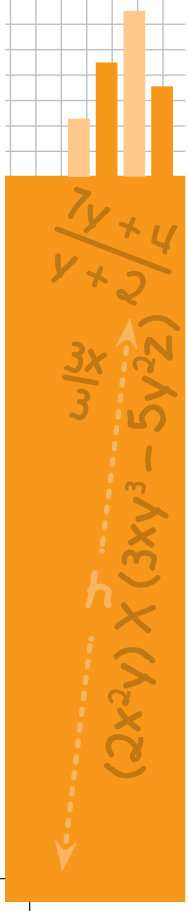
4. Using the identity, find:

(i) $(998)^2$

(ii) $(105)^2$

(iii) 78×82

(iv) 104×106



UNIT 4

Factorisation and Division of Algebraic Expressions

Structure

- Introduction
- Main Concepts and Sub-concepts
- Objectives
- Teaching Points
  1. Factorisation
  2. Division of Algebraic Expressions using Factors
- Common Errors
- Exercise

Introduction

Students are familiar with algebraic expressions and operations of addition, subtraction and multiplication on them. They are also familiar with some standard identities. In this unit, factorisation of algebraic expressions including factorisation using identities will be discussed. Factorisation of algebraic expressions will also be used in the division of polynomials.

Main Concepts and Sub-concepts

- Meaning of factorisation of an expression
- Method of common factors
- Regrouping of terms
- Using identities
- Division of a monomial by a monomial
- Division of a polynomial by a monomial
- Division of a polynomial by a binomial

Objectives

After teaching this unit, the students can

- find factors of a given expression by
  (i) taking common factors;
  (ii) regrouping of terms;
  (iii) using identities;
- Find division of an expression by another expression using factors.

Teaching Points

1. Factorisation

You may introduce the idea of factorisation of an algebraic expression as a reverse process of multiplication of two or

more algebraic expressions. Let the student recall it for whole numbers, i.e. since 7 × 11 = 77, 7 and 11 are factors of 77.

Similarly, since $(x + 1)(x - 1) = x^2 - 1$,

Therefore, $(x + 1)$ and $(x - 1)$ are called factors of $x^2 - 1$.

The process of finding factors of a given expression is called factorisation. Teacher may ask the students to recall factorisation of numbers studied in Class VI.

Teacher may explain different methods of factorisation through some examples as given below:

1.1 Method of Common Factors

Students are very familiar with distributive law of whole numbers.

$$5 \times 11 + 5 \times 17 = 5 \times (11+17)$$

The teacher may explain that the number whose multiplication is being distributed over 11 and 17 is called a common factor of two numbers 5, 11 and 5, 17.

Similarly in algebraic expression

$$5 \times x + 5 \times 2 = 5\,(x+2)$$

the teacher may explain that 5 is the common factor and 5 and $(x+2)$ are the factors of $5x + 5 \times 2$ and $5\ (x+2)$ is the required factorisation.

Expression	Common factor	Factorisation (by distributive law)
$a \times x + a \times y$	a	$a\,(x + y)$
$a \times x^2 + b \times x^2$	x^2	$x^2\,(a + b)$

It is important to note that the common factor may be one or more than one, i.e.

$a \times x \times x \times y + b \times x \times x \times y = x \times x \times y\,(a + b) = x^2y\,(a + b)$

In this expression x, x and y are the common factors.

Also common factor may be an algebraic expression, e.g.

$$5x\,(x+y) + y\,(x+y) = (x+y)\,(5x+y)$$

In this expression $(x+y)$ is the common factor.

1.2 Regrouping of Terms

Sometimes an expression may be of the form

$$ax + by + bx + ay$$

To factorise this, first look for the common factor of each term. In this case, all the terms have no common factor other than 1.

Now regroup the terms as

$$(ax + ay) + (bx + by)$$

$= a\,(x + y) + b\,(x + y)$ [Here we take the binomial $(x+y)$ common]

$= (x + y)\,(a + b)$

Students may also regroup it as

$$(ax + bx) + (ay + by)$$

$= x\,(a + b) + y\,(a + b)$ [Here, we take the binomial $(a+b)$ common]

$= (a + b)\,(x + y)$

You may take one more algebraic expression for factorisation, say $4ax + 4ay + 6bx + 6by$. Observe that 2 is a factor common to each term.

So, $4ax + 4ay + 6bx + 6by = 2(2ax + 2ay + 3bx + 3by)$

$= 2\,[x\,(2a + 3b) + y\,(2a + 3b)]$

$= 2\,[(2a + 3b)\,(x + y)]$

$= 2\,[(2a + 3b)\,(x + y)]$

Alternatively, this can also by done as

$4ax + 4ay + 6bx + 6by = 2\,(2ax + 2ay + 3bx + 3by)$

$= 2\,[2a\,(x+y) + 3b\,(x+y)]$

$= 2\,[(x + y)\,(2a + 3b)]$

(One) 1 is common to all the terms. Sometimes, it is required to take 1 as a common factor, e.g.

$x_2 + xy + x + y = x.x + x.y + 1.x + y.1 = x\,(x + y) + 1(x + y) = (x + y)\,(x + 1)$

Students may be asked to solve questions of Exercise 14.1, of ClassVIII, Mathematics, NCERT.

1.3 Use of Identities

The students have already learnt the following identities:

(i) $(a + b)^2 = a^2+2ab+b^2$

(ii) $(a - b)^2 = a^2 - 2ab + b^2$

(iii) $(a - b)\,(a + b) = a^2 - b^2$

(iv) $(x + a)\,(x + b) = x^2 + (a + b)\,x + ab$

You may explain the use of these identities in factorisation of algebraic expressions through the following examples.

Teacher may emphasise upon identification of problem as observing square terms and with identities (i) and (ii).

Example: Factorise $4x^2 - 20x + 25$

Solution: Here both $4x^2$ and 25 are perfect squares of $2x$ and 5, respectively. Also, middle term $-20x = -2\ (2x)\ (5)$. So, help the students to use identity (ii) of above.

So, $4x^2 - 20x + 25$

$= (2x)^2 - 2 \times (2x) \times 5 + 5^2$

$= (2x-5)^2$, using identity (ii) i.e. $a^2-2ab+b^2 = (a-b)^2$

$= (2x - 5)\,(2x - 5)$ Advise the students to write factors in this form.

Example: Factorise $(2x + y)^2 - 16$

Solution: $(2x + y)^2 - 16$ is of the form $a^2 - b^2$.

So, help the students to use identity (iii).

$(2x + y)^2 - 16 = (2x + y)^2 - 4^2$

$= (2x + y - 4)(2x + y + 4)$, using Identity (iii)

Example: Factorise $x^2 + 4xy + 4y^2 - 4z^2$

Solution: $x^2 + 4xy + 4y^2$ can be expressed as $x^2 + 2 \times x \times 2y + (2y)^2$

$= (x + 2y)^2$, using Identity (i)
i.e., $a^2 + 2ab + b^2 = (a + b)^2$

Now $x2 + 4xy + 4y^2 - 4z^2 = (x+2y)^2 - (2z)^2$

$= (x + 2y - 2z)(x + 2y + 2z)$, using Identity (iii)

Example: Factorise $9x^2 - 4y^2 + 4yz - z^2$

Solution: $9x^2 - (4y^2 - 4yz + z^2)$

$= 9x^2 - [(2y)^2 - 2 \times (2y) \times z + (z)^2]$
[using identity (ii)]

$= (3x)^2 - (2y - z)^2$

$= [(3x) + (2y - z)][(3x) - (2y - z)]$

$= (3x + 2y - z)(3x - 2y + z)$, using Identity (iii)
i.e., $a2 - b2 = (a + b)(a - b)$

Example: Factorise $x^2 - 2x - 63$

Solution: Teacher can ask students to get factors of 63 as

$63 = 1 \times 63$

$= 3 \times 21$

$= 7 \times 9$

But since it is '–63' so both factors should have opposite sign. Again since middle term has negative sign, so, larger factor amongst two should have negative sign.

But on the same side $a + b = -2$

So, 7 and –9 can work (3 and –21 cannot work, 1 and –63 cannot work)

Likewise other cases should be discussed.

Here – 63 is not a perfect square. So, let us try to use identity (iv). Comparing the expression $x^2 - 2x - 63$ with $x^2 + (a + b)x + ab$, we get

$a + b = -2$ and $ab = -63$

$-63 = -3 \times 21 = -9 \times 7$

So choose $a = -9$, $b = 7$ so that $a + b = -2$, $ab = -63$

Therefore, $x^2 - 2x - 63 = x^2 + (-9 + 7)x + (-9)(7)$

$= [x + (-9)](x + 7) = (x - 9)(x + 7)$, using Identity (iv)

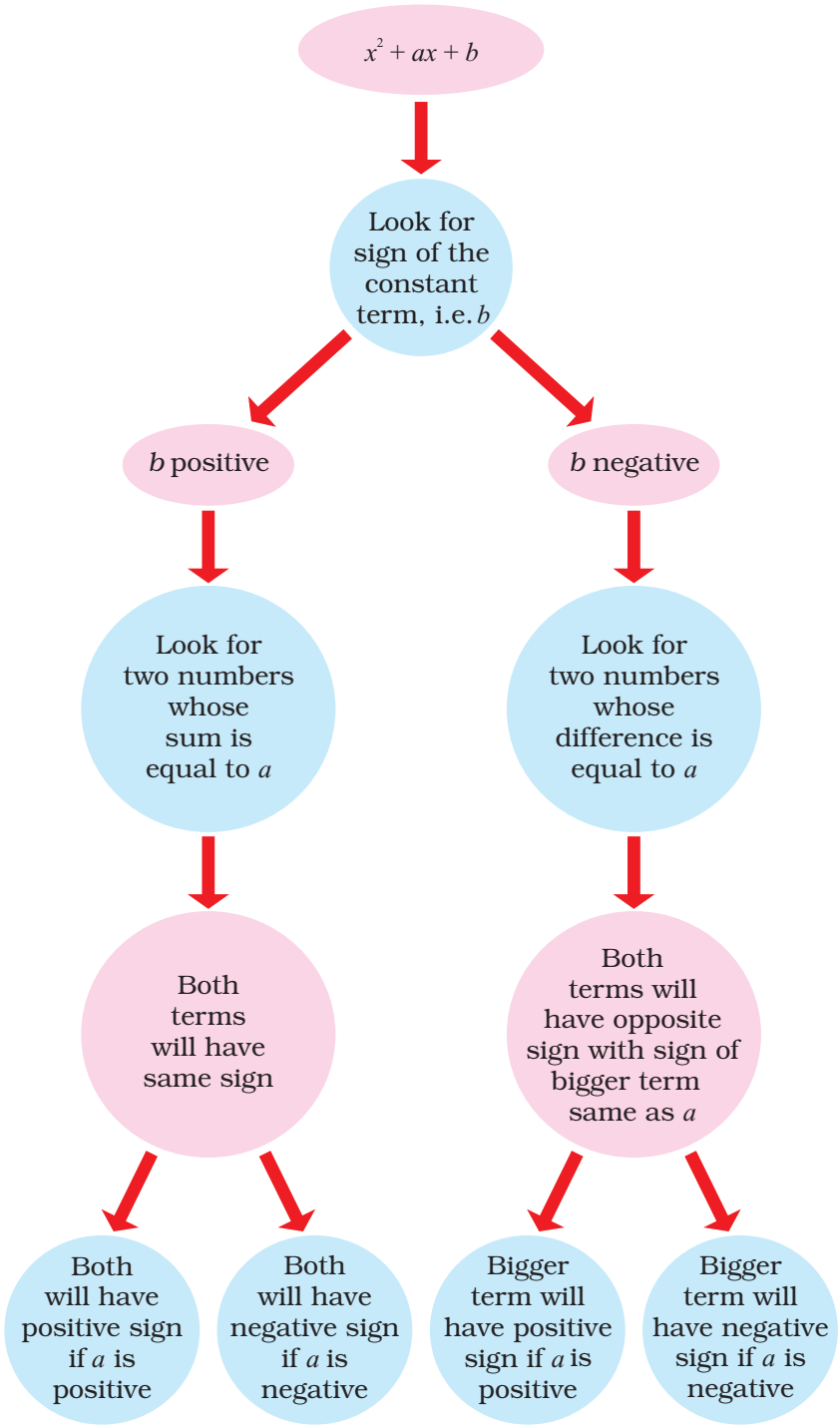
The given expression can also be factorised by splitting the middle term $(-2x)$ as follows:

$$x^2 - 2x - 63 = x^2 - 9x + 7x - 63$$
$$[-9x + 7x = -2x, -9 \times 7 = -63]$$
$$= x(x - 9) + 7(x - 9)$$
$$= (x - 9)(x + 7)$$

The coefficient of middle term must be splitted in two terms such that their product is equal to the product of coefficient of x^2 and the constant term, i.e., $(-9 + 7) = -2$ and $-9 \times 7 = 1 \times (-63) = -63$.

Teacher may further reinforce the concept of splitting the middle term by taking different examples as given on page 222-223 of Class VIII Mathematics, NCERT.

You may help the students to do Exercise 14.2 on page 223-224 of Class VIII Mathematics, NCERT.


$x^2 + ax + b$
Look for sign of the constant term, i.e. b
b positive
b negative
Look for two numbers whose sum is equal to a
Look for two numbers whose difference is equal to a
Both terms will have same sign
Both terms will have opposite sign with sign of bigger term same as a
Both will have positive sign if a is positive
Both will have negative sign if a is negative
Bigger term will have positive sign if a is positive
Bigger term will have negative sign if a is negative

2. DIVISION OF ALGEBRAIC EXPRESSIONS USING FACTORS

Multiplication of algebraic expressions has been discussed in earlier unit. Division of algebraic expression has been discussed in Class VIII Mathematics Textbook in a limited sense i.e., division of an algebraic expression using factors only.

This process may be explained to the students in phased manner as given below:

2.1 Division of monomial by another monomial

Suppose you have to divide $32x^2y^3z$ by $4xyz$. Then, write

$32x^2y^3z = 4 \times 8 \times x \times x \times y \times y \times y \times z$

and $4xyz = 4 \times x \times y \times z$

Now, each expression has been factorised
Therefore, $32x^2y^3z \div 4xyz =$

$$\frac{32x^2y^2z}{4xyz} = \frac{4\times 8\times x\times x\times y\times y\times y\times z}{4\times x\times y\times z} = 8xy^2$$

2.2 Division of polynomial by a monomial

Suppose you want to divide $3x^2y + 6xy$ by $3y$. Using factorisation, you can write $3x^2y + 6xy$ as $3xy\,(x + 2)$. Dividing it by $3y$, you get $x(x + 2)$ as the quotient. The same process can also be done as follows:

$$(3x^2y+6xy)\div 3y = \frac{3x^2y+6xy}{3y} = \frac{3x^2y}{3y} + \frac{6xy}{3y} = x^2+2x$$

2.3 Division of polynomial by a binomial

Teacher may explain the process of division of a polynomial by a binomial by using factors through the following examples:

Example: Divide $x^2 - 3x - 54$ by $x + 6$

Solution: Help the students to compare $x^2 - 3x - 54$ with $x^2 + (a + b)\,x + ab$ and get

$a + b = -3$, $ab = -54 = -9 \times 6$. Thus, $a = 6$, $b = -9$

So, $x^2 - 3x - 54 = x^2 - 9x + 6x - 54$

$= x(x-9) + 6(x-9)$

$= (x - 9)(x + 6)$

Now $(x^2 - 9x + 6x - 54) \div (x + 6) = (x + 6)(x - 9) \div (x + 6)$

$= \dfrac{(x+6)(x-9)}{(x-6)} = x - 9$, the quotient

Note: You can also use identity (iv) to find factors of $x^2 - 3x - 54$ as $(x + 6)(x - 9)$.

Example: Divide $x^4 - 81$ by $x + 3$

Solution: $x^4 - 81$

$= (x^2)^2 - 9^2$

$= (x^2 - 9)(x^2 + 9)$, using Identity (iii),

i.e., $a^2 - b^2 = (a - b)(a + b)$

$= (x^2 - 3^2)(x^2 + 9)$

$= (x + 3)(x - 3)(x^2 + 9)$, using identity (iii) again

So, $(x^4 - 81) \div (x + 3) = \dfrac{(x^2+9)(x-3)(x+3)}{(x+3)} =$

$(x - 3)(x^2 + 9)$, the quotient

Example: Divide $4x^2 - 36$ by $4x - 12$

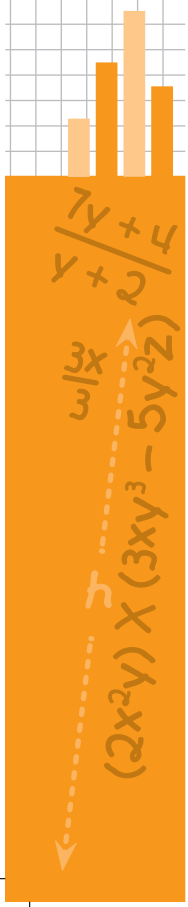
Solution: $4x^2 - 36 = 4(x^2 - 9) = 4(x^2 - 3^2)$

$= 4(x - 3)(x + 3)$, using identity (iii)

Also $4x - 12 = 4(x - 3)$

So, $(4x^2 - 36) \div (4x - 12) = \dfrac{4(x-3)(x+3)}{4(x-3)} = x + 3$, the quotient.

Students may be asked to do Exercise 14.3 on page 227 of Class VIII, Mathematics, NCERT.



Application of factorisation

A teacher can give area of rectangle and then ask the students to find length and breadth of that rectangle.

(This question has been given in different manner but it literally demands factorisation)

e.g. Suppose Area of Rectangle = $x^2 + 16x + 63$

To find: Length and breadth

$(x^2 + 16x + 63) = x^2 + (9+7)x + (9)(7)$

$= (x + 9)(x + 7)$

Since $[(x + a)(x + b) = x^2 + (a+b)x + ab]$

So length = $x + 9$ and breadth = $x + 7$.

Common Errors

1. Some students may write

   $\frac{4x^2-36}{4x-12} = \frac{4x^2}{4x} + \frac{-36}{-12} = x+3$. Although, answer is correct but method is incorrect.

2. One may write $3x^2 + 6x + 4 = (3x + 2)^2$, which is not correct.

3. Sometimes students write $\frac{3x}{4x+5} = \frac{3}{4+5} = \frac{1}{3}$ which is wrong.

4. Sometimes students write $a^2 + 8 = (a + 4)(a + 2)$, which is wrong.

You may evaluate learning of the concepts through the following exercise:

Exercise

1. Factorise the following expressions:

   (i) $14pq + 35pqr$ (ii) $15a^2 + 20b^2$

   (iii) $z - 7 + 7xy - xyz$ (iv) $121b^2 - 88bc + 16c^2$

   (v) $x^2 - 2xy + y^2 - z^2$ (vi) $10ab + 4a + 5b + 2$

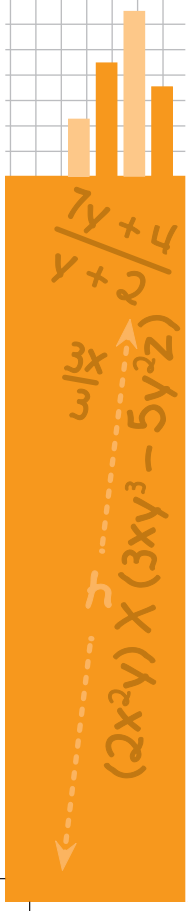
   (vii) $x^2 - y + 2yz - z^2$

2. (i) Divide $12a^8b^8$ by $-6a^6b^4$

   (ii) Divide $x^3 + 2x^2 + 3x$ by $2x$

   (iii) Divide $12xy\ (9x^2 - 16y^2)$ by $4xy\ (3x + 4y)$

   (iv) Divide $39y^3\ (50y^2 - 98)$ by $26y^2(5y + 7)$



Block

Teaching of Ratio and Proportion

Ratio and proportional reasoning is at the heart of mathematics in the upper primary stage. In principle, proportional reasoning deals with mathematical relations, which are multiplicative in nature, in contrast to additive mathematical relations that are typical for many young children in elementary schools. The ability to reason proportionally develops in students throughout Classes VI–VIII. It is of such great importance that it merits whatever time and effort that must be spent to assure its careful development. Therefore, the topics of ratio and proportion have central part in mathematics curriculum for children in school as well as for pre-service mathematics teacher education. So building an intuitive understanding of the meaning of ratio and proportionality must be emphasised.

In fact, two quantities of the same kind can be compared with respect to magnitude either by their **difference** or by **division.** The latter gives the concept of a **ratio** which is very helpful in solving problems related to everyday life. Other **concepts** related to ratio are proportion, unitary method, direct and inverse proportions.

The concept of **per cent** is also related to a ratio. The concept of per cent is useful in the problems on profit

and loss, discounts, simple and compound interest, etc.

This block contains three units.

Unit 1: Introduction to Ratio and Proportion

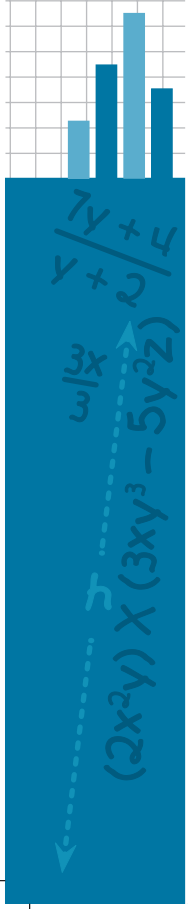
Unit 2: Comparing Quantities

Unit 3: Simple and Compound Interest

Unit 1 introduces the concepts of ratio and proportion, unitary method, direct and inverse proportions.

Unit 2 deals with percentages and applications of percentages to the problems of profit and loss, discount, etc.

Unit 3 introduces the idea of simple and compound interest, applications of compound interest formula to the problems of appreciations and depreciations.



UNIT 1

Introduction to Ratio and Proportion

Structure

- Introduction
- Main Concepts and Sub-concepts
- Objectives
- Teaching Points
  1. Ratio
  2. Equivalent Ratio
  3. Proportion
  4. Unitary Method
  5. Direct Proportion
  6. Inverse Proportion
- Common Errors
- Exercise

Introduction

In the primary classes, students have already learnt to convert metres into centimetres, kilograms into grams and litres into millilitres and vice versa. In the block on Number System, besides numbers and operations on them, fractions and their comparison have also been discussed.

In daily life situations, sometimes it is required to compare two or more quantities. For that purpose, the concepts of ratios and equivalent ratios are needed. This unit will start with the discussion on ratios and equivalent ratios. The concepts closely related with ratios such as proportion, unitary method, direct and inverse proportions will also be discussed.

Main Concepts and Sub-concepts

- Comparison of two quantities
- Ratios
- Equivalent Ratios
- Proportion
- Unitary Method
- Direct Proportion
- Inverse Proportion

Objectives

After teaching these concepts and sub concepts, students can:

- express some daily life situations in the form of ratios and vice versa;
- compare two given ratios and write equivalent ratios;
- check whether given four numbers are in proportion or not;
- solve daily life problems using unitary method;

- identify whether two given quantities vary in direct or inverse proportion; and
- solve daily life problems based on direct and inverse proportions.

Teaching Points

1. Ratio

The concept of ratio should be explained by taking some examples from the child's immediate environment. For example, a farmer produces 50 quintals of wheat and 30 quintals of rice in his field. Teacher may now ask the students: "In what way can the two quantities be compared?"

One of the responses from them would be that production of wheat is more than production of rice. Other response could be that production of wheat is 50–30, i.e. 20 quintals more than that of rice. This is known as comparison by difference.

Some students may come forward with a response that production of wheat is $\frac{50}{30}$, i.e. $\frac{5}{3}$ times the production of rice. This is known as comparison by division. You may now explain the two ways of comparison of quantities: (i) by difference (ii) by division. The comparison of two quantities by division is called a **ratio.** We use the symbol ':' to express a ratio.

This ratio is expressed as $\frac{5}{3}$ or 5 : 3 and is read as 5 is to 3. It should be emphasised that the ratio 5 : 3 is not the same as the ratio 3 : 5 as $\frac{5}{3}$ is not the same as $\frac{3}{5}$.

While explaining the concept of ratio, it should be explained that the ratio of similar quantities can be taken only when they are in same units. It can be explained by taking examples of the type:

(i) The height of a flower plant is 70 cm and that of a mango tree is 15 m. Then the ratio of their heights will not be 70 : 15 as their units are different. They have to be first converted into the same units, say in centimetres and then the ratio would be found. Thus, required ratio is

$$70 : 1500 = \frac{70}{1500} = \frac{7}{150} = 7 : 150 \text{ [as 1 m = 100 cm]}$$

(ii) The height of a person is 1.75 m and his weight is 75 kg. Can we find the ratio of his height and weight? The ratio cannot be found since the units of height and weight are different and they cannot be converted into the same unit. But in case of proportion, we can take the ratio in different quantities as per context.

(iii) It may be explained to students that ratio is always a number and it does not have any unit.

2. EQUIVALENT RATIOS

The students are already familiar with the comparison of two fractions. Since the ratio of two quantities can be represented by a fraction, they can also be compared like fractions. The teacher may explain this process through some examples as given below:

Example: Compare the following ratios:

(i) 2 : 3 and 3 : 4

(ii) 6 : 8 and 9 : 12

Solution: (i) $2 : 3 = \frac{2}{3}$ and $3 : 4 = \frac{3}{4}$

To compare the ratios, we compare $\frac{2}{3}$ and $\frac{3}{4}$

$$2 : 3 = \frac{2}{3} = \frac{2\times4}{3\times4} = \frac{8}{12} \quad \text{[LCM of 3 and 4 is 12]}$$

$$3 : 4 = \frac{3}{4} = \frac{3\times3}{4\times3} = \frac{9}{12}$$

Since $\frac{8}{12} < \frac{9}{12}$, therefore $2 : 3 < 3 : 4$

(ii) $6 : 8 = \frac{6}{8} = \frac{3}{4}$, $9 : 12 = \frac{9}{12} = \frac{3}{4}$

Since $\frac{3}{4} = \frac{3}{4}$, therefore $6 : 8 = 9 : 12$

Ratios such as 6 : 8 and 9 : 12 are known as equivalent (or equal) ratios.

Example: Write two equivalent ratios of 5 : 7.

Solution: $5 : 7 = \frac{5}{7} = \frac{5\times 2}{7\times 2} = \frac{10}{14} = 10 : 14$

again, $5 : 7 = \frac{5}{7} = \frac{5\times 3}{7\times 3} = \frac{15}{21} = 15 : 21$

Therefore, two equivalent ratios of the ratio 5 : 7 are 10 : 14 and 15 : 21.

It should be explained to the students that we can write many equivalent ratios of a given ratio which can be obtained by multiplying (or dividing) its numerator and denominator by a positive number.

3. PROPORTION

The concept of proportion may be introduced through the situations as given on Page 252 of Class VI Mathematics, NCERT.

If two ratios $a : b$ and $c : d$ are equal, we say that the numbers a, b, c, d taken in this order are in proportion. We use the symbol :: or '=' to equate two ratios $a : b$ and $c : d$. Thus, $a : b = c : d$ or $a : b :: c : d$ and is read as a is to b as c is to d. The terms a and d are called **extreme terms** and b and c are called **middle terms**.

Since $a : b = c : d \Rightarrow a \times d = b \times c$, we say that product of extreme terms is equal to the product of middle terms.

Teacher may explain this concept through examples.

Example: Do 30 mL : 25 mL and 48 kg : 40 kg form a proportion?

Solution: 30 mL : 25 mL = $\frac{30}{25}=\frac{6}{5}=6:5$

and 48 kg : 40 kg = $\frac{48}{40}=\frac{6}{5}=6:5$

Therefore, the ratios 30 mL : 25 mL and 48 kg : 40 kg form a proportion.

Teacher may give one example in which two given ratios do not form a proportion as given on page 255 of Class VI Mathematics, NCERT.

Example: Find the value of x so that 5, x, 2, 8 are in proportion.

Solution: Given that 5, x, 2, 8 are in proportion.

Therefore, 5 : x = 2 : 8

or $\frac{5}{x}=\frac{2}{8}$

or $\frac{5\times 8}{2} = x$ or $x = 20$

4. UNITARY METHOD

In unitary method, we first find the value of one unit and then the value of the required number of units. The students may be explained the meaning of unitary method through some situations and examples as given on page 256 of Class VI, Mathematics, NCERT.

Some more examples are given below:

Example: The cost of 5 m cloth is Rs 175. Find the cost of 8 m cloth.

Solution: Since, cost of 5 m cloth is Rs 175

Therefore, cost of 1m cloth $\frac{175}{5}$ = Rs 35

and cost of 8 m cloth is Rs 35 × 8 = Rs 280

Thus, cost of 8 m cloth is Rs 280.

Explanation: Students should be explained that if the cost of 5m cloth is Rs 175, then the cost of 1m cloth will be less. Hence to find the cost of 1m of cloth, we divide Rs 175 by 5. Once we know the cost of 1m cloth, the cost of 8 m cloth will be 8 times the cost of 1 m cloth. Hence, we multiply it by 8.

Example: A car takes 3 hours to reach a destination by travelling at a speed of 40 km/h. How much time will it take when the speed of car is 60 km/h?

Solution: When speed of car is 40 km/h, time taken = 3 hours.

Therefore, when speed of car is 1km/h, time taken = $40 \times 3 = 120$ hours.

Therefore, when speed of car is 60 km/h time taken = $\frac{120}{60}$

= 2 hours.

Hence, time taken to reach the destination is 2 hours.

Explanation: It should be explained to the students that when speed reduces to 1km/h from 40 km/h, the time taken to cover same distance will be more. That is why, we multiply by 40. Again, when speed increases from 1km/h to 60km/h, the time taken to reach the destination will be less. Therefore, we divide by 60. The teacher may highlight the above two "explanations" in different situations involving unitary method. Section 13.3 of Class VIII, Mathematics, NCERT explains more such examples. More over this example can be taken after doing Exercise 13.2, Class VIII, Mathematics, NCERT to give an idea that problems based on inverse proportion can be solved using unitary method.

While discussing unitary method, we have seen how the change in one quantity affects the change in the other quantity. We come across three types of situations in our daily life.

(i) As one quantity increases (or decreases), the other quantity also increases (or decreases) in the same ratio. Here the two quantities are said to be in direct proportion. For example, number of books and its cost, volume of Petrol and distance travelled, etc.

(ii) As one quantity increases (or decreases), the other quantity decreases (or increases) in the same ratio. Here the two quantities are said to be in inverse proportion. For example, speed and time, number of days and number of workers, etc.

(iii) As one quantity changes, the other quantity also changes, however the ratio in change may not be the same. For example, if the side of a square is 2 cm then its area will be $(\text{side})^2 = 2^2 = 4$ cm^2. If its side changes to 3 cm then its area will be 3^2, i.e. 9 cm^2. But here the sides change in the ratio 2:3 and the area changes in the ratio 4:9. So we cannot use the unitary method in such cases.

Many such situations have been discussed on page 202 Class VIII Mathematics, NCERT. The students should be encouraged to discuss among themselves similar situations from their environment.

5. DIRECT PROPORTION

Teacher may introduce the concepts of direct proportion by taking some examples from the surroundings as given in section 13.1 of Class VIII Mathematics, NCERT. With the help of section 13.2 of Class VIII Mathematics, NCERT concept of direct proportion can be explained.

Let us suppose that the cost of one onion, capsicum, mushroom is ₹ 70. Then, the cost of 2 pizzas will be ₹ 140 and the cost of 3 pizzas will be ₹ 210 and so on. Let us represent this information in the form of a table.

No. of pizzas	1	2	3	4	5	6	7
Cost in Rs.	70	140	210	280	350	420	490
Ratio of no. of pizzas and cost	$\frac{1}{70}$	$\frac{2}{140}$	$\frac{3}{210}$	$\frac{4}{280}$	$\frac{5}{350}$	$\frac{6}{420}$	$\frac{7}{490}$
Above ratio when expressed in its lowest term	$\frac{1}{70}$	$\frac{1}{70}$	$\frac{1}{70}$	$\frac{1}{70}$	$\frac{1}{70}$	$\frac{1}{70}$	$\frac{1}{70}$

We notice that when the number of pizzas increases (decreases) their total cost increases (decreases), i.e. the quantities are varying but in such a manner that the ratio always remains constant. Such a variation is called the **direct variation**. The constant ratio, i.e. $\frac{1}{70}$ in this case is called the constant of variation. The symbol used to denote variation is 'α'. When a and b vary directly with each other we can express this fact in mathematical language as $a \alpha b$. If k is the constant of variation then $\frac{a}{b} = k$ or $a = kb$. If a and b vary directly to each other, then the ratio of any two values of a is same as the ratio of corresponding two values of b.

In the above example, ratio of number of pizzas is $\frac{a_1}{a_2} = \frac{1}{3}$.

Cost in Rs = $\frac{b_1}{b_2} = \frac{70}{210} = \frac{1}{3}$

In other words, when x and y are in direct proportion, then $\frac{x_1}{y_1} = \frac{x_2}{y_2}$ where y_1, y_2 are values of y corresponding to the values x_1, x_2 of x, respectively.

Activity under the heading 'Do This' on page 203 of Class VIII Mathematics, NCERT could be a good example of direct proportion.

Teacher may also perform the following activity using few counters or bottle caps or pebbles.

Suppose teacher wants to arrange 5 counters in a row. So as the number of rows increases, the number of counter increases in the same proportion. Teacher may ask a student to record observations in a table drawn on the blackboard. Remaining students may draw inferences from this (as given on page 203 under 'Do This' activity).

Number of counters	5	10	.....	.....	.....	.....
Number of rows	1	2	.....	.....	.....	.....

It may also be noted that this type of activity can be performed to demonstrate the concept of inverse proportion as shown on page 211 under 'Do This' in Class VIII Mathematics, NCERT.

In solving the following examples, the above relation has been used.

Example: The cost of 3 kg sugar is ₹ 75.

(i) Find the cost of 5 kg sugar.
(ii) How much sugar can be purchased for ₹ 225?

Solution: Let the cost of 5 kg sugar be ₹ y_1 and x_2 kg sugar can be purchased for ₹ 225. We make the following table:

Weight of sugar (in kg) x_1	3	5	x_2
Cost of sugar (in Rupees) y_1	75	y_1	225

(i) As the amount of sugar increases, its cost also increases in the same ratio. Therefore, this is a case of direct proportion. Therefore,

$$\frac{3}{75}=\frac{5}{y_1} \text{ or } 3y_1 = 5 \times 75$$

or $y_1 = \frac{5\times75}{3} = 125$

Hence, cost of 5 kg sugar is Rs 125.

(ii) Again $\frac{3}{75}=\frac{x_2}{225}$ or $x_2=\frac{3\times225}{75}=9$

Therefore, 9 kg sugar can be purchased for Rs 225.

Example: Check if two quantities x and y are for direct proportion:

x	18	12	30	25
y	3	2	5	4

Solution:

Here $\frac{x_1}{y_1}=\frac{18}{3}=6$

$\frac{x_2}{y_2}=\frac{12}{2}=6$

$\frac{x_3}{y_3}=\frac{30}{5}=6$

$\frac{x_4}{y_4}=\frac{25}{4}\neq 6$

So, x and y are not in direct proportion.

6. INVERSE PROPORTION

Many situations of inverse proportion have been discussed in Class VIII Mathematics, NCERT. The students may be encouraged to discuss similar situations from their environment. In one of the above examples (given in Unitary Method) of 'speed of car and the time taken' the values are

in inverse proportion. Through such examples, it should be explained that two quantities x and y are said to be in inverse proportion or to vary in inverse proportion, if an increase in x causes a decrease in y and vice versa in such a manner that the product of their corresponding values remains constant. That is if $xy = k$, then x and y are said to vary inversely. In this case if y_1 and y_2 are the values of y corresponding to the values x_1 and x_2 of x, respectively, then $x_1 y_1 = x_2 y_2$.

Let us suppose that we have 10 pizzas. We wish to distribute these equally among a certain number of children. If the number of children is 10, obviously each child will get one pizza. If there are 5 children then each child will get 2 pizzas. If there are 20 children, each child will get $\frac{1}{2}$ pizza. Let us write this in tabular form.

No. of children	10	5	20	2	40	80
No. of pizzas each gets	1	2	$\frac{1}{2}$	5	$\frac{1}{4}$	$\frac{1}{8}$
xy	10	10	10	10	10	10

From the above table, it is clear that as the number of children increases the number of pizzas per child decreases, whereas when the number of children decreases the number of pizzas per child increases. However, the product of x and y, being always $xy = 10$ remains constant.

Constant of variation: This constant product $xy = 10$ is the constant of variation for the inverse variation.

In solving the following examples, the above relation has been used.

Example: Eight persons can do a work in 20 days. In how many days can 10 persons do the same work?

Solution: As the number of persons increases, the time taken to complete the work will decrease in the same ratio. Hence, it is a case of inverse proportion. Let the number of days required to complete the work be y. The situation can be displayed in the form of the following table:

Number of persons	8	10
Number of days	20	y

Now, since it is a case of inverse proportion, therefore

$$8 \times 20 = 10 \times y$$

or $$y = \frac{8 \times 20}{10} = 16$$

Thus, the work will be completed in 16 days.

Example: If x and y are in inverse proportion, complete the following table:

x	8	3	4	—
y	6	—	—	8

Solution: Let the entries in the table be as shown below:

x	8	3	4	x_3
y	6	y_1	y_2	8

Since, x and y are in inverse proportion, therefore, we have

$$8 \times 6 = 3 \times y_1 = 4 \times y_2 = 8 \times x_3$$

Thus, $y_1 = \frac{8 \times 6}{3} = 16$, $y_2 = \frac{8 \times 6}{4} = 12$, $x_3 = \frac{8 \times 6}{8} = 6$

Hence, the required table is:

x	8	3	4	6
y	6	16	12	8

We can therefore summarise that

(i) Two quantities 'a' and 'b' are said to vary directly, if the ratio $\frac{a}{b}$ remains constant.

(ii) Two quantities 'a' and 'b' are said to vary inversely, if the product 'ab' remains constant.

Common Errors

(i) Ratio of 50 cm and 2m is taken as 50 : 2 in place of 50 : 200.

(ii) Ratio of 10 mL and 5 kg is taken as 10 : 5, which has no meaning.

(iii) Ratio of 5 kg and 10 kg is taken as 10 : 5 in place of 5 : 10.

(iv) If 5 persons can do a work in 3 days, then 1 person will do the same work in $\frac{3}{5}$ days in place of 5 × 3 = 15 days.

(v) If two quantities x and y are in direct proportion, then $xy = k$ is taken in place of $\frac{x}{y} = k$.

(vi) If two quantities x and y are in inverse proportion, then $\frac{x}{y} = k$ is taken in place of $xy = k$.

You may evaluate the students through the following exercise:

Exercise

1. Find the ratio of the following:
   (a) 20 minutes to 2 hours
   (b) 25 km to 30 km
2. Is it possible to find the ratio of 20 km/h to 30 km/h?
   (a) If yes find the ratio.
   (b) If no, give reason.
3. In a college, out of 4000 students, 2500 are girls. Find the ratio of
   (a) number of girls to the number of students.
   (b) number of boys to the number of girls.
4. Determine if the following ratios form a proportion:
   (a) 25 cm : 1 m and ₹ 40 : ₹ 160
   (b) 2 kg : 80 kg and 25 g : 625 g
5. The weight of 72 books is 9 kg. What is the weight of 40 such books?
6. Cost of 5 kg wheat is ₹ 80. What quantity of wheat can be purchased for ₹ 65?
7. A truck travels 14 km in 25 minutes. If the speed remains the same, how far can it travel in 5 hours?
8. A machine in a soft drink factory fills 840 bottles in 6 hours. How many bottles will it fill in
   (a) 5 hours? (b) 3 hours?
9. If 15 workers can build a wall in 48 hours, how many workers will be required to do the same work in 30 hours?
10. A farmer has enough food to feed 90 animals in his cattle for 6 days. How long would the food last, if there were 10 more animals in his cattle?

Time and distance problems

Special Cases

Example: A train 300m long is running at a speed of 60 km/h. How long will it take to cross a tree?

Solution: When a train has to cross a stationary object whose length is negligible, e.g tree, pole, etc. it has to cover a distance equal to its own length.

$$d = 300 \text{ m} = \frac{300}{1000}\text{km} = \frac{3}{10}\text{km}$$

speed = 60 km/hr.

$$\text{Time} = \frac{\text{distance}}{\text{speed}} = \frac{3/10}{60}\text{hr} = \frac{3}{10}\text{min}$$

Example: The speed of train 150m long is 50 km/h How much time will it take to pass a platform 750 m long?

Solution: When a train has to cross a stationary object with some length of its own, then the distance, which the moving train has to cover is equal to the sum of the lengths of the train itself and the stationary object.

d = 750m +150m = 900m.

$$= \frac{900}{1000}\text{km} = \frac{9}{10}\text{km}$$

$$\text{Time} = \frac{9/10}{50}\text{h}\,\frac{9}{500} \times 60\,\text{min} = \frac{27}{25}\text{min}$$

Example: Two trains 100m and 120m long are moving at the speed of 34 km/h and 32 km/h respectively in the opposite directions. How long would it take them to pass each other.

Solution: When two trains are moving in the opposite directions then the speed is taken as the sum of the speeds of the two trains (why?). Distance covered is the sum of the lengths of the two trains.

$$d = (120+100)\ m = 220\ m = \frac{220}{100} km$$

New speed = (34 + 32) km/h = 66km/h

$$\text{Time} = \frac{220}{66\times1000} h = \frac{220\times60}{1000\times66} min = \frac{1}{5} min = 12 \text{ seconds}$$

Note: The speed found in this case is called the relative speed.

Example: Two trains 300 m and 450 m long are going in the same direction at the speed of 20 km/h and 15km/h on parallel tracks respectively. The shorter train is behind the longer train. How long would they take to pass each other?

Solution: Here resultant speed is the difference of their speeds and d = Sum of the lengths of the trains.

2 UNIT

Comparing Quantities

Structure

- Introduction
- Main Concepts and Sub-concepts
- Objectives
- Teaching Points
  1. Meaning of percentage
  2. Converting fractions and decimals into percentage and vice versa
  3. Application of percentage
  4. Profit and Loss
  5. Discount
  6. Sales tax/VAT
- Common Errors
- Exercise

Introduction

In Unit 1 'Introduction to Ratio and Proportion', Unitary Method and its use in solving daily life problems have been discussed. Also, ratio has been discussed as a method of comparison by division. One of the very common methods to compare two quantities is by using percentages. In this unit, the meaning of percentage, converting fractional and decimal numbers into percentages and vice versa will be discussed. Using percentage and unitary method, many real life problems related to profit and loss, discount and sales tax/VAT will also be discussed.

Main Concepts and Sub-concepts

- Per cent
- Profit and Loss
- Discount
- Sales Tax and VAT

Objectives

After teaching these concepts and sub concepts, students will be able to

- understand the meaning of per cent;
- convert a fraction into percentage and vice versa;
- convert a decimal into percentage and vice versa; and
- solve problems on percentage, profit and loss, discount, sales tax/VAT.

Teaching Points

1. Meaning of Percentage

Before introducing the concept of a per cent, the teacher may ask the students to recall what they have studied about fractions and decimals. Then she can introduce

percentage as a fraction whose denominator is 100.

Teacher may use a square grid to recall previous knowledge as given below:

Here students can easily answer that the shaded portion of this grid represents $\frac{46}{100}$ or $\frac{23}{50}$. Decimal representation of this portion is 0.46 [Fig. 1(i)].

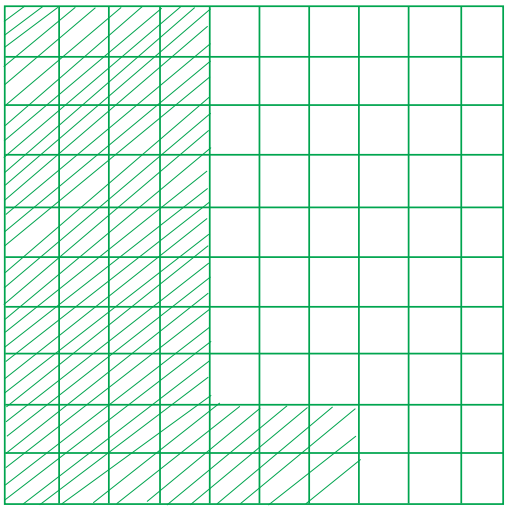


Fig. 1(i)

Here teacher can point out that, this can also be written as 46%. Hence a relationship between fractions, decimals and percentage can be established.

Further 5 × 10 square grid or 5 × 5 square grid can be used to establish a relationship between fractions, decimals and percentage.

For example

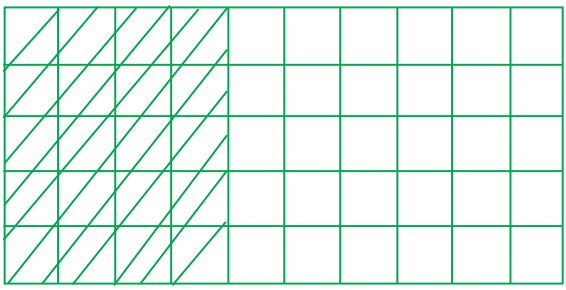


Fig. 1(ii)

Here in Fig 1(ii) shaded portion of the grid = $\frac{20}{50}$, which is equal to $\frac{40}{100}$ or 0.40 or 0.4 or 40%.

This concept and its need has been explained in the textbook on page 157 of Class VII Mathematics, NCERT.

The word per cent is derived from Latin word 'per centum' meaning per one hundred. Per cent is denoted by the symbol '%' meaning hundredths i.e. per hundred. You may supplement it by taking one example from 'Try these' in the concerned portion of Class VII Mathematics, NCERT.

Example: Mala has a collection of gold and silver bangles. She has 20 gold and 10 silver bangles. What is the percentage of bangles of each type?

Solution: Mala has a total of 20 + 10 = 30 bangles. Out of 30 bangles, she has 20 gold bangles. Thus, out of 1 bangle, Mala has $\frac{20}{30}$ gold bangles. Therefore, out of 100 bangles, Mala has $\frac{20}{30} \times 100 = \frac{200}{3}$ gold bangles.

Hence, percentage of gold bangles = $\frac{200}{3}\% = 66\frac{2}{3}\%$.

Again, out of 30 bangles, Mala has 10 silver bangles. Therefore, out of 100 bangles, Mala has $\frac{10}{30} \times 100$ silver bangles. Hence, percentage of silver bangles = $\frac{100}{3}\% = 33\frac{1}{3}\%$.

Note: Here teacher can make students realise that sum of all components will be 100%.

2. CONVERTING FRACTIONS AND DECIMALS INTO PERCENTAGES AND VICE VERSA

Process of converting fractions and decimals into percentages and vice versa may be explained through examples as given below.

To convert $\frac{2}{3}$ into percent, we write

$$\frac{2}{3} = \frac{2}{3} \times \frac{100}{100} = \frac{200}{3} \times \frac{1}{100}$$

Therefore, $\frac{2}{3}$ is $\left(\frac{200}{3}\right)^{th}$ part of 100.

Hence, $\frac{2}{3} = \frac{200}{3}\% = 66\frac{2}{3}\%$.

In short, to convert a fraction into percentage, we multiply the fraction by 100 and express it as a per cent. Thus,

$$\frac{2}{5} = \left(\frac{2}{5} \times 100\right)\% = 40\%$$

To convert a decimal number into per cent, we can either convert it into fraction and find its percentage or simply multiply the decimal number by 100 and express it as a per cent. Thus, 0.231 = (0.231 × 100)% = 23.1%.

To convert a percent into a fraction or a decimal, we just divide it by 100 and write the result in terms of a fraction or a decimal removing the % symbol. This may be explained through the following examples:

Example: Convert 25% into fraction as well as into decimal.

Solution: $25\% = \frac{25}{100} = \frac{1}{4}$ (fraction)

Also, $25\% = \frac{25}{100} = 0.25$ (decimal)

3. APPLICATION OF PERCENTAGE

The percentage is a very important concept of arithmetic and has various applications. Once the students have

understood the concept of per cent, it will be easy to apply this concept to many real life situations such as 'calculating from Percentage to how many', 'Ratio to Per cent' and 'Increase or Decrease as Per cent'. Examples from the Textbook can be taken to explain these concepts. After explaining these concepts, students should be encouraged to attempt questions given under the heading 'Try These' on page 119 of Class VIII Mathematics, NCERT.

Example: Monthly income of Reena is ₹ 15000. She saves 15% of her income. How much does she save in a month?

Solution: The problem can be solved using unitary method as follows:

Reena saves 15% of her income. It means that when income is ₹ 100, she saves ₹ 15.
Therefore, when income is ₹ 1, she saves Rs $\frac{15}{100}$ and when income is ₹ 15000, she saves

₹ $\frac{15}{100} \times 15000$ = ₹ 2250

Thus, Reena saves ₹ 2250 per month.

Remark: In the above example, savings of Reena is 15% of ₹ 15000, which comes out to be ₹ $\frac{15}{100} \times 15000$. It means

15% of ₹ 15000 = ₹ $\frac{15}{100} \times 15000$.

Similarly, 10% of 2000 = $\frac{10}{100} \times 2000$

And 12% of 30000 = $\frac{12}{100} \times 30000$ and so on.

From such examples, it should be made clear to the students that x% of $y = \frac{x}{100} \times y$

What is y % of x ? It is $\frac{y}{100} \times x$. Both are same. It has to be highlighted.

This formula may be explained to the students, which can be used in solving problems based on it.

Sometimes it is easy to use algebra to solve many problems on per cent, which is discussed in the following example.

Example: What per cent of 60 is 15?

Solution: Let x % of 60 be 15.

This means $\frac{x}{100} \times 60 = 15$

or $x = \frac{15 \times 100}{60} = 25$

Therefore, 25% of 60 is 15.

This can also be done using unitary method. Students should be encouraged to attempt questions by different methods. Many other situations have been discussed in the textbook for Class VII, Mathematics, NCERT and many problems have been given under the heading 'Try These' on page 164. Students should be advised to attempt these questions and discuss among the peer group.

4. PROFIT AND LOSS

The meaning of the terms Cost Price (CP), Selling Price (SP), Profit and Loss, overhead expenses have been explained on page 167 of Class VII Mathematics, NCERT and on pages 120-124 of Class VIII Mathematics, NCERT.

Sometimes, after purchasing an article, some extra money has to be paid for things like transportation, storage, labour, repairing, etc. These extra expenses are called overhead expenses. For calculating the total cost price, we add the overhead charges to the purchase price or cost price. Profit and loss including overhead charges have been discussed in Class VIII. Through different examples, you may ask the students to tell whether there is profit or loss in a given transaction and help them list the following formulas:

Profit = S.P. – C.P.

Loss = C.P. – S.P.

$$\text{Profit per cent} = \frac{\text{Profit}}{\text{CP}} \times 100$$

$$\text{Loss per cent} = \frac{\text{Loss}}{\text{CP}} \times 100$$

where Cost Price (CP) includes overhead expenses, if any.

It should be stressed that profit or loss per cent is always calculated on **cost price** and not on selling price. The concept of profit and loss per cent should be first explained by using unitary method and then the formula should be explained for its application, as given in the following example:

Example: An item was sold for ₹ 540 at a loss of 10%. What was its cost price?

Solution: Let the cost price be ₹ 100. Then loss will be ₹ 10. Therefore, selling price is ₹ (100 – 10) = ₹ 90. We now use unitary method.

When SP = ₹ 90, CP = ₹ 100

When SP = ₹ 1, CP = ₹ $\frac{100}{90}$

When SP = ₹ 540, CP= ₹ $\frac{100 \times 540}{90}$ = ₹ 600

Therefore, CP of the item is ₹ 600.

Alternatively, let the cost price = ₹ x

Therefore, loss = 10% of x = $\frac{10x}{100} = \frac{x}{10}$

Hence, SP = CP – Loss = $x - \frac{x}{10} = \frac{9x}{10}$

But selling price is given to be ₹ 540

Therefore, $\frac{9x}{10} = 540$

or $x = \frac{540 \times 10}{9} = 600$

Hence, cost price of the item = ₹ 600.

5. DISCOUNT

Teacher may collect some advertisements from news paper and magazines as teaching aids. These advertisements are the strategies adopted by the shopkeepers to attract customers. Usually they offer discount during festival season. All this is done to increase sales. Like in winter, they offer discounts on fans, cooler, ACs and similarly in summers they offer discount on woollen clothes and geysers. These discounts are called off season discounts. In these advertisements you can see the different strategies adopted by different companies.

The price printed on an article is called its marked price or sometimes is also called list price.

Discount is the deduction mode of the marked price.

Successive discount

Suppose discount is of the type 50% + 20%. This means 50% discount is on M.R.P. Then on the reduced price a discount of 20%. Note that here 50%+20% ≠ 70%.

Thus, discount is given on the marked price or list price of an article to attract the customers. The marked price is the price marked on an article and sale price is the price at which it is sold to the customer. So,

Discount = Marked price – Sale price (Selling Price)

Discount is generally given as a per cent on the marked price. The problems based on discounts are simple applications of percentage.

Example: The marked price of a table is ₹ 5600. It is sold for ₹ 4480. Find the discount and discount per cent.

Solution: We are given that MP = ₹ 5600 and sale price = ₹ 4480. Therefore,

discount = ₹ (5600 – 4480) = ₹ 1120.

Hence, discount per cent = $\frac{\text{discount}}{\text{MP}} \times 100 = \frac{1120}{5600} \times 100 = 20$

Therefore, discount is 20%.

Remark: Discount per cent can also be calculated using unitary method.

6. SALES TAX/VAT

The meaning of sales tax and VAT (value added tax) should be explained to students through examples as given in Class VIII Mathematics, NCERT. The sales tax/VAT is collected by the shopkeeper from the customer and given to the government. Sales tax is added to sale price of an article, whereas discount is reduced from the marked price. This difference of discount and sales tax should be clarified to the students. The sales tax is calculated on the sale price of the article.

Example: The marked price of a TV set is ₹ 12000. If the sales tax is 7%, find the selling price of the TV set.

Solution: The marked price of TV set = ₹ 12000.

Sales tax = 7% of ₹ 12000 = ₹ $\frac{7}{100} \times$ 12000 = ₹ 840

Therefore, selling price = ₹ (12000 + 840) = ₹ 12840.

Common Errors

1. 15% of 200 = $\frac{15}{200} \times 100 = \frac{15}{2}$ in place of $\frac{15}{100} \times 200 = 30$
2. If SP = ₹ 210, CP = ₹ 200, then profit per cent = $\frac{10}{210} \times 100$, in place of $\frac{10}{200} \times$ 100%, i.e. while finding

profit and loss per cent, students sometimes use SP in place of CP.

3. The overhead expenses are not included in the cost price, whereas overhead expenses should be added to the cost price to get the actual or total cost price.

4. Discount is calculated on sale price in place of marked price.

You may evaluate the students through the following exercise:

Exercise

1. Convert each of the following into per cent:

   (i) $\frac{3}{4}$ (ii) $\frac{5}{7}$ (iii) 0.32 (iv) 2.5

2. Convert the following into fractions and decimals:

   (i) 32% (ii) 5.2%

3. Compute 15% of 300.

4. What per cent of 400 is 8?

5. 8% of which number is 25?

6. A shopkeeper bought a chair for ₹ 375 and sold it for ₹ 400. Find the profit percentage.

7. An item was sold for ₹ 300 at a loss of 5%. What was its cost price?

8. An almirah is sold at ₹ 5225 after allowing a discount of 5%. Find its marked price.

9. The marked price of a table is ₹ 15000. It was sold for ₹ 14400. Find the discount per cent.

10. The sale price of an article is ₹ 250. If sales tax is 6%, how much one has to pay for it?

11. The price of a shampoo bottle is ₹ 180 including 8% VAT. Find the original price of shampoo bottle without VAT.

UNIT 3

Simple and Compound Interest

Structure

- Introduction
- Main Concepts and Sub-concepts
- Objectives
- Teaching Points
  1. Simple Interest
  2. Compound Interest
  3. Applications of Compound Interest formula
- Common Errors
- Exercise

Introduction

In Unit-1 'Introduction to Ratio and Proportion', the concept of unitary method was discussed. In this unit, the concept of simple and compound interest will be introduced and the formulae for calculating the simple and compound interest will also be explained. Finally, the application of compound interest formula for solving problems based on rate of growth (appreciation) and depreciation will be discussed.

Main Concepts and Sub-concepts

- Simple Interest
- Amount
- Conversion period
- Compound interest compounded annually, half yearly, quarterly
- Application of compound interest formula.

Objectives

After teaching these concepts and sub concepts, students will be able to

- find simple interest by using unitary method as well as by using formula;
- apply simple interest formula for different situations;
- find compound interest by using the concept of simple interest;
- find conversion periods and rate of interest per conversion period;
- calculate compound interest by using the compound interest formula; and
- apply compound interest formula for solving problems of rate of growth and depreciation.

Teaching Points

1. Simple Interest

Sometimes we need to borrow money from someone (lender) or bank for a specified period of time. At the end of this period the money borrowed has to be paid back along with some additional money for using the lender's money. The terms involved in lending or borrowing money are **Principal (P)**, **Rate of interest (R)**, **time period (T), Interest (I),** and **Amount (A)**. These terms should be explained through different situations/examples as given in Mathematics textbook for Class VII. In brief;

Principal: The money lent or borrowed.

Interest: Extra money paid in addition to the principal.

Rate of Interest: Rate at which interest is charged on the principal. This is usually in per cent per year.

Time Period: Time (in years) for which money is lent or borrowed.

Amount: Sum of Principal and Interest.

Simple Interest: Interest, when the principal does not change.

You may first explain the method of calculating simple interest by using unitary method through some examples and then arrive at the formula for simple interest:

$$S.I. = \frac{PRT}{100}$$

These terms may be reinforced with the help of examples.

Example: A sum of ₹ 5000 is invested at 5% per annum (p.a.) for 3 years. Find the interest and the amount at the end of 3 years.

Solution: Here, principal (P) = ₹ 5000, rate of interest ($R\%$) = 5 % per annum and time (T) = 3 years. Using unitary method:

On ₹100, interest for 1 year = ₹ 5

Therefore, on ₹ 100, interest for 3 years = ₹ 5 × 3

Therefore, on ₹ 1, interest for 3 years = ₹ $\frac{5\times3}{100}$

and, on ₹ 5000, interest for 3 years = ₹ $\frac{5000\times5\times3}{100}$ = ₹ 750.

Hence, interest = ₹ 750 and

Amount (A) = $P + I$ = ₹ (5000 + 750) = ₹ 5750.

Here, the students may be asked to see that simple interest (I) = ₹ $\frac{5000\times5\times3}{100}$, which can be written as $\frac{P\times R\times T}{100}$.

In the formula for simple interest, there are four unknowns I, P, R and T. Given any three of these, the fourth can be found using the above formula. This may be explained through the following example:

Example: On a certain sum of money, the interest paid after 3 years at 7% per annum is ₹ 126. Find the sum of money.

Solution: Let the sum of money (in rupees) = P. Here, time period (T) = 3 years
Interest (I) = ₹ 126.

Using the formula, $I = \frac{PRT}{100}$

$$126 = \frac{P\times7\times3}{100}$$

or $$P = \frac{126\times100}{7\times3} = 600$$

Hence, sum of money is ₹ 600.

2. Compound Interest

When money is borrowed on simple interest, then the interest is calculated uniformly on the original principal throughout the loan period. But in certain cases, the amount of the previous unit of time with interest is taken as the principal for the second unit of time. The amount with interest at the end of the second unit of time becomes the principal for the third unit of time, and so on.

After a certain specified time, the difference between the amount and the money borrowed is called the **compound interest** or we can say that the interest calculated in this manner is called the **compound interest**.

The fixed unit of time is known as the **conversion period.**

One such situation has been discussed on page 126 of Class VIII, Mathematics, NCERT.

Teacher may explain that

(i) When the interest is added to the principal every year and the interest is calculated thereon, then interest is said to be **compounded annually**.

(ii) If the interest is added to the principal after every 6 months, the interest is said to be **compounded half-yearly** or **semi-annually**.

(iii) If the interest is added to the principal, after every 3 months, the interest is said to be **compounded quarterly**.

Conversion Period

(i) When interest is compounded annually

The conversion period is one year. If the time period is 3 years, then the number of conversion periods is 3. If the rate of interest is 8% per annum, the rate of interest per conversion period will be 8%.

(ii) When interest is compounded half yearly

The conversion period is 6 months or $\frac{1}{2}$ years. If the time period is 3 years, then the number of conversion

periods is 3 × 2 = 6 (half years). If the rate of interest is 8% per annum, the rate of interest per conversion period (half yearly) is $\frac{8}{2}\% = 4\ \%$.

(iii) When interest is compounded quarterly

The conversion period is 3 months or $\frac{1}{4}$ years. If the time period is 3 years, then the number of conversion periods is 3 × 4 = 12 (quarter years). If the rate of interest is 8% per annum, the rate of interest per conversion period (quarterly) is $\frac{8}{4}\% = 2\ \%$.

Formula for calculating **Amount** (A) in case of compound interest is given by $A = P\left(1+\frac{R}{100}\right)^n$, where n is the number of conversion periods and R is the rate of interest per conversion period.

Teacher may explain this formula through examples as given on page 128 of Class VIII Mathematics Textbook, i.e. Compound Interest (C I) = A – P

Before using the formula for Amount and C I, it is advisable that sufficient practice should be provided to the students for finding rate per conversion period and the number of conversion periods. The number of conversion periods should not exceed 3.

Now the teacher may explain Examples 11 and 12 given on page 129–130 of Class VIII Mathematics, NCERT. After that, students may be asked to do direct questions based on the C I formula given in Exercise 8.3.

You may now discuss Example 13, where the number of conversion periods is not a natural number. In such situation, the Amount is calculated for complete conversion periods and then simple interest on this amount for remaining part and obtain the final amount accordingly.

3. Application of Compound Interest Formula

There are certain situations such as rate of growth of population, growth of bacteria, depreciation of a machine, etc. where the formula of compound interest can be applied. The compound interest formula gives the growth of money at a given rate. The formula for compound interest is

$$A = P\left(1 + \frac{R}{100}\right)^n$$

If the problem is based on rate of growth, we apply the above formula. In case of depreciation, the growth becomes negative and the formula becomes

$$A = P\left(1 - \frac{R}{100}\right)^n$$

Here, R is the rate of depreciation per conversion period and other symbols have their usual meanings.

It may be explained by using the examples given in the book as well as the one given below:

Example: A motorcycle was bought for ₹ 50000. Its value depreciates at the rate of 10% per annum. Find its value at the end of 3 years.

Solution: Here, P = ₹ 50000, R = 10% p.a. and n = 3 years. Since value of motorcycle depreciates at 10% p.a, we use the formula:

$$A = P\left(1 - \frac{R}{100}\right)^n$$

$$A = 50000\left(1 - \frac{10}{100}\right)^3 = 50000 \times \left(\frac{9}{10}\right)^3$$

$$= \frac{5000 \times 9 \times 9 \times 9}{10 \times 10 \times 10} = 36450$$

Therefore, value of motor cycle at the end of 3 years is Rs 36540.

Common Errors

If P = ₹ 1000, R = 8% per annum, T = 2 years and interest is compounded half yearly, then some students may write

$$A = 1000\left(1+\frac{8}{100}\right)^2$$

in place of $A = 1000\left(1+\frac{4}{100}\right)^4$

In question 10, Exercise 8.3, some students may be tempted to find population of 2001, by using the formula P = $5400\left(1-\frac{5}{100}\right)^2$, which is wrong.

You may evaluate the students through the following exercise:

Exercise

1. A sum of ₹ 3500 is invested at 7% p.a. Find the simple interest at the end of 3 years. Also, find the amount.
2. In how many years will the interest on ₹ 1200 at 5% per annum become ₹ 240?
3. A sum of ₹ 2400 is invested at 8% p.a. compounded annually for 2 years. Find the amount and the compound interest at the end of two years.
4. The population of a town is 56,000. If the population grows at the rate of 5% per annum, what will be the population after 3 years?
5. A scooter was bought at ₹ 42,000. Its value depreciates at the rate of 10% per annum. Find its value after 2 years.

Block

Teaching of Mensuration

Measurement is one of the most fundamental of all mathematical processes, permeating not only in all branches of mathematics but in everyday activities as well. It is an area of study that must begin early in the schools and continue to develop in depth throughout all levels of learning.

At an early stage, the students should become proficient in converting one unit to another. Within a system of measurement, they should know the equivalence and convert easily. Although students do computations of measurements such as areas and volumes, we need frequent hands-on measurement experiences, such as when we look around some 2D objects, we get the feel of their boundaries and the regions enclosed. To measure and compare them, we need the concepts of perimeter and areas. The concept of perimeter is needed to fence a field, to construct a boundary wall around a house, etc., and concept of area is needed in tiling a floor of a room, painting a wall, constructing roads, etc. Similarly, the concept of volume/ capacity arises when we come across 3D objects which is needed if one wants to know the quantity of water in a swimming pool, quantity of wood needed to make a box,

number of bricks needed to construct a house, etc. This block deals with these concepts—perimeter, area and volume, etc.

In this Block, there are two units:

Unit 1: Perimeter and Areas

Unit 2: Surface Areas and Volumes

Unit 1 deals with perimeters and areas of some simple closed regions such as rectangle, triangle, square, trapezium, polygon, circle, etc.

Unit 2 deals with surface areas and volumes of some 3D objects such as cube, cuboids and cylinder, etc.

UNIT 1

Perimeters and Areas

Structure

- Introduction
- Main Concepts and Sub-concepts
- Objectives
- Teaching Points
  1. Perimeter and area
  2. Perimeters and areas of Rectangles and Squares
  3. Area of a Parallelogram and a Triangle
  4. Area of a Trapezium
  5. Area of a Polygon
  6. Circumference and Area of a Circle
  7. Areas of Rectangular and Circular paths
- Common Errors
- Exercise

Introduction

These topics have been distributed over three Classes, i.e. VI, VII and VIII. In primary classes, students have already gathered some knowledge about these concepts through activities such as fencing a field or tiling a floor, etc. In this unit, these ideas shall be consolidated and extended to the study of perimeters and areas of parallelograms, triangles, trapeziums, polygons and circles. A discussion on rectangular and circular paths has also been taken up.

Main Concepts and Sub-concepts

- Perimeters and areas of some simple closed figures
- Standard units of measurement for perimeters and areas
- Perimeter of a rectangle = 2 (length + breadth)
- Area of a rectangle = length × breadth
- Area of a square = $(\text{side})^2$
- Area of a parallelogram = base × corresponding altitude
- Area of a triangle = $\frac{1}{2}$ base × corresponding altitude
- Area of a trapezium = $\frac{1}{2}$ (sum of the two parallel sides) × perpendicular distance between them
- Area of a polygon = sum of the areas of all the known figures into which it is splitted
- Circumference of a circle = $2\pi r$, where r is radius of the circle
- Area of circle = πr^2, where r is the radius of the circle
- Area of a rectangular path = Area of outer rectangle – Area of inner rectangle
- Area of cross rectangular paths = Sum of areas of the two paths – Area of the common (intersection) portion
- Area of a circular path = Area of the outer circle – Area of the inner circle

Objectives

After teaching this unit, the students will be able to

- cite examples from their surroundings indicating the use of perimeters and areas of plane figures;
- calculate the perimeters of some simple closed figures;
- calculate the perimeter and area of a rectangle and a square;
- calculate the area of a parallelogram, triangle and trapezium;
- calculate the circumference and area of a circle;
- calculate the area of a polygon or an irregular rectilinear figure by splitting it into known figures like triangles, rectangles, trapeziums, etc., and
- calculate the areas of rectangular and circular paths.

Teaching Points

1. Perimeter and Area

Students are familiar with the terms—perimeter and area of some simple closed figures—from the primary classes. They have also learnt to fence a field and also to tile a given floor, which ultimately lead to the ideas of perimeter and area, respectively. The teacher may emphasise that perimeter is the distance moved around a simple closed figure once. On the other hand, the area of a simple closed figure is the amount of the region enclosed by that figure.

Here, the teacher may give some situations requiring the use of perimeter (such as fencing a field, preparing race tracks for sports, constructing a boundary wall, putting border on a table cloth, etc.) and also situations requiring the use of area (such as seeds required for sowing in a field, cost of manuring a field, cost of building roads, etc.). Then, the teacher may encourage the students to give their own examples for these two types of situations. It may be stressed that the units of measuring the perimeter is the

same as that of length. They are cm, mm, m and so on. In the beginning itself teacher may explain the need of converting the numbers into same unit before actually adding for obtaining perimeter. To explain this she may tell a student to get up and demonstrate a step of 20 cm and 20 inch, through which children can realise that both numbers differ largely, so units should be same to have significant answer. However, the unit for measuring area is a square of side of unit length. Thus, if this square is of side 1 cm, then unit for area is cm^2; if it is a square of side 1m, the unit of area is m^2; and so on.

Now, the teacher may ask the students to attempt questions on calculation of perimeters given on pages 206 and 207 of Class VI, Mathematics, NCERT.

Teacher may reinforce the concepts through some examples as given below:

Find the perimeter of the figure ABCDEFGHIJKLA given below :

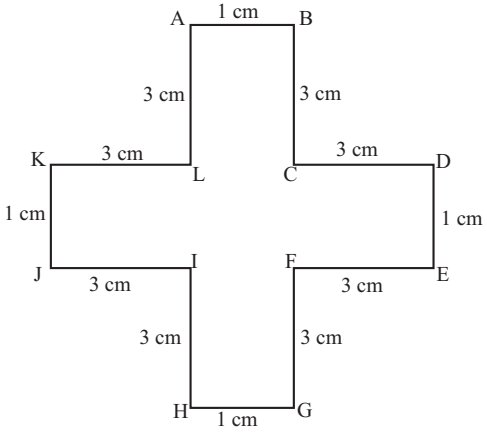


Fig. 1(i)

Teacher may explain its solution by stating that perimeter of a simple closed figure is the distance moved around the figure once. So, required perimeter = AB + BC + CD + DE + EF + FG + GH + HI + IJ + JK + KL + LA = 1 cm + 3 cm + 3 cm + 1 cm + 3 cm + 3 cm + 1 cm + 3 cm + 3 cm + 1 cm + 3 cm + 3 cm = 28 cm.

Find the perimeter of the following:

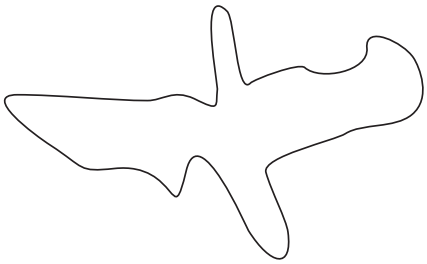


Fig. 1(ii)

A child can use a thread to measure the perimeter of the above figure.

Fig. 1.1(iii)

Note that we cannot have perimeter of an open curve like Fig. 1.1(iii).

For areas, the teacher may take up some irregular figures and find their areas by the method of counting squares, as discussed on pages 214 and 215, specially Example 10 of Class VI, Mathematics, NCERT. The students may take some objects like leaves of different plants. A geoboard may also be used for this purpose. See Mathematics Kit, Activity 8, 'Exploring Area with Geoboard'.

In a similar activity, students find interesting to calculate area of their own hand-span using grid sheet as given below in Fig. 1.1(iv)

The student can make similar table of full squares, half squares, etc. to calculate the area of his own hand.

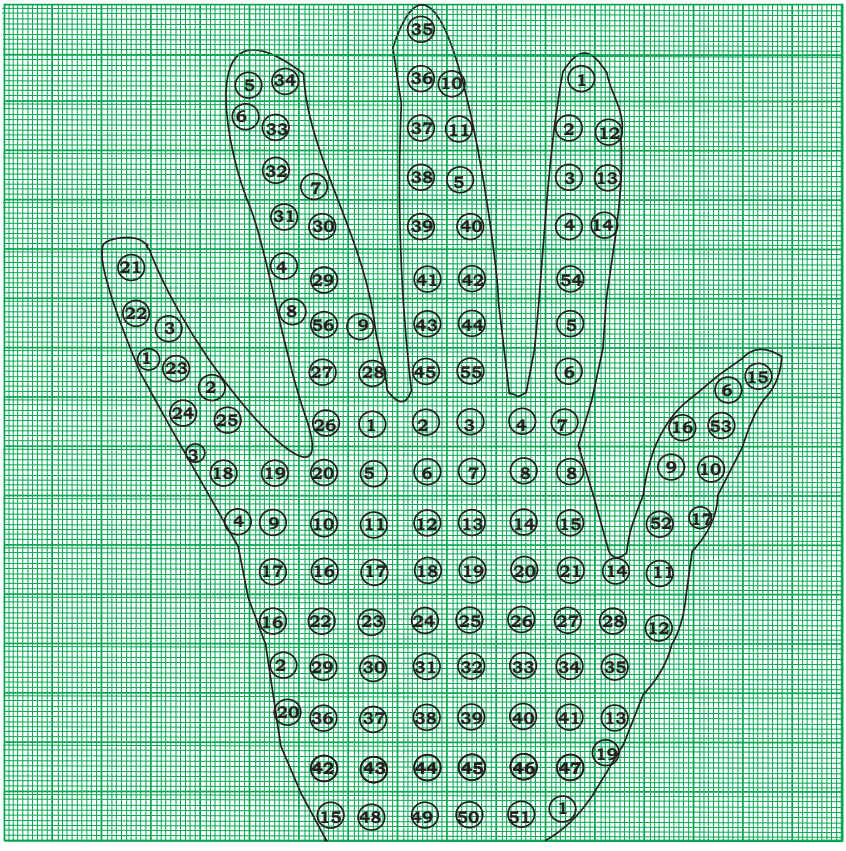


Fig. 1.1(iv)

Example: Find the area of the following figure by the method of counting squares.

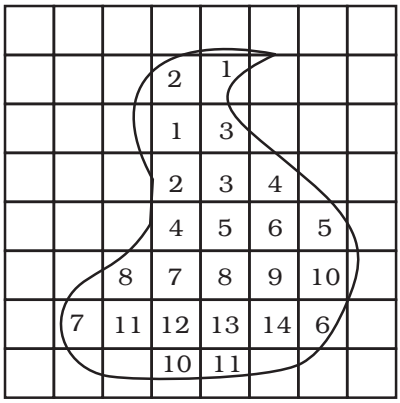


Fig. 1.2

Solution: Number of full squares enclosed by the figure = 14

Number of squares which are more than half but not fully enclosed by the figure = 11

Number of half squares enclosed by the figure = 0

Less than half squares enclosed are ignored.

So, area of the figure = (14 + 11 + 0) square units

= 25 square units

2. Perimeters and Areas of Rectangles and Squares

Teacher can distribute cut-out of squares, rectangles, circles, triangles of different perimeters and some irregular plane figures and simple closed curves to students and a piece of thread for measurement. After telling students that perimeter is the length of boundary, teacher can help students to measure the length of boundary of their cut-outs with the help of thread.

Now the teacher can make the following table on the blackboard and ask each student to come on board with their observations.

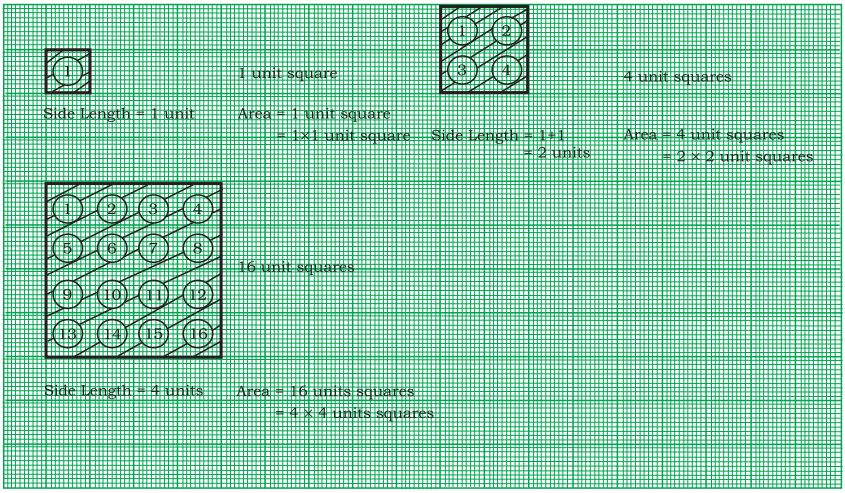
Figures	Length	Breadth	Perimeter	Suggestive formula
Square				
Rectangle				

A class discussion can be done to arrive at an expression for perimeter of regular plane figures.

This can help students to feel that all simple closed figures have perimeter whether we have a formula or not. A similar discussion may also be performed for concept of area.

Thus, teacher can help the students to arrive at the standard formulae for perimeters and areas of rectangles and squares through a number of examples.

Area of Square



Some more activities can be done for calculating area of rectangle as well as for perimeter.

After this, they may be asked to attempt questions given on pages 212 and 213 (Exercise 10.1) and pages 219 and 220 (Exercise 10.3) of Class VI Mathematics, NCERT. In many of these questions, the students will have to split the given figure into rectangles and squares. Let them split the given figure into rectangles and squares in their own way and calculate the areas or perimeters. The teacher should just act as a facilitator in the process. Some of the examples are given below:

Example: A rectangular piece of land measures 0.7 km by 0.5 km. It is to be fenced with four rows of wires. Find the length of wire required for fencing.

Solution: Perimeter of the land = 2 (l + b)

= 2 (0.7 km + 0.5 km) = 2 × 1.2 km = 2.4 km

So, length of the wire required for fencing with one row of wire = perimeter of the land = 2.4 km

Therefore, the length of the wire required for fencing with four rows of wire = 4 × 2.4 km = 9.6 km

Example: Five square flower beds each of sides 2m are dug on a piece of rectangular land of length 10 m and breadth 8 m. Find the area of the remaining land.

Solution: Area of the land = $l \times b$ = (10 × 8)m^2 = 80 m^2

Area of five square flower beds = 5 × (2 × 2)m^2 = 20 m^2

So, area of the remaining land = 80 m^2 – 20 m^2 = 60 m^2

A challenge as an activity

The teacher may take up the challenge given on page 220 of Class VI Mathematics, NCERT as an interesting activity. Let the students consider the rectangles of the following lengths and breadths:

(i) Length = 16 cm; Breadth = 1 cm

(ii) Length = 8 cm; Breadth = 2 cm

(iii) Length = 4 cm; Breadth = 4 cm

Students may observe that area of each of the above rectangles = (16 ×1) cm^2 = (8 × 2) cm^2 = (4 × 4) cm^2 = 16 cm^2

But, in (i), perimeter = 2 (16 + 1) cm = 34 cm (Maximum)
in (ii), perimeter = 2 (8 + 2) cm = 20 cm
in (iii), perimeter = 2 (4 + 4) cm = 16 cm (Least)

Through such type of examples, the teacher may help the students to accept the challenge and state the following result:

(i) Out of the rectangles of a given area, the perimeter is the least when the rectangle is a square.

(ii) Out of the rectangles of a given area, the perimeter is maximum, if the breadth of the rectangle is 1 unit

(say, 1 cm here), provided the length and breadth are measured into the same units and their numerical values are natural numbers only.

Mathematics kit may be used for hands-on activity to verify the above facts.

In the similar manner, as an activity, the teacher may ask the students to find the areas and perimeters of the following rectangles:

(i) Length = 10 cm; Breadth = 6 cm

(ii) Length = 8 cm; Breadth = 8 cm

(iii) Length = 9 cm; Breadth = 7 cm

(iv) Length = 15 cm; Breadth = 1 cm

(v) Length = 15.5 cm; Breadth = 0.5 cm

Clearly, in (i), Area = 60 cm^2, Perimeter = 32 cm

in (ii), Area = 64 cm^2, Perimeter = 32 cm

in (iii), Area = 63 cm^2, Perimeter = 32 cm

in (iv), Area = 15 cm^2, Perimeter = 32 cm

in (v), Area = 7.75 cm^2, Perimeter = 32 cm

From the above observations, the teacher may help the students to arrive at the following results :

Of all the rectangles with a given perimeter, area is maximum when the rectangle is a square.

The students may also be encouraged to note that the area will be minimum, if the breadth of the rectangle is 1 unit (say, 1 cm here), provided the length and breadth are measured into the same units and their numerical values are natural numbers only, otherwise nothing can be said of the case, when area is minimum.

3. AREA OF A PARALLELOGRAM AND A TRIANGLE

These concepts have been taken up in Class VII, Mathematics, NCERT. By now, the students have been exposed to the congruency of two figures. At this stage,

the teacher may encourage the students through activities as suggested on pages 210, 213 (Chapter 11, Fig. 11.15) of Class VII, Mathematics, NCERT, to understand the following which is tacitly used in solving many problems relating to areas:

All the congruent figures are equal in area but figures equal in area need not be congruent.

This idea has been used in arriving at the formula for area of a parallelogram from the known formula for the area of a triangle. The teacher may point out to the students that here we have used the result that two congruent triangles have equal areas.

The teacher may perform the following activity to calculate area of any parallelogram (Fig. 1.3):

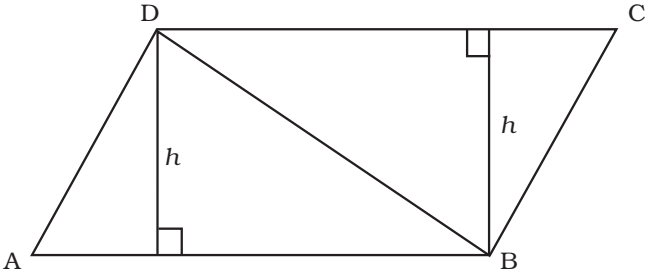


Fig. 1.3

The diagonal DB divides the parallelogram [Fig 1.4(i) and (ii)] into two triangles. Now cutting the parallelogram into two triangles, students can be made to realise that two triangles are congruent as they overlap each other.

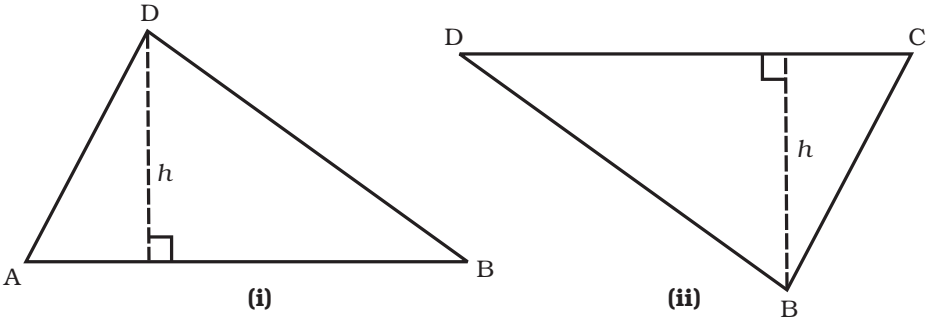


Fig. 1.4

Since area of two congruent triangles are equal,

we have 2 × Area ΔABD = Area of parallelogram

i.e., 2 × Area ΔABD = Area of parallelogram

i.e., $2 \times \frac{1}{2} \times AB \times h$ = Area of parallelogram

Hence, Area of Parallelogram = AB × h = Base × Height

Thus, the area (A) of a parallelogram = Base × Corresponding Altitude.

Here, it may be stressed that any side of the parallelogram can be taken as the base. The altitude will change according to the base.

For example, in the following parallelogram ABCD, AQ is the corresponding altitude for base (side) CD but it is not altitude corresponding to BC. Altitude corresponding to BC

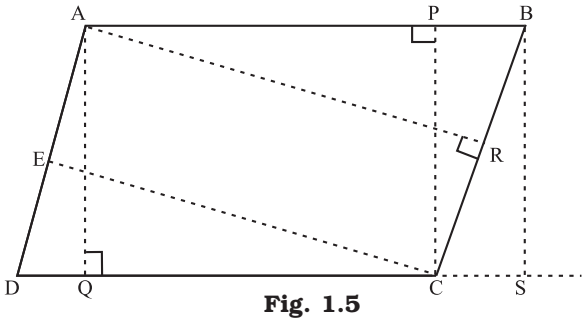


Fig. 1.5

is AR. Similarly, altitude corresponding to AB is CP or QA, while altitude corresponding to AD is CE. It may also be noted that BS is also an altitude corresponding to CD (Fig. 1.5). Through such examples, it may also be made clear to the students that an altitude need not wholly lie in the interior of the parallelogram.

After getting mastery on the use of the formula for area of a parallelogram, teacher may help the students to arrive at the formula for area of a triangle from that of the area of a parallelogram. It is suggested that teacher may have

some paper cut-outs for square, rectangle, parallelograms and perform an activity by cutting these figures across any diagonal to verify the fact by overlapping them.

Area of parallelogram = 2 × Area of Δ (associated)

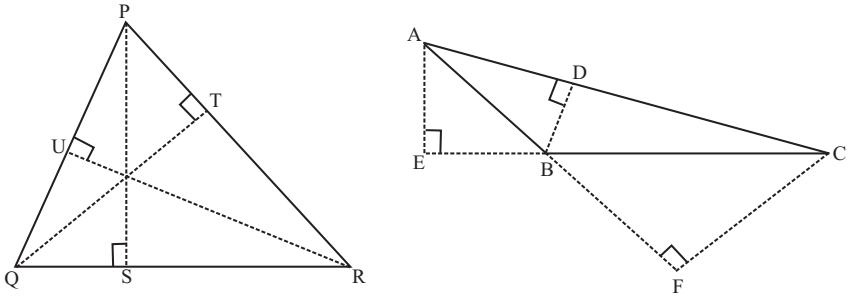
and

Area of a triangle = $\frac{1}{2}$ base × corresponding altitude

After this, the students may be asked to attempt some direct questions based on this formula.

Here, again it may be stressed that any side of the triangle can be taken as the base. The altitude will change accordingly.

For example, in the following triangle PQR [Fig. 1.6(i)]



(i) (ii)

Fig. 1.6

PS is the altitude corresponding to base QR, QT is the altitude corresponding to base PR and RU is the altitude corresponding to base PQ. Further, in triangle ABC, [Fig. 1.6(ii)] altitude corresponding to base AC is BD, altitude corresponding to base AB is CF and altitude corresponding to base BC is AE. It may again be noted that in case of triangles

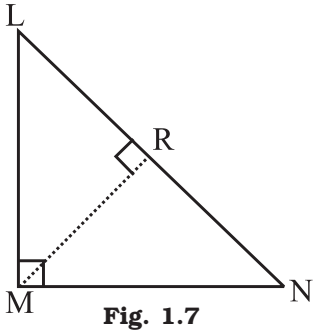


Fig. 1.7

also, an altitude need not lie wholly in the interior of the triangle.

In case of a right triangle LMN (See Fig. 1.7), altitude corresponding to base MN is LM for base LM it is NM; and for base NL it is MR.

After this explanation, students may be asked to attempt questions in Exercise 11.2 of Class VII Mathematics, NCERT. One such example is given below :

Example: In (Fig. 1.8), PQR is a triangle, PR = 8 cm, QR = 5cm,

QM ⊥ PR, PN ⊥ NR and PN = 4 cm. Find the area of the triangle PQR and also calculate QM.

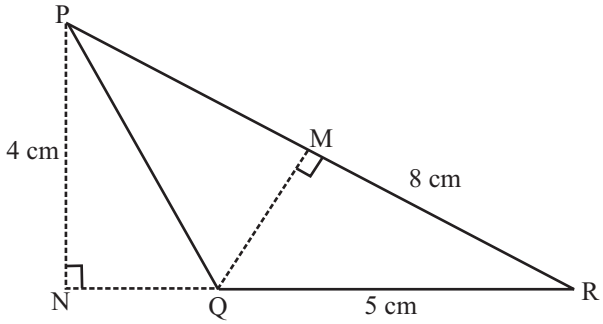


Fig. 1.8

(Sometimes in these type of examples, students get confused that the required length of base of triangle is NR, but they should be explained that it is not so and QR itself remains the base. They should also be told that in a question of finding area, if length of two sides of a triangle is given, then they should prefer that side of the triangle for which corresponding altitude is known.)

Solution: Here, two sides PR and QR are given and any of these can be taken as base. If we take base PR, then length of QM must be known to us for finding the area of the triangle, which is not known. So, here QR must be taken as base.

Therefore, area of the triangle PQR

$$= \frac{1}{2} \text{ QR} \times \text{PN}$$

$$= \frac{1}{2} \times 5 \times 4 \text{ cm}^2 = 10 \text{ cm}^2$$

Now, of course, the students can find QM

as $\frac{1}{2}$ QM × PR = 10, $\frac{1}{2}$ × 8 × QM = 10,

$$\text{QM} = \frac{10}{4} \text{ cm}, = 2.5 \text{ cm}$$

4. AREA OF A TRAPEZIUM

Teacher should encourage the students to formulate some expressions for the area of given geometrical figures,rather than giving direct formula to them. Area of trapezium can be deduced by doing the activities given in section 11.3 (Page 171-173) of Class VIII Mathematics, NCERT.

In the continuation with the help of the formula for the area of a triangle, the teacher may help the students to arrive at the formula for the area of a trapezium, which is as follows:

Area of a trapezium = $\frac{1}{2}$ (sum of the two parallel sides) × distance between them

For the reference of teachers, details are given as follows:

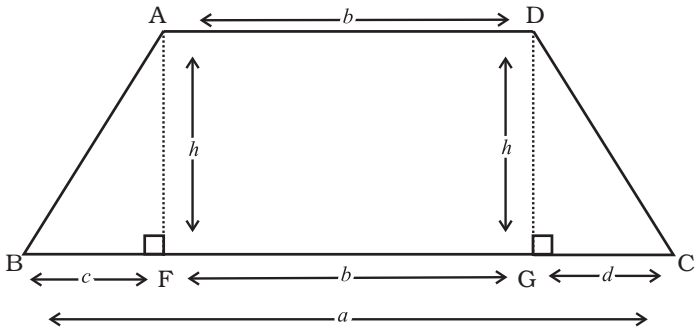


Fig. 1.9

In (Fig. 1.9)

Area of Δ AFB = $\frac{1}{2} \times h \times c$ (1)

Area of Δ DGC = $\frac{1}{2} \times h \times d$ (2)

Area of Rectangle ADGF = $b \times h$ (3)

Area of Trapezium ABCD = (1) + (2) + (3)

$$= \frac{1}{2} hc + \frac{1}{2} hd + bh$$

$$= \frac{1}{2} hc + \frac{1}{2} hd + \frac{1}{2} bh + \frac{1}{2} bh$$

$$= \frac{1}{2} h\,(c + d + b + b)$$

$$= \frac{1}{2} h\,[(c + d + b) + b]$$

$$= \frac{1}{2} h\,(a + b) \text{ (as } c + d + b = a)$$

For verifying this, following activity may also be performed:

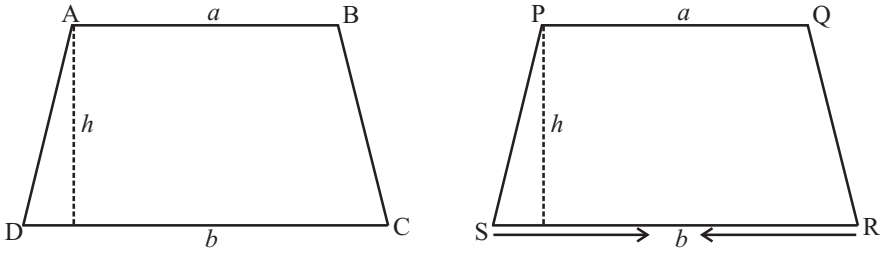


Fig. 1.10

Cut out two congruent trapeziums ABCD and PQRS with parallel sides of lengths a and b and distance between them as h (See Fig. 1.10) from a cardboard.

Now, arrange the two trapeziums as shown below (Fig. 1.11):

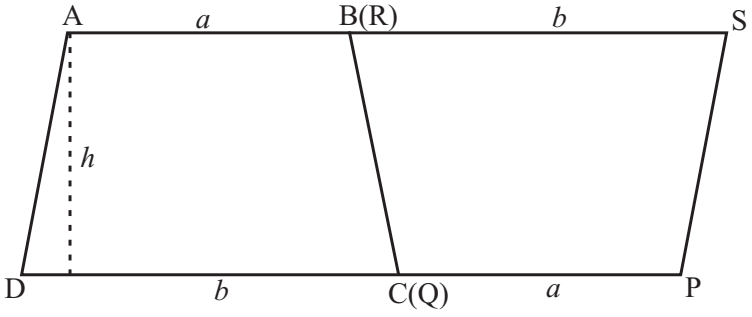


Fig. 1.11

Ask the students to observe that the figure so obtained is a parallelogram ADPS whose one pair of parallel sides is of length $(a + b)$ each and altitude corresponding to base DP is h.

So, Area of the parallelogram ADPS,

= Area of trapezium ABCD + Area of trapezium PQRS

= Area of trapezium ABCD + Area of trapezium ABCD

[As Area trapezium ABCD = Area of trapezium PQRS as congruent figures are equal in area.]

= 2 × area of trapezium ABCD = DP × h

= $(a + b) \times h$

So, 2 × Area of trapezium ABCD = $(a + b) \times h$, i.e., Area of the trapezium ABCD = $\frac{1}{2}(a + b) \times h$

Initially, students may be asked to attempt some direct questions on area of a trapezium and then they may attempt some questions involving trapeziums and right triangles as given in 'Try These' on page 176 (ii) and (iii) of Class VIII Mathematics, NCERT.

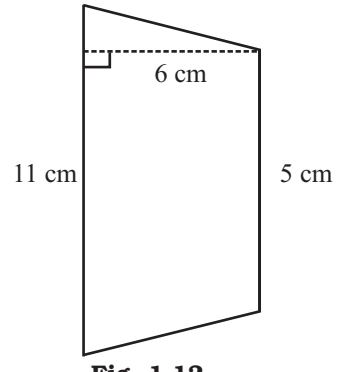


Fig. 1.12

You may make use of the items in the Mathematics kit of Activity 9 Area of Parallelogram, Triangle and Trapezium to demonstrate these concepts to the students. One such example is given below:

Example: Find the area of the trapezium given below :

Solution: Sum of two parallel sides = (5 + 11) cm = 16 cm

Distance between them = 6 cm

Area of the trapezium

$$= \frac{1}{2}(a + b) \times h$$

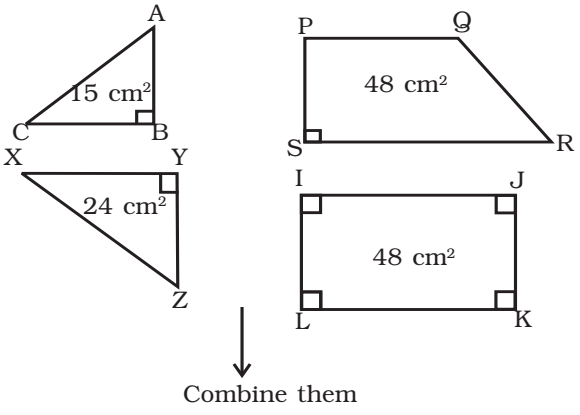
$$= \frac{1}{2}(16) \times 6 \text{ cm}^2$$

$$= 48 \text{ cm}^2$$

In fact, area of a triangle may be the key factor in finding the areas of different figures. Enough practice may be given to the students to utilise this fact.

5. AREA OF A POLYGON

Initially teacher can ask the students to bring cut-outs of polygons whose area is already known to them and then the students can be encouraged to combine these polygons to make a bigger polygon, e.g.



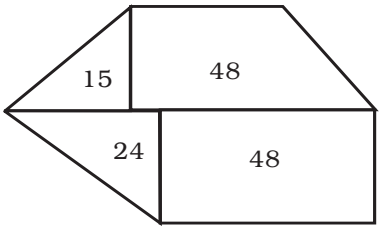


Fig. 1.13

They can easily find out the total area.

Now the students may be given exposure to observe a bigger polygon in different known polygons.

As already stated in the previous section, area of a triangle is the key factor in finding the areas of other figures, specially various polygons. For example, area of the quadrilateral PQRS (Fig.1.14) can be easily found by drawing its diagonal QS and drawing perpendicular on it from P and R.

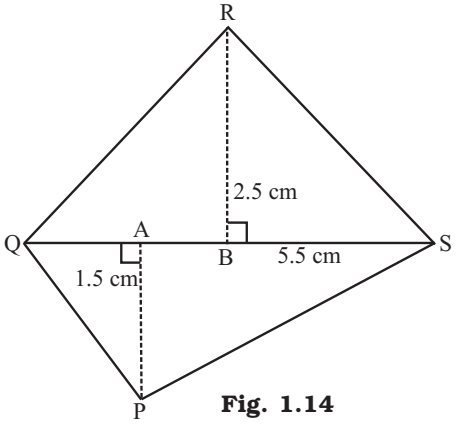


Fig. 1.14

If the heights AP and RB are known, then area of the quadrilateral PQRS can be found using the formula for area of a triangle.

Clearly, area of quadrilateral PQRS

= area of Δ PQS + area of Δ RQS

$$= \left(\frac{1}{2} \times 5.5 \times 1.5 + \frac{1}{2} \times 5.5 \times 2.5\right) \text{cm}^2$$

$$= \frac{1}{2} \times 5.5 \times [1.5 + 2.5] \text{ cm}^2$$

$$= \frac{1}{2} \times 5.5 \times 4\text{cm}^2 = 11 \text{ cm}^2$$

In the similar manner, any polygon or a rectilinear figure can be splitted into triangles, trapeziums and other known figures (some simple cases have already been taken in the previous section), and then its area can be found by first finding the areas of the individual figures and then adding them.

The questions mentioned in 'Try these' on page 176 of Class VIII Mathematics, NCERT may now be attempted. During this process, the teacher may emphasise that area of a figure can be found in several ways (See Example 3 on page 177 of Class VIII Mathematics Textbook). Now the teacher may help the students to attempt questions of Exercise 11.2 on pages 177 and 178 of Class VIII Mathematics, NCERT.

(Area of special quadrilateral such as 'Rhombus' may be explained as on page 174, Class VIII Mathematics, NCERT under the heading 'Area of special quadrilateral'.)

6. CIRCUMFERENCE AND AREA OF A CIRCLE

Students have already learnt some basic terms related to circles such as centre, radius, diameter, chord, circumference, and so on. Before starting this topic, students may be asked to recall these terms. Here, through a simple activity (as suggested on page 218 of Class VII, Mathematics, NCERT) explain that **ratio of circumference and diameter of a circle is a constant**. This **constant is denoted by a Greek letter π** (pronounced as pie). Its approximate value is usually taken as $\frac{22}{7}$ or 3.14 for calculation purpose in mensuration problems. Thus, we

have $\frac{c}{d} = \pi$, where c and d are respectively the circumference and diameter of the circle.

So, $\frac{c}{2r} = \pi$, where r is the radius of the circle.

i.e. $c = 2\pi r$

Thus, it may be stated that circumference of a circle is $2\pi r$, r being the radius. It may be mentioned here that circumference is, in fact, the perimeter of the circle.

Formula for area of a circle may be explained through an activity as suggested on page 221 of Class VII Mathematics Textbook, NCERT (cutting the circular disc in two equal sectors) (See Fig. 11.35 and 11.36 of the book on page 221). You may show this through activity 10 'Area of a circle' in the Mathematics Kit given with this package. Now, the students may be asked to attempt some direct questions based on the following two formulae:

Circumference of a circle = $2\pi r$, and

Area of a circle = πr^2, where r is the radius of the circle.

Teacher may ask the students to attempt Questions 1 and 2 of Exercise 11.3 of Class VII Mathematics, NCERT. Some examples are given below:

Example: Find the circumference and area of a circle of radius 14 cm.

Solution: Circumference = $2\pi r$

$$= 2 \times \frac{22}{7} \times 14 \text{ cm} = 88 \text{ cm}$$

$$\text{Area} = \pi r^2 = \frac{22}{7} \times 14 \times 14 \text{ cm}^2 = 616 \text{ cm}^2$$

Example: Find the cost of polishing a circular table top of radius 2 m at the rate of ₹ 10 per m². (Use $\pi = 3.14$)

Solution: Area of the table top = πr^2

$$= 3.14 \times 2 \times 2 \text{ m}^2$$

$= 12.56 \text{ m}^2$

So, cost at ₹ 10 per m^2 $= ₹ 10 \times 12.56$

$= ₹ 125.60$

7. AREA OF RECTANGULAR AND CIRCULAR PATHS

For understanding these types of problems, students must be encouraged to look around their surroundings and observe various paths made inside, outside and in the middle of the parks. You may help them to draw their figures. Once the figures are drawn, there will be no problem in answering the questions. To understand the concept of circular path there are many objects in our surroundings. Teacher can prominently ask them to bring a CD on which circular path can be shown (as drawn below Fig. 1.15). Most of the CDs have diameter of outer circle as 12 cm and inner circle as 5 cm (or 4 cm) and the circular path portion has the labelling. So, this activity helps them to determine how much area is occupied by the label on a CD.

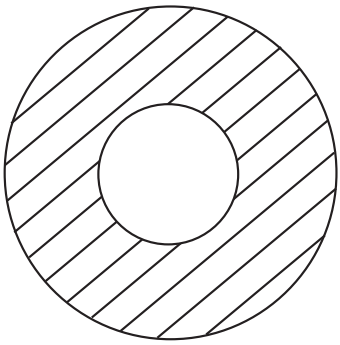


Fig. 1.15

Once a student has observed rectangular paths in his surroundings, the teacher can generalize her explanation for few of most common amongst them such as (Fig. 1.16):

(i) Area of Shaded Portion

$$= \begin{pmatrix}\text{Area of}\\ \text{rectangle}\\ \text{ABCD}\end{pmatrix} - \begin{pmatrix}\text{Area of}\\ \text{rectangle}\\ \text{EFGH}\end{pmatrix}$$

(ii) Area of Shaded Portion

$$= \begin{pmatrix}\text{Area of}\\ \text{rectangle}\\ \text{ABCD}\end{pmatrix} - 4\begin{pmatrix}\text{Area of}\\ \text{square}\end{pmatrix}$$

(iii) Area of Shaded Portion

$$= \begin{pmatrix}\text{Area of}\\ \text{rectangle}\\ \text{EFGH}\end{pmatrix} + \begin{pmatrix}\text{Area of}\\ \text{IJKL}\end{pmatrix} - \begin{pmatrix}\text{Area of}\\ \text{MNOP}\end{pmatrix}$$

↓

MNOP is being subtracted once, because it has been added twice, once with EFGH and then with IJKL.

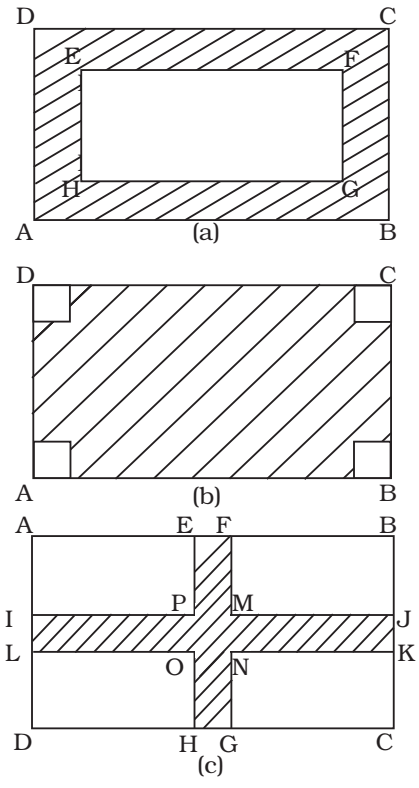


Fig. 1.16

* Note that MNOP might not be a square always.

* Also, labelling with accuracy is very essential for question of this type.

At this stage, only simple and direct questions should be taken up. Here, the main emphasis should be on drawing correct figures. The teacher may also make the students aware that the questions like making borders on a floor, leaving margins in a photo frame or for doing some embroidery work for table cloths, constructing grassy lawns, making washer, etc. can be considered under the category of rectangular or circular path problems. You may first explain the solved Examples 19, 20, 21 and 22 of Class VII Mathematics Textbook (Chapter 11) and then ask the students to attempt questions of Exercise 11.3 and 11.4 of the same book. Do not give much complicated questions at this stage to students.

You may help the students to find area of rectangular paths through the example given below :

Example: A photograph is to be framed leaving a margin of 5 cm on all the sides. If the length and breadth of the photograph are 30 cm and 25 cm respectively, find

(i) length and breadth of the frame.
(ii) area of the margin.

Solution: See Fig. 1.17

(i) Required length

= 30 cm + 5 cm + 5 cm = 40 cm

and breadth

= 25 cm + 5 cm + 5 cm = 35 cm

These steps should be made clear to the students i.e. why 5 cm is added two times.

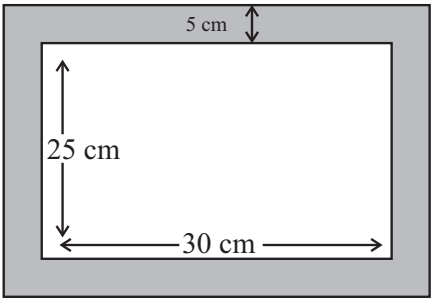


Fig. 1.17

(ii) Area of the margin = Area of the outer rectangle – Area of the inner rectangle

= (40 × 35 – 30 × 25) cm²

= (1400 – 750) cm²

= 650 cm²

Common Errors

(i) Many a times, students do not make any distinction between perimeter and area and as a result of it, they calculate area instead of perimeter and perimeter instead of area.

(ii) While performing calculations, it is necessary that length and breadth or base and altitude,etc. must be expressed in the same units. Many a times, students do not keep this thing in mind.

(iii) Many a times, students use linear units for area. This fact should be emphasized that area is always measured in square units and perimeter in linear units.

(iv) Sometimes in finding the area of a parallelogram, students find the product of the two adjacent sides as area instead of base × corresponding altitude.

(v) Sometimes the students use the value of diameter for radius while calculating the circumference and area of a circle.

(vi) Many a times, in the questions involving rectangular or circular paths, width of the path is not subtracted from both the sides for arriving at the dimensions of the internal figure.

You may evaluate your students through the following Exercise :

Exercise

1. A room is 4 m long and 3 m 50 cm wide. How many square metre of carpet will be required to cover the floor of the room?

2. Find the area and perimeter of the Fig. 1.18:

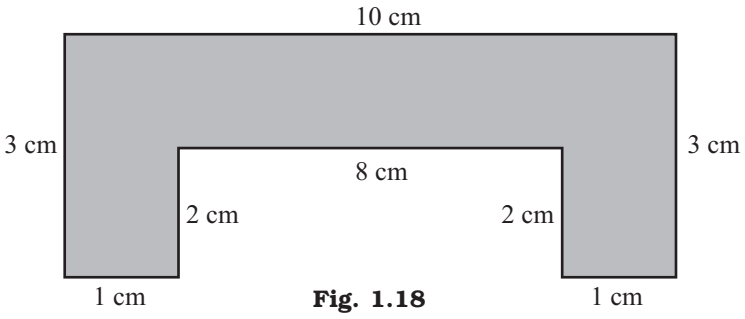


Fig. 1.18

3. Find the area of the following Fig. 1.19 :

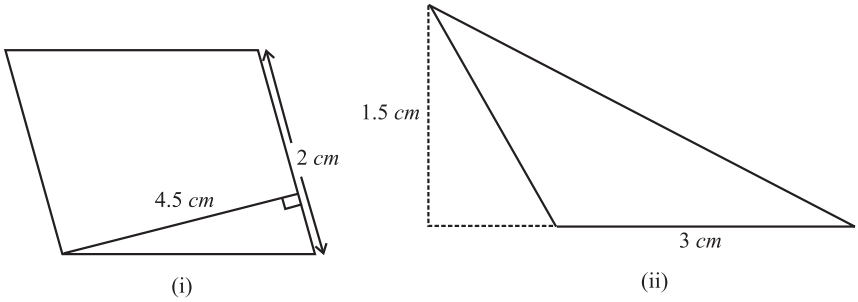


Fig. 1.19

4. Find the cost of polishing a table top of a round table of diameter 1.6 m at the rate of ₹ 15 per m^2 (use $\pi = 3.14$).

5. A path 1m wide is built along the border and inside lawn in the form of a square of side 30 m. Find :

   (i) Area of the path.

   (ii) Cost of planting grass in the remaining portion of the lawn at the rate of ₹ 40 per m^2.

6. Find the area of a trapezium with parallel sides 15 cm and 10 cm and the distance between them as 6 cm.

7. Find the area of a rhombus whose diagonals are 16 cm and 12 cm.

8. Find the area of the field ABCDEF of the following figure in which AP = 2.5 m, AQ = 3 m, AR = 5 m, AS = 7.5 m and AD = 10 m.

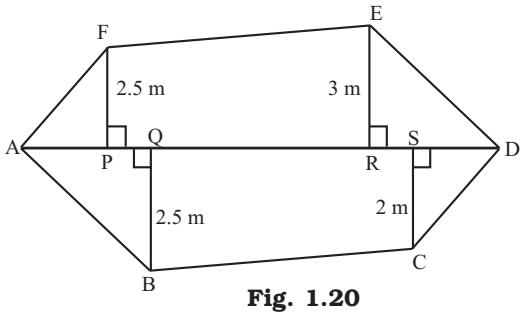


Fig. 1.20

(Hint : Find the area of each of the triangles and trapeziums)

UNIT 2

Surface Areas and Volumes

Structure

- Introduction
- Main Concepts and Sub-concepts
- Objectives
- Teaching Points
  1. From area to surface area
  2. Surface Area and Volume
  3. Surface Areas and Volumes of Cuboids and Cubes
  4. Surface Area and Volume of a Cylinder
- Common Errors
- Exercise

Introduction

These topics have been discussed in Chapter 11, Class VIII Mathematics textbook. Discussion is limited to only surface areas and volumes of cuboids, cubes and cylinders. Students are already aware of these 3D figures from the chapters on visualisation of solid shapes in different classes. They have also acquired some knowledge of volume (capacity) by performing the activity of filling the space in some containers in Primary classes. Here, this knowledge has been consolidated and extended to surface areas also.

Main Concepts and Sub-concepts

- Standard units of measurement of surface area and that of volume
- Surface areas and volumes of 3D figures
- Surface area of a cuboid = $2\,(lb + bh + hl)$, and
- Volume of a cuboid = $l \times b \times h$, where l, b and h are respectively the length, breadth and height of the cuboid
- Surface area of a cube = $6a^2$, and
- Volume of a cube = a^3, where a is the side or edge of the cube.
- Lateral surface area of a cuboid = $2\,(l + b)\,h$, and
- Lateral surface area of a cube = $4a^2$
- Curved surface area of a cylinder = $2\pi rh$
- Total surface area of a cylinder = $2\pi rh + 2\pi r^2 = 2\pi r\,(h + r)$, and
- Volume of a cylinder = $\pi r^2 h$, where r is the base radius and h is the height of the cylinder.

Objectives

After teaching this unit, the students can:

- give some examples from the environment or surroundings, where it is required to calculate the surface areas and volumes;
- calculate the surface areas and volumes of cuboids and cubes of given measurements;

- calculate the surface areas and volumes of cylinders of given measurements; and
- apply the knowledge of surface area and volume in solving some daily life problems.

Teaching Points

1. From Area to Surface Area

Teacher can take nets of various solids like cube, cuboids and cylinder. Apart from this, teacher can take wet clay to make irregular solids.

Teacher should ask: Can you name the surfaces of these solids? Though the students will be able to name the surfaces of cuboid, cube, and cylinder as squares, rectangles and circles, they will not be able to name the surfaces of irregular solids. Now teacher tells when we add the area of all planes of a solid, we get area of solid and since it is the area of their surfaces, we call it as surface area, not simply area.

Teacher should explain that we do have surface area of irregular solids. Whatever outer part of a solid that you can touch provides its surface area, whether you have a method to find it numerically or not.

2. Surface Area and Volume

Students are already aware of many 3D objects or figures from the chapters on visualisation of solid shapes. They are also aware with the nets of some of these shapes. Teacher may realize the need of finding surface area or volume of a solid. For example, the teacher may ask the measurements of a cuboidal tank in order to find the cost of its painting. Similarly the measurements are required to find the amount of water stored in the tank. Teacher may give some more such situations.

With this knowledge and also with the knowledge of perimeter and area, the teacher may come up with an idea that by a surface, we actually mean the surface constituting

the 3D (or solid) shape. Then, explain the idea that surface area of a 3D shape is, in fact, the measure of the 3D shape itself and the volume of a 3D shape is the amount (or measure) of space enclosed by the 3D shape. Thus, by the surface area of a cuboid, we mean the measure of the cuboid and by volume of a cuboid, we mean the measure of the corresponding space enclosed by the cuboid and so on. Again, linking it with the ideas of perimeters and areas, you may explain that standard unit for surface areas is square units and that of volume is cubic units. It may also be explained that a 'unit cube', i.e. a cube of unit length (say 1 cm, 1 m) is taken as the unit of measurement of volume. Thus, if the unit cube is of length 1 cm, then the unit of volume is cm^3 if it is 1 m, then the unit of volume is m^3 and so on.

3. SURFACE AREAS AND VOLUMES OF CUBOIDS AND CUBES

The surface areas of cuboids and cubes may be explained through the idea of their nets (already learnt in visualisation of solid shapes). Students may also be encouraged to use the nets of cuboids and cubes given in Mathematics Kit for Upper Primary Stage (NCERT) and arrive at the following two formulae:

Surface area of a cuboid = $2(lb + bh + hl)$, and

Surface area of a cube = $6a^2$, where l, b, h and a have their usual meanings.

Let the students play with the nets of cuboids, cubes and unit cubes given in Activity 11 'View of solid shapes, their surface areas and volumes', Activity 12 'Nets of solid shapes' to arrive at the formula of surface areas and volumes of cubes and cuboids given in the Mathematics kit.

Here, the teacher may also explain that if we ignore the base and top of the cuboid (or cube), then the surface area so obtained is called the **lateral surface area**. Thus,

Lateral surface area of a cuboid = $2lh + 2bh = 2(l + b)h$, and

Lateral surface area of a cube = $4a^2$

For arriving at the formula for volume of a cuboid, students may be allowed to play with unit cubes given in the Mathematics Kit for Upper Primary Stage and form cuboids of different dimensions. By counting the number of unit cubes, they can obtain the volume of the cuboid formed by them and by observing the concerned cuboid, they can find the length (l), breadth (b) and height (h) of the cuboid. With these observations, teacher may help the students to arrive at the following formula for volume of a cuboid:

Volume of a cuboid = $l \times b \times h$

For this, you may use the table given in Section 11.8.1 on Page 187 of Class VIII Mathematics, NCERT.

Further, as cube is a special cuboid in which $l = b = h = a$ (say), then

Volume of a cube = $a \times a \times a = a^3$

The teacher may also explain that in both the cases,

Volume = Area of the base × height

Now, you may ask the students to attempt questions given under 'Try These' (Page 188) and 'Try These' (Page 189) of Class VIII Mathematics Textbook. They may also be asked to attempt some questions related to cuboids and cubes of Exercise 11.3 and Exercise 11.4 of Class VIII Mathematics, NCERT.

Teacher may explain the use of formulae discussed above in solving problems of surface area and volume through the following examples:

Example: Find the length of cloth of width 90 cm required to cover a suitcase of dimensions 80 cm × 50 cm × 30 cm.

Solution: Cloth required = Surface area of the cuboidal suitcase

$= 2(lb + bh + hl)$

$= 2(80 \times 50 + 50 \times 30 + 30 \times 80)\ \text{cm}^2$

$= 2(4000 + 1500 + 2400)\ \text{cm}^2$

$= 2 \times 7900 \text{ cm}^2$

$= 15800 \text{ cm}^2$

Therefore, length of the cloth $= \dfrac{\text{Area}}{\text{Breadth}}$

$= \dfrac{15800}{90} \text{ cm}$

$= \dfrac{1580}{9} \text{ cm}$

$= 175\dfrac{5}{8} \text{ cm}$

Example: Find the volume of a cube of edge 7 cm.

Solution: Required volume $= (\text{edge})^3$

$= (7 \times 7 \times 7) \text{ cm}^3$

$= 343 \text{ cm}^3$

4. SURFACE AREA AND VOLUME OF A CYLINDER

The teacher may encourage the students to perform the activity suggested under 'Do This' on Page 183 of Class VIII, Mathematics, NCERT and help them to arrive at the following formulae for surface area of a cylinder:

Curved surface area of a cylinder = $2\pi rh$, and

Total surface area of a cylinder = $2\pi rh + 2\pi r^2 = 2\pi r (r + h)$, where r and h are respectively the base radius and height of the cylinder.

In this connection, the teacher may also ask the students to recall knowledge of net of a cylinder. Then, they may be asked to attempt questions under 'Try These' (Page 184) and questions related to surface area of a cylinder of Exercise 11.3 of Class VIII, Mathematics, NCERT. Initially, direct questions based on the formula may be given and

after enough practice, inverse problems may also be taken up (for example, finding the height, if radius and surface area of a cylinder are given). Also any side of the cuboid can be calculated if other two sides and surface area are given.

For volume of a cylinder, it is appropriate at this stage to arrive at the formula for volume of a cylinder through the volume of a cuboid, considering it as area of the base × height. With this assumption, students may be helped to arrive at the following formula :

Volume of a cylinder = area of the base × height = $\pi r^2 h$

For practice, students may be asked to attempt questions under 'Try These' (Page 189) and questions related to cylinders in Exercise 11.4 of Class VIII Mathematics, NCERT.

Some Examples are given below:

Example: Radius of base and height of a cylinder are 28 cm and 14 cm, respectively. Find its

(i) curved surface area,

(ii) total surface area,

(iii) volume

Solution: (i) Curved surface area = $2\pi rh$

$$= 2 \times \frac{22}{7} \times 28 \times 14 \text{ cm}^2$$
$$= 2464 \text{ cm}^2$$

(ii) Total surface area = $2\pi rh + 2\pi r^2$

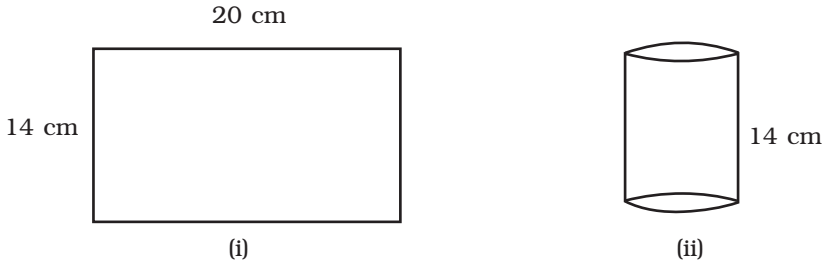
$$= \left(2 \times \frac{22}{7} \times 28 \times 14 + 2 \times \frac{22}{7} \times 28 \times 28\right) \text{cm}^2$$
$$= (2464 + 4928) \text{ cm}^2 = 7392 \text{ cm}^2$$

(iii) Volume = $\pi r^2 h$

$$= \frac{22}{7} \times 28 \times 28 \times 14 \text{ cm}^3$$
$$= 34496 \text{ cm}^3$$

Example: A piece of paper of length 20 cm and breadth 14 cm is folded about its breadth. Find the surface area of the cylinders so formed.

Solution:



Cylinder formed by folding the paper is shown in Fig. (ii).

Here, $2\pi r = 20$ cm and $h = 14$ cm.

It will have only curved surface area.

So, required surface area $= 2\pi rh$

$= 20 \times 14 \text{ cm}^2$

$= 280 \text{ cm}^2$

Note: Had the paper been folded about its length, then $2\pi r$ would have been 14 cm and $h = 20$ cm.

In that case, surface area

$= 2\pi rh$

$= 14 \times 20 \text{ cm}^2$

$= 280 \text{ cm}^2$

i.e. the same as obtained earlier and also equal to the area of the paper.

Volume and Capacity

The difference between 'volume' and 'capacity' requires special mention. Students are aware with the fact that most of the liquids are kept in containers or vessels. The amount

of the liquid that can be kept in a container is equal to the inner volume of the container and this volume is usually referred to as the **capacity of the container**. We generally refer the term volume of a solid object. If an object is hollow, we talk about the volume of the material of which the solid is made and its capacity for the volume of liquid it can hold. Generally, capacity is expressed in litres, millilitres, and so on. It is also instructive to note that litres, etc. are also used to denote the volumes of liquids. Students may also be asked to note the following:

$1 \text{ mL} = 1 \text{ cm}^3$

$1 \text{ litre} = 1000 \text{ mL} = 1000 \text{ cm}^3$

$1\text{KL} = 1000 \text{ litres} = 1\text{m}^3$

Students may also be asked to attempt some questions related to litres and millilitres (some questions are there in Exercise 11.4).

Common Errors

(i) Students forget to take l, b and h in the same units for calculating the surface areas and volumes of solid shapes.

(ii) Many a times, the students calculate volume instead of surface area and calculate surface area instead of volume.

(iii) Many times, the students take 100 millilitres = 1 litre, which is not correct.

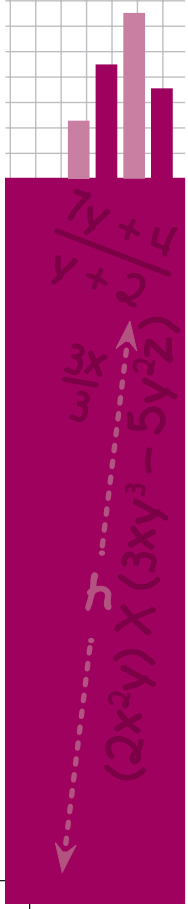
(iv) Sometimes students do error in mentioning the units of surface area or volume.

(v) Some students do not know that total surface area is written as surface area.

You may evaluate the students through the following exercise:

Exercise

1. Find the surface areas of the following :
   (i) Cuboid of length 10 cm, breadth 8 cm and height 3 cm
   (ii) Cube of edge 6 cm
2. Find the volume of the solids given in Question 1 above
3. Find the curved surface area of a cylinder which has radius 5 cm and height 10 cm. Also find its volume. (use π = 3.14)
4. Find the capacity of a container of internal dimensions 15 cm, 6 cm, 8 cm in litres.
5. Find the area of the four walls and the ceiling of a room of dimensions 5 m × 4 m × 3.5 m.
6. Compare the volume of the following cylinders :
   (a) Cylinder of radius 5 cm and height 8 cm, and
   (b) Cylinder of radius 10 cm and height 4 cm.



Block

Teaching of Data Handling

Children come to school with lots of information/data with them under different contexts. They can list names of their favourite movies, TV programmes, favourite dishes, favourite vehicles, telephone numbers, etc. Sometimes children are expected to read railway timetables, bus schedules, telephone directory, etc. and interpret them. They also come across a wide variety of information in the form of figures, tables and graphs, etc. But all this is available in an unorganised way. These information are referred to as **data.** These data help one in planning life better and to budget one's time and resources to optimum use. The data in the progress report of a child is an indicator of the academic performance of the child in the school. The activities for data handling should proceed from concrete to semi concrete and semi concrete to abstract. It is, therefore, essential to know how to extract or take out relevant information from such data and make these data meaningful.

Data Handling concerns itself not only with collection, organisation, presentation and tabulation of data but with their interpretation and drawing of meaningful inferences as well.

Finally, importance of data collection, its representation and interpretation in every field in the society may be linked

to the classroom experiences in this regard for better appreciation of the topic. Activities can be used to assess the learning.

This block has four units:

Unit 1: Introduction to Data Handling

Unit 2: Pictorial Representation of Data

Unit 3: Introduction to Measures of Central Tendency and Probability

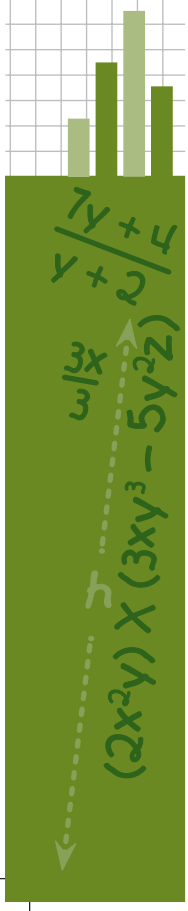
Unit 4: Introduction to Graphs

Unit 1 deals with collection of data and organisation of data in the form of tables.

Unit 2 deals with pictorial representation of data in the form of pictographs, bar-graphs, histograms and pie-charts.

Unit 3 introduces the concept of measures of central tendency namely, mean, mode and median of the data. The probability of occurrence of an event is also discussed briefly.

Unit 4 deals with different types of graphs such as line graphs, linear graphs and plotting of points in a Cartesian plane.



UNIT 1

Introduction to Data Handling

Structure

- Introduction
- Main Concepts and Sub-concepts
- Objectives
- Teaching Points
  1. Data and collection of data
  2. Organisation of data
  3. Grouping of data
- Common Errors
- Exercise

Introduction

In day-to-day life, children come across a wide variety of information, through newspapers, magazines, radio, television, etc., such as

1. Today, temperature of Delhi is the maximum in the last 51 years.
2. Kerala is the most literate state of all the Indian States.
3. Indian economy will grow 9% in 2010–11 — A Report
4. This year, the rains will be normal.
5. Smokers are more prone to lung cancer than those who do not smoke, etc.

Such statements are made on the basis of some information collected through various sources like field survey or information collected in the past years by some agency. Information collected is presented in the form of numbers. These facts/figures so presented are what we call the **data.** Thus, several investigations lead to a lot of data. To make this data meaningful, this unit will help you to collect, organise and interpret the data effectively.

Data is the plural form of the latin word 'datum'. We will discuss about data, collection of data and presentation of data in the form of tables. Each numerical figure in the data is referred as an **observation.**

Main Concepts and Sub-concepts

- Data-collection of data, raw data
- Organisation of data
- Preparation of tables using tally marks.
- Frequency of an observation in the data and making frequency distribution table.
- Class interval (class), size (width) of a class interval or class size

- Lower/upper class limits of a class
- Grouping data – grouped frequency distribution
- Making inferences from the frequency distribution tables.

Objectives

After teaching this unit, the students can

- collect the data from different sources around them;
- find the frequency of an observation in the data collected by them or in the given data;
- make frequency distribution table;
- understand the meanings of a class interval, class size and lower and upper class limits of a class;
- group the given data into class intervals of a convenient and equal size; and
- draw meaningful inferences from a given frequency distribution table,

Teaching Points

1. Data and Collection of Data

Teacher may divide the class into groups each of 4 or 5 students and keep each group engaged in collecting information from their immediate surroundings. Collecting information from the situations around the child/student will trigger interest and motivate the student to learn the topic. Some activities for which data may be collected are listed below. Some of them are given in the book. Teacher may make and add some more activities to this list.

(i) Names of the most favourite fruits of students.

(ii) Names of the games most preferred by the students.

(iii) Names of the localities from where the students come to attend the school.

(iv) Types of transport used by each student in coming to/going back from school.

(v) Blood group of students.

(vi) Days of the week of the births of students.

(vii) Marks obtained by students in mathematics in the last test.

(viii) Number of absentees in the class each day for a month.

(ix) Favourite colour of each student.

(x) Heights (in cm) of students in the class.

The data collected by students in each of the above activities is in an unorganised form and is called **raw data.**

Need for Data

Teacher may explain 'why do we need data'? by taking a few activities listed above.

For example:

(i) Class teacher is planning to go for a picnic with the students. The teacher may like to know the names of the fruits which the students like and arrange the quantity of each type of fruit accordingly. Thus, information of Activity (i) will be useful to her.

(ii) On the basis of the data collected in Activity (ii), the Principal of the school may arrange different games for the students.

(iii) On the basis of the data collected in Activity (iv), the transport incharge of the school may arrange transport for the students and fix monthly fares accordingly and so on.

In this way, using various examples, the teacher may explain that data is usually collected with some definite purpose in mind.

2. Organisation of Data

Teacher can initially ask a few students (around 10 or 12) to record the name of their favourite fruits, on the black board. Then at random she can ask any other student to quickly respond to 'How many students like mangoes?' She can do the same with few other students but at the same time she will address the problem that it is an undoubtedly cumbersome task to stand and count and it also leaves space for too many errors. In this way, she can emphasise that not only collection but organisation as well is important if we want to extract useful information from the data.

Such discussions motivate the children to organise the data first so that the possibilities of errors might be reduced.

Certainly for any required information each group will come up with a list.For example, suppose for Activity (i), the following is the list which one group has prepared (page 185 of Class VI, Mathematics, NCERT).

Name	Fruit
Mary	Banana
Preeti	Apple
Neha	Guava
Fatima	Orange
Amita	Banana
Radha	Orange
Farida	Guava
----------	----------
----------	----------

Now, teacher may discuss different ways to organise the data as follows:

(i) Teacher may now ask the group or the students to find out the number of each kind of fruit needed for picnic purpose. One choice will be that a student may encircle 'banana' with one colour and count

them. Similarly encircle 'apple' with another colour and then count their number and so on.

(ii) Ungrouped Data

Another way would be to write the names of fruits in the following way (one below the other) and mark a tick (✓) against each fruit as and when it comes. Then write the total number of ticks against each fruit as shown below:

Banana	✓	✓	✓	✓	✓	✓	✓	✓	8
Orange	✓	✓	✓						3
Apple	✓	✓	✓	✓	✓				5
Guava	✓	✓	✓	✓					4

(iii) Yet another way is to put a vertical bar 'I' in place of a tick mark(✓), as shown below:

Banana	I I I I I I I I	8
Orange	I I I	3
Apple	I I I I I	5
Guava	I I I I	4

(iv) It can also be written using $\cancel{\text{IIII}}$ in place of five vertical bars I I I I I, so that the above table becomes

Banana	$\cancel{\text{IIII}}$ I I I	8
Orange	I I I	3
Apple	$\cancel{\text{IIII}}$	5
Guava	I I I I	4

Why have we done so? The teacher may explain that it is convenient to count when the number of observations

(here it is banana, orange etc.) is large, These bars are called **tally marks.**

In the table above, 8 is the number of students who liked banana, 3 is the number of students who liked orange and so on. Here 8 is known as the **frequency** of banana, 3 is the **frequency** of orange and so on.

Thus, **frequency of an observation means the number of times that observation occurs for a particular data.** The table so obained is called **ungrouped frequency distribution table**.

Through the following example, teacher may reinforce the idea of frequency and frequency distribution table.

Example

An election was held in Class VI having 31 students, to select a Class Monitor. The candidates were Amana, Jaina, Anvi, Mala and Rubi. The students voted as follows:

Rubi,	Jaina,	Anvi,	Mala,	Amana,	Anvi,	Amana
Jaina,	Anvi,	Rubi,	Mala,	Amana,	Anvi	
Jaina,	Rubi,	Mala,	Anvi,	Anvi	Mala	
Amana,	Jaina,	Jaina,	Anvi,	Mala,	Jaina	
Anvi,	Anvi,	Amana,	Mala,	Jaina,	Rubi	

Make a table and enter the data using tally marks. Find the name of the student who got maximum votes and the name of the student who got the least votes.

Ask the students to prepare a table as given below:

Name	Tally Marks	Number of Votes
Amana		
Jaina		
Anvi		
Mala		
Rubi		

Start from the first name (of the earlier table)- Rubi (top left). Put a tally mark against her name. Then go to second name Jaina (in the first row), put a tally mark against the name Jaina. Go on like this till all the names are exhausted. If there are four tally marks (𝍸) against a name and there is one more tally mark against that name then make a bunch of five as 𝍸 .

Finally the table may be completed as follows:

Name	Tally Marks	Number of Students
Amana	𝍸	5
Jaina	𝍸 |	7
Anvi	𝍸 ||||	9
Mala	𝍸 |	6
Rubi	||||	4
Total		**31**

After completing this activity, teacher can ask students as to what useful information they can extract from this data. She can ask them to list a few things as:

- Votes– Highest, Lowest, Comparison between any two (by how much) (one lesser, one greater),
- Which two people got votes having sum total equal to votes by Anvi?

From the above table, let the students find out who got the maximum votes (Here it is Anvi) and who got the minimum votes (Here it is Rubi). The teacher may help them to see that Amana has got 5 votes. So, frequency is 5 for Amana. Similarly, frequency of Jaina is 7 and so on.

The teacher should also highlight the importance of this frequency distribution. From the table, it is very easy to find out who got how many votes, who got the minimum or maximum votes, etc.

Ask the students to enter the data of the activities suggested, i.e. Activities (i) to (x) in a table using tally marks and ask some questions of the type as discussed in the above example.

Further, let the students try questions 1 to 3 of Exercise 9.1, page 192 of Class VI Mathematics, NCERT and also Example 2 on page 187.

Upto Class VI, the use of terms such as frequency, frequency distribution, etc. may be avoided. However, these terms have to be used in Class VIII.

3. Grouping of Data

In Class VI, the students have learnt to:

(i) collect data;

(ii) enter the data in a table using tally marks; and

(iii) draw inference from the table.

No grouping of data has been done at this stage. In Class VIII, the students will learn grouping of data. Also the need for grouping has to be emphasised by the teacher.

Let the students feel the need of grouping the data themselves. Go back to Activity vii. Let this activity be done for two sections of Class VI. Suppose the marks (out of 100) obtained by the students of both the sections in Mathematics are as follows:

Section A

16, 13, 5, 80, 86, 7, 51, 48, 24, 56, 70, 19, 61, 17,
16, 36, 34, 42, 34, 35, 72, 55, 75, 31, 52, 28, 72,
97, 94, 45

Section B

62, 68, 86, 34, 86, 36, 81, 74, 54, 26, 94,
31, 7, 79, 92, 62, 52, 55, 14, 62, 24, 36,
55, 44, 47, 27, 72, 17, 4, 30

Ask the students to enter the data in a table using tally

marks as they have done in Class VI. Let them try! See, if they have some problems. If they proceed in this way, the table will be very lengthy and not manageable also. Moreover, it will be difficult to draw meaningful inferences from that list.

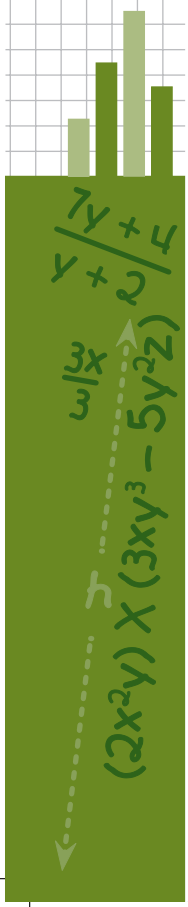
Now tell the students to make groups of observations, as follows:

0 - 10, 10 - 20, 20 - 30, 30 - 40, 40 -50, 50 - 60,
60 - 70, 70 - 80, 80 - 90, 90 -100,

In the group, 0 - 10, include all those observations which are less than 10. In the group 10 - 20, include all those observations which are equal to and above10 but less than 20. Similarly, in the group 20 - 30, include all those observations which are equal to and above 20 but less than 30 and so on.

Ask the students to obtain the following distribution table:

Group	Tally Marks	Frequency
0 - 10	𝍸	4
10 - 20	𝍸 |	7
20 - 30	𝍸	5
30 - 40	𝍸 𝍸	10
40 - 50	𝍸	5
50 - 60	𝍸 |||	8
60 - 70	𝍸	5
70 - 80	𝍸 |	7
80 - 90	𝍸	5
90 - 100	𝍸	4
Total		60



Tell the students that:

(i) Data presented in this form is said to be **grouped data** and the distribution so obtained is called **grouped frequency distribution.*

(ii) The groups 0 - 10, 10 - 20, 20 - 30, etc. are called the **class intervals** or simply **classes**.

(iii) For the class 0 - 10, 0 is called the lower class limit and 10 is called the upper class limit. For the class 10 - 20, 10 is the lower class limit and 20 is the upper class limit and so on.

(iv) The numbers 4, 7, 5, ... are the frequencies of the classes 0 - 10, 10 - 20, 20 -30,..... respectively.

(v) The difference between the highest and the lowest observation in the data is called the **range** of the data. **It helps us in deciding the number of classes to be made for forming a grouped frequency distribution for the given data.**

(vi) Difference between the upper class limit and the lower class limit for each of the classes 0 - 10, 10-20, 20 - 30, is the same (10 in this case). This difference is called the **width** or **size** of the class.

(vii) The sum of all the frequencies in the table is the same as the total number of observations in the given data.

Ask the students to observe the table and answer the questions like:

(a) How many students have scored marks below 10?

(b) How many students have scored marks 10 and above but below 20?

(c) How many students have scored marks 40 and above but less than 50?

(d) Which group has the maximum frequency?

(e) Which group has the least frequency?
and so on.

* Such type of grouped frequency distribution is also called a Continuous grouped frequency distribution.

The data above can also be grouped into classes as 0 - 9, 10 - 19, 20 - 29, etc. and grouped frequency distribution** will be as follows:

Class	Tally Marks	Frequency
0 - 9	𝍸	4
10 - 19	𝍸 |	7
20 - 29	𝍸	5
30 - 39	𝍸 𝍸	10
40 - 49	𝍸	5
50 - 59	𝍸 |||	8
60 - 69	𝍸	5
70 - 79	𝍸 |	7
80 - 89	𝍸	5
90 - 99	𝍸	4
Total		60

Here in the class 0 - 9, all the observations from 0 to 9 have been taken. Similarly in the class 10 - 19, all the observations from 10 to 19 have been taken and so on.

It differs from the way when we take classes as 0-10, 10-20, etc. Teacher may ask students here to explain the difference in the groupings in the two cases (i.e. when we record as 0-10, 10-20, --- and as 0-9, 10-19, ------,).

Tell them to group the same data with classes as

(i) 0-5, 5-10, ..., 95-100

(ii) 0-20, 20-40, ..., 80-100

and compare which class intervals interpretation was best for the data.

Ask the students to do Questions 1 and 2 on pages 73 and 74 of Mathematics Textbook for Class VIII.

** Such distribution is called a **discrete** grouped frequency distribution.

Common Errors

(i) In making a bunch of 5 tally marks, the students sometimes write it as 𝍸 or 𝍸 in place of 𝍸 .

(ii) 𝍸 is sometimes counted as 4 instead of 5.

(iii) In the classes 10 - 20, 20 - 30, etc. 20 is sometimes taken in the class 10 - 20 and sometimes in both the classes 10 - 20 and 20 - 30 some students. It has to be taken in class 20 - 30 only.

(iv) Emphasise students to carefully observe what class size (or width) is provided for these groupings: Students are often in the habit of choosing class intervals of width10.

You may evaluate the students through the following exercise.

Exercise

1. Sara did a survey of the buildings in her locality. The information was recorded in the following table. Complete the table.

Buildings	Tally Marks	Number of Buildings
Shops	𝍸	
Houses	𝍸 𝍸 𝍸 𝍸 𝍸 𝍸 ||	
Offices	𝍸 ||||	
Places of Worship	||	

2. Students of Class VI were asked to name their favourite pets. The obtained information is given as follows:

Rabbit, Cat, Dog, Cat, Rabbit, Fish, Pigeon,
Cat, Fish, Dog, Cat, Fish, Cat, Pigeon,
Parrot, Cat, Dog, Rabbit, Pigeon, Rabbit,

Enter the data in a table using tally marks.

3. "MATHEMATICS SHOULD NOT ONLY BE TAUGHT BECAUSE IT IS USEFUL. IT SHOULD BE A SOURCE OF DELIGHT AND WONDER TO ALL CHILDREN, WHATEVER BE THEIR AGE".

Using tally marks, record in the following table, the number of times each vowel appears in the above paragraph:

Table

Vowel	Tally Marks	Number of times the vowel appears
A		
E		
I		
O		
U		

4. Each student in a class of 30 students was asked to tell whether he/she would like to be a police officer, a teacher, a doctor, a lawyer or an engineer. The list below shows their choices:

Police officer, Teacher, Doctor, Lawyer, Teacher,
Engineer, Doctor, Doctor, Police officer, Teacher
Doctor, Lawyer, Doctor, Engineer, Police officer
Police officer, Teacher, Lawyer, Engineer, Doctor
Engineer, Teacher, Doctor, Lawyer, Police officer

Using tally marks, record the above information in a table. From the table, tell, which profession is liked by most of the students.

5. The heights (in cm) of 40 persons are listed below:

139,	165,	151,	131,	144,	125,	150,	158	146,
159,	140,	148,	136,	148,	150,	144,	168,	125,
139,	176,	163,	153,	156,	160,	146,	170,	142,
147,	138,	153,	140,	170,	160,	145,	138,	148,
150,	154,	149,	148.					

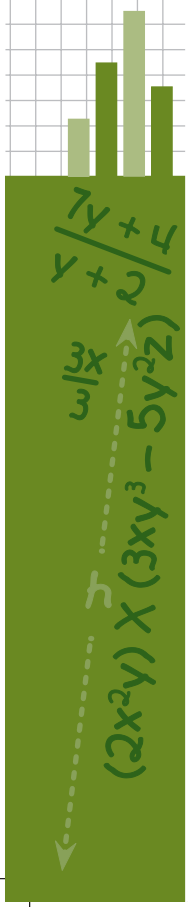
Using the classes 125-130, 130-35 etc. make a grouped frequency distribution of the data.

2 UNIT

Pictorial Representation of Data

Structure

- Introduction
- Main Concepts and Sub-concepts
- Objectives
- Teaching Points
  1. Pictograph
  2. Bar graph
  3. Histogram
  4. Pie Chart or Circle Graph
- Common Errors
- Exercise



Introduction

In Unit I, the following aspects of Data Handling were discussed:

(i) Data and collection of data

(ii) Organisation of data in the form of ungrouped frequency distribution table/grouped frequency distribution table.

Once the data is collected/presented in tabular form, we need to represent it in the form of a diagram/picture.

In fact, to study the characteristics of the data, it is quite helpful to use pictures. Representing numerical data through pictures is called pictorial or graphical representation of data.

In this Unit, the following forms of pictorial representation of the data will be discussed:

(i) Pictograph

(ii) Bar Graph

(iii) Histogram

(iv) Pie Chart

Main Concepts and Sub-concepts

- Pictograph – reading and drawing
- Bar graph – reading and drawing
- Histogram – reading and drawing
- Pie chart – reading and drawing

Objectives

After teaching this unit the students can

- interpret and draw a pictograph;
- interpret and draw a bar graph;
- draw a double bar graph and draw inferences from it;

- read a pie chart and draw inferences from it;
- draw a pie chart;
- read a histogram and draw inferences from it; and
- draw a histogram.

Teaching Points

Teacher can pick up some newspaper or magazine which shows different ways of representing a data pictorially, e.g.

(i) Pictograph – Example of a Pictograph and its appropriateness for given data

(ii) Bar graph – Some examples of Bar graph and its relevance for data

(iii) Histogram – Relevance for data

(iv) Pie Chart

Initially a teacher should avoid giving them the names and let them discover best possible name on their own realizing its due significance.

But showing all these together might not be good for learners. So, each of them should be introduced at due stage clearly explaining its relevance and need.

1. PICTOGRAPH

A pictograph represents data through pictures of objects. It helps in answering the questions about the data at a glance.

Teacher may ask the students to collect some pictographs from newspapers and magazines as a home-work activity.

Now the teacher may refer to pictograph given in Example 3 on Page 189 of Class VI Mathematics, NCERT and ask questions (a), (b) and (c).

Here, one picture represents 1 absentee. So, it is easy to answer all the questions. But in next Example 4, one picture represents 10 people.

i.e.,  = 10 people

So,  will represent $\frac{10}{2} = 5$ people.

Similarly in Example 6, half photo of a wrist watch represents $\frac{100}{2} = 50$ wrist watches.

It is easy to visualise $\frac{1}{2}$ part of the picture but if it is $\frac{1}{4}$ or some other part of the picture, it is difficult for the students to read the picture and draw conclusions. So, in reading a pictograph, **teacher should restrict to $\frac{1}{2}$ of the picture.**

After discussing solved examples, teacher should help the students in reading pictographs given in Questions 4 to 7, page 192-194 of Class VI Mathematics, NCERT. There should be no difficulty for the students in these pictographs because the pictures are restricted to half only.

Drawing a pictograph

(i) To draw a pictograph using picture of an elephant, tractor, etc. is difficult for the students. Here, teacher may emphasise that the students can choose a simple picture like a circle, star, smiley face, triangle, square, etc. in place of an elephant, tractor, etc.

(ii) Different students may use different symbols (pictures) for the same item but in a particular question, a student must use the same symbol throughout.

(This point should be emphasised more as students have tendency of choosing different pictures).

(iii) In Example 7, data given (Number of students present) are such that they do not have a common multiple. Using the picture to represent 6 students,

24 will be represented by four such pictures. But what about 29? 29 is a little less than 30. So, to represent 29, 5 such pictures are needed with little cut in one picture. Such minor/little cut is sometimes not even visible. Same type of difficulty will come in Question 2 of Exercise 9.2 when 535 has to be shown through pictures. **Through such examples, let the students realise that representing data by pictograph is not only time consuming but at many times difficult also. So, there is a need to find other ways of representing data pictorially.**

2. Bar Graph

(i) A bar graph is a pictorial representation of data in which bars of uniform width and varying heights are drawn with equal spacing between them (no picture/ symbol as in the case of pictograph). Here, teacher may explain the benefit of bar graph over pictograph.

Bars are drawn at equal spaces either on the horizontal line or on the vertical line, as shown on page 197 of Class VI, Mathematics, NCERT. Length of each bar shows the frequency of each observation. Bars can be shaded or coloured to look more attractive. Sometimes vertical line segments/ horizontal line segments are drawn in place of bars/ rectangles. In that case, it is referred to as line-chart.

A line-chart (also a bar graph) for data representing population of India in different years as given on page 197 of Class VI Mathematics, NCERT is shown in Fig.1.

(ii) Let the students observe the bar graph given on page 197 (Class VI, Mathematics, NCERT). On the top of each bar, a number is written. Why is it so? It is so, because without this number it will be very difficult to read population of India say in 1951 as 36 comes between 30 and 40. So, 36 is little more than 35.

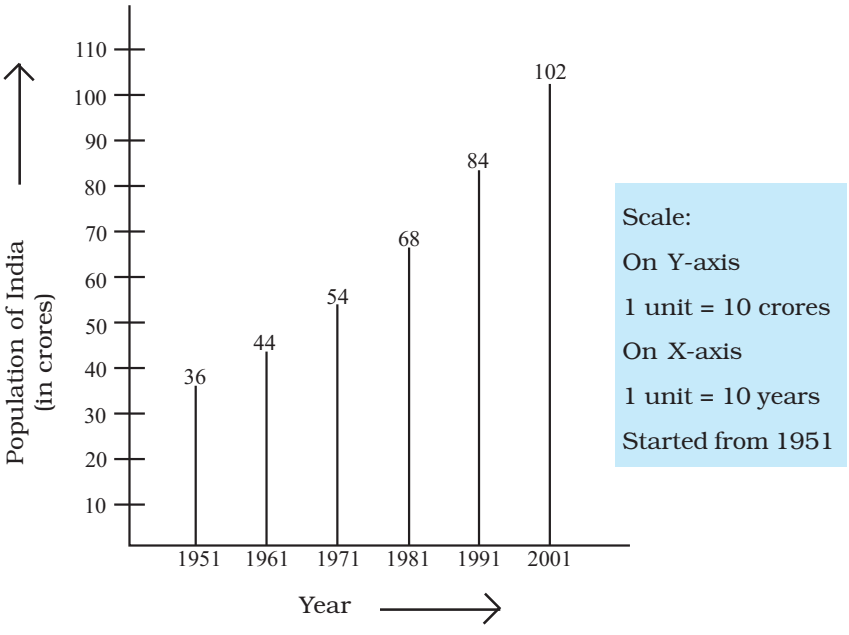


Fig. 1

The bar or rectangle will have length/height slightly more than 35 on the vertical line.

(iii) Tell the students that the width of a rectangle/ bar has no meaning to convey except to make the presentation more attractive.

(iv) Bars can be drawn horizontally also. However, generally vertical bars are preferred. Ask the students to attempt Questions 1 to 3 in Exercise 9.3 of Class VI Mathematics, NCERT.

Here students may be asked to draw in one of the question as horizontal bar, so that they will be familiar with both types of bar graphs.

Drawing a bar graph

(i) Students may be suggested to use initially a graph paper for drawing a bar graph, as drawing bars on a graph paper is convenient. But slowly and slowly, they should be asked to draw bar graphs without using a graph paper.

(ii) Choose a convenient scale keeping in view the range of the given data and write it on the top of the bar graph. Teacher should explain the scale clearly, specifically addressing if 10 units mean 50 books, then what will 1 unit mean?

Teacher may ask the students to draw bar graphs for questions in Exercise 9.4 (Class VI) using bars horizontally as well as vertically in each question.

Why a double bar graph?

In Class VII, double bar graphs have been discussed. Double bar graph has a different purpose. For example, consider the following table which gives maximum and minimum temperatures of Delhi on April 30 for different years:

Year	Maximum temperature (°C)	Minimum temperature (°C)
2004	37	26
2005	35	22
2006	41	25
2007	42	24
2008	41	21
2009	43.6	24.4

Students know how to draw individual bar graphs (one for maximum temperature and another for minimum temperature). From the bar graphs drawn, the students can answer question like:

(i) In which year was the temperature of Delhi maximum?

or

(ii) In which year was the temperature of Delhi minimum?

However, to answer questions like,

"In which year, the variation in the two temperatures was maximum or minimum? It will be helpful to draw both the bar graphs on the same sheet/graph paper as shown below:

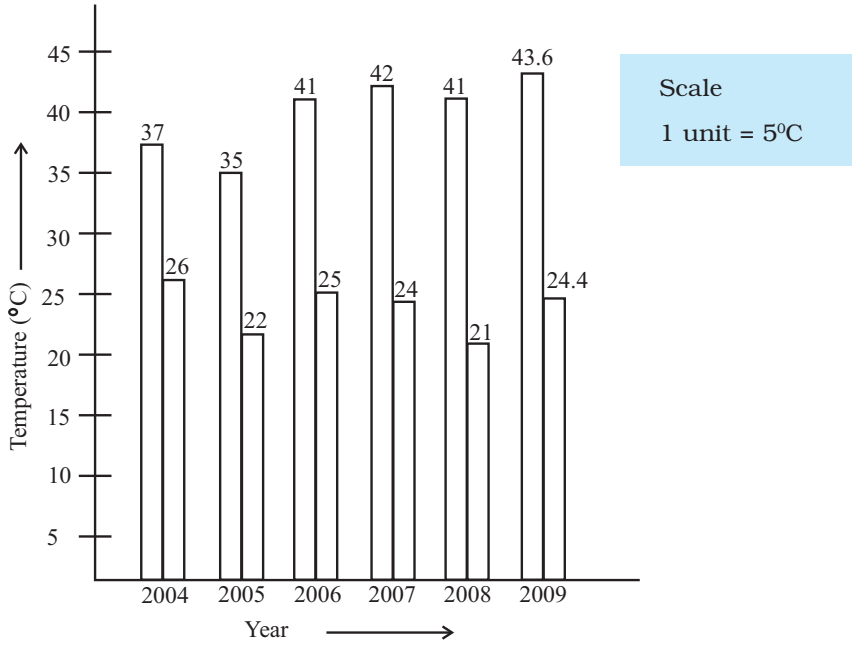


Fig. 2

This is called a **double bar graph**. Just by looking at the graph, one can say that the variation in the two temperatures is maximum in 2008 and minimum in 2004.

Double bar graphs are useful in situations like data on import and export of a country in different years, production of two crops such as wheat and rice of different states in a particular year, etc.

Teacher may ask the students to collect data of above types and construct double bar graphs. Discuss solved Example 10 on page 71 of Class VII, Mathematics, NCERT and highlight the importance of the double bar graph in the context. Let the students draw double bar graphs for the data given in Questions 4 and 5 of Exercise 3.3 of Class VII Mathematics, NCERT and answer the corresponding questions.

(In Question 5, students should be given the idea about plotting of 423 and 105 after rounding them off, e.g. 423 ⟶ 420, 105 ⟶ 110 because the scale would be 1 unit = 100).

3. HISTOGRAM

In Unit 1, we have discussed grouping of data and construction of grouped frequency distribution. A grouped frequency distribution of the type as given on page 74 of Mathematics Textbook for Class VIII, can be represented pictorially as in Fig. 5.1 of the book. This pictorial representation is known as histogram.

Ask the students to find the difference between a bar graph and a **histogram**. Tell them that in a histogram:

(i) Group of observations (class intervals) are represented along the horizontal axis.

(ii) Unlike bar graph, there is no gap between the rectangles (bars) in a histogram as there is no gap between the class intervals.

(iii) The height of each rectangle shows the frequency of its class interval.

(iv) The area of the rectangle on any class interval is proportional to the frequency in that class interval. Since all the class intervals are of equal width, therefore heights of the rectangles are kept proportional to the corresponding frequencies.

In case, the class widths are different, we have to adjust heights of the rectangles accordingly which will be discussed in Class IX.

(v) In Fig 5.2 on page 75 of Class VIII, Mathematics, NCERT, a jagged line/jumping line of the type "⩘" has been shown on the horizontal axis. Meaning of this symbol has to be explained to the students as follows:

Since the scale on the horizontal axis starts at 20, this symbol is shown near the origin to signify that

the graph is drawn to scale beginning with 20 and not with the origin (0).

Morever, we are compressing the length along horizontal axis from 0 to 20 (since there is no teacher between the age 0 to 20, so taking 0-5, 5-10, 10-15, 15-20 with no bar above horizontal is merely a waste of paper), therefore ' ⩘ ' is drawn to show that the length is compressed. This symbol is read as 'kink'. Students can also be told that class size in Fig. 5.2 is actually 5 but it appears as 20 for the interval 0-20 and so we need a kink to avoid the discrepancy.

(vi) Students get confused about whether for the first rectangle, there would be a line coinciding with vertical axis or not. So, they should be told that there would be a line to coincide with the vertical axis only if the class interval starting from origin have a frequency to be represented, as in Fig. 5.1. But in Question 5 of Exercise 5.1, since there are no students who watch T.V. for 1 hour, so there will not be a line to coincide with vertical axis.

Help the students to do Questions 4 and 5 of Exercise 5.1, Class VIII Mathematics, NCERT.

4. PIE CHART OR CIRCLE GRAPH

In bar graphs, we have compared the individual items with one another. Sometimes, it is necessary to compare parts (or proportion) of the individual with reference to total as shown in Fig 5.4 (i) and (ii) on page 77 of Class VIII, Mathematics, NCERT.

In Fig 5.4 (i), total time is 24 hours (a day).

In Fig 5.4 (ii), total population is 100 thousand.

Fig 5.4 (i) and (ii) are called Pie charts or Circle graphs.

In a pie chart, the various observations or components are represented by the sectors of a circle and the whole circle represents the sum of the values of the components.

Teacher may give some situations which require the use of a pie chart when information is required to be shown pictorially. Some situations are given below:

(i) 50% students of a class are boys and 50% are girls.

(ii) The way of coming to school by students of a class:

Walking :	40%
Bus :	30%
Cycle :	30%

(iii) In a particular day, the sales in a Bakery shop are:

White bread	:	₹ 2600
Fruit bread	:	₹ 400
Cakes/pastries	:	₹ 1000
Biscuits	:	₹ 520
Others	:	₹ 280
	Total	₹ 4800

(iv) Total strength of a school is 720. The following are the votes in favour of

Sushma	200
Ramana	240
Sunita	120
Reshma	160

Teacher may ask the students to give some more situations in day-to-day life which require the use of pie charts. Example 1 on page 79 of Class VIII Mathematics, NCERT deals with reading a pie chart and then answering some questions based on it.

Looking at the pie chart in Fig 5.7, one can see that expenditure is maximum on food because expenditure on food is 25% which is more than on any other item. Also, from the pie chart, one can see that the area of the sector*

* The term "sector" should be discussed before procedings.

related to food is the biggest. In the same Example 1, Fig. 5.7, part (iii) needs discussion to be initiated by the teacher, since it involves some part of calculation which is not directly visible in the pie chart.

For example, it is given that monthly savings = Rs 3000

Let the total money with family = ₹ x

Then,

15% of ₹ x = ₹ 3000

i.e. $\frac{15}{100} \times x = 3000$

or $x =$ ₹ $\frac{3000 \times 100}{15} = 20000$

So, the total money is ₹ 20000.

Now this total money with the family can help us to find the money corresponding to the other sectors.

e.g. House Rent = 10% of ₹ x = ₹ $\frac{10}{100} \times 20000$

= ₹ 2000

Clothes = 10% of ₹ x = ₹ 2000

Drawing a pie chart

Teacher should tell the students that before constructing a pie chart, it is important for us to know the central angle of each sector which is a part of complete angle (360°). So, we will first make a table which will give us the fraction represented by each sector that will further help us to find the central angle of each sector.

(i) When information is given in percentages

Example A survey was made to know the favourite flavours of ice cream for students of a school. Information received is as below:

Flavour	**Percentage**
Chocolate	50%

Vanilla	30%
Strawberry	10%
Other	10%

To draw pie chart for this information (data), teacher may ask the students to prepare a table of the type as given on page 79 of Class VIII, Mathematics, NCERT. The percentage of each flavour has to be converted into a fraction of the total. Since total angle at the centre of a circle is 360^o, so the central angle of each of the sectors will be a fraction of 360^o. This fraction is called the central angle of the sector. The central angle has to be calculated for each observation. All the angles calculated will add up to 360^o, Larger the observation, larger the central angle in the pie chart.

(ii) When information is given in terms of units such as Rs, kg, metres, hours, etc.

Example: The number of hours spent by a student on different activities on a working day is given below:

Activity	Number of hours
Sleeping	8
Schooling	7
Homework	4
Playing	2
Others	3
Total	**24**

To draw a pie chart for this type of data, find central angle of the sector for each activity as given here:

Activity	Number of hours	Fraction	Central angle
Sleeping	8	$\frac{8}{24}=\frac{1}{3}$	$\frac{1}{3}\times360°=120°$
Schooling	7	$\frac{7}{24}$	$\frac{7}{24}\times360°=105°$
Homework	4	$\frac{4}{24}=\frac{1}{6}$	$\frac{1}{6}\times360°=60°$
Playing	2	$\frac{2}{24}=\frac{1}{12}$	$\frac{1}{12}\times360°=30°$
Others	3	$\frac{3}{24}=\frac{1}{8}$	$\frac{1}{8}\times360°=45°$
Total	**24**		**360°**

Once the central angles are known, then proceed to draw the pie chart accordingly as discussed in the textbook.

Teacher may now ask the students to do Examples 1 and 2 on pages 79-80 and Excerise 5.2 of Class VIII Mathematics, NCERT. Teacher may also give some data and ask which form of graph would be appropriate to display that data. For example, refer to Questions 1 to 3 on page 81 of Class VIII Mathematics, NCERT.

Common Errors

(i) In a bar graph/histogram, rectangles (bars) of uniform width are not drawn.

(ii) In a bar graph, equal spacing between rectangles (bars) is not taken.

(iii) Labelling on horizontal and vertical lines is not done in a bar graph/histogram.

(iv) Scale of the graph is not mentioned.

(v) In a pie chart, proper labelling is not done.

(vi) Sometimes, students draw histogram, just like bar graph. So it needs to be emphasised that there are no gaps in histograms.

The teacher may evaluate the students by the following exercise.

Exercise

1. The number of students admitted in different classes of a school are given below:

Class	VI	VII	VIII	IX	X
Number of students	200	240	140	80	60

Draw a pictograph to represent this information.

2. Students of Class VI were asked about the months of their births and the following bar graph was prepared for the data so obtained.

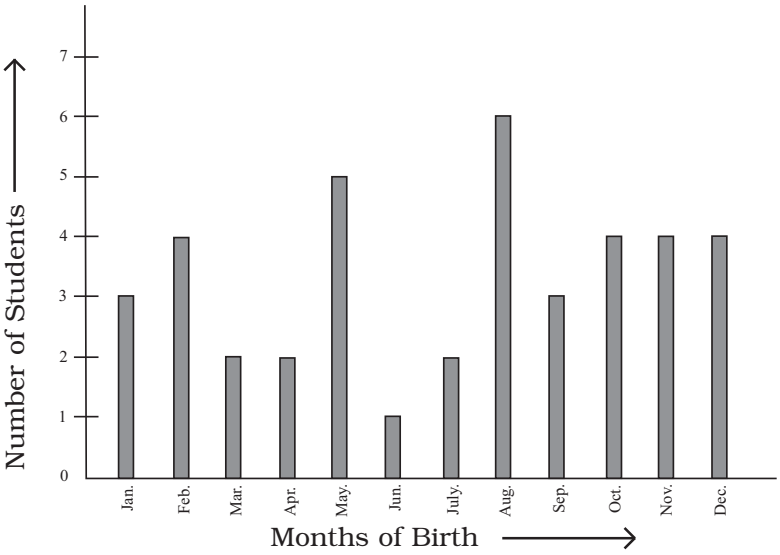


Fig. 3

Observe the bar graph given above and answer the following questions:

(i) How many students were born in the month of May?

(ii) In which month, were the maximum number of students born?

(iii) In which month, were the minimum number of students born?

(iv) What was the total number of students in the class?

3. The bar graph given below shows the number of bottles of soft drink bought by a group of friends in a certain week:

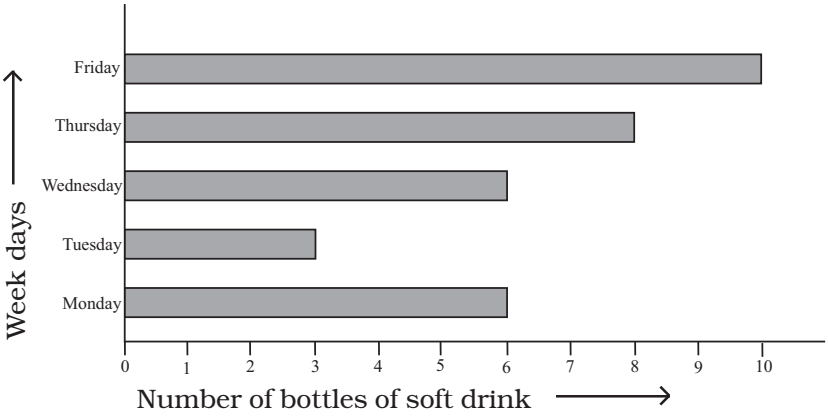


Fig. 4

Answer the following questions:

(i) How many bottles of soft drink were bought on Friday?

(ii) How many more bottles were bought on Thursday than on Tuesday?

(iii) If a bottle of soft drink costs ₹ 10, how much was spent on soft drink on Monday?

4. The following data gives the amount of loans given by a bank in different years:

Year	**Loan (in crores of rupees)**
2002	20
2003	25
2004	30
2005	40
2006	65

Construct a bar graph representing the above data.

5. The population (in lakhs) of a city in different years is as given below:

Year	**Population (in lakhs)**
2001	15
2002	16
2003	18
2004	22
2005	30
2006	35
2007	38
2008	42

Represent the above information with the help of a bar graph. From the graph, find in which year the increase in population was maximum and in which year it was minimum?

6. Bar graph of the production of rice and wheat crops in some states in the year 1984-85 is given below:

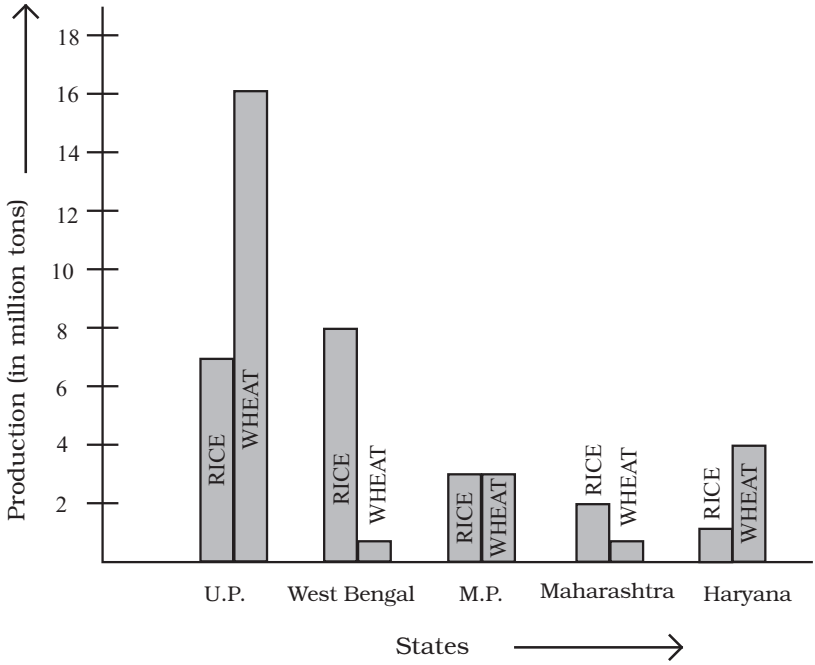


Fig. 5

Read the bar graph given above and answer the following questions:

(i) What information is given by the bar graph?

(ii) Which of the above states is the largest producer of wheat?

(iii) Which of the above states is the largest producer of rice?

(iv) In which of the above states, the total production of rice and wheat is the least? Also, in which state, it is the largest?

7. Study the histogram given below and answer the following questions:

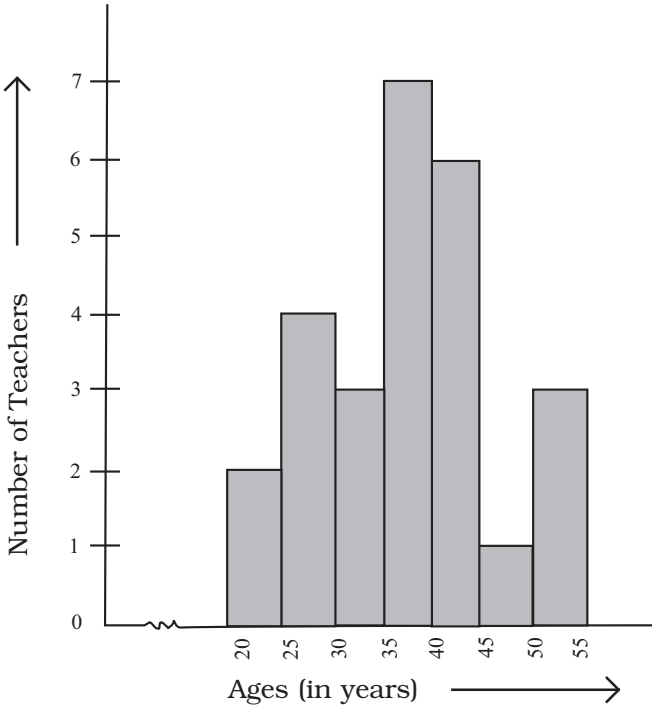


Fig. 6

(i) What information is depicted by the above histogram?

(ii) What is the number of teachers which are in the oldest age group in the school?

(iii) Write the number of teachers in the youngest age group in the school?

(iv) In which age group, the number of teachers is the least?

(v) In which age group, the number of teachers is the maximum?

8. The following table gives the weights (in kg) of 100 students:

Weight (in kg)	Number of Students
40 – 45	10
45 – 50	30
50 – 55	20
55 – 60	25
60 – 65	15

Construct a histogram for the above data.

9. The adjoining pie chart gives the expenditure on various items and savings of a family during a month

Answer the following questions based on the pie chart.

(i) On which item, the expenditure is maximum?

(ii) Expenditure on which two items, is the same?

(iii) If the monthly savings is ₹ 1500, what is the monthly expenditure on food?

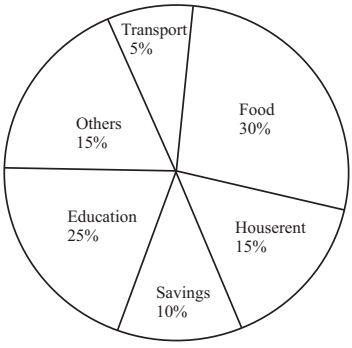


Fig. 7

(iv) Combined expenditure on which two items is equal to the expenditure on house rent?

10. The following pie chart represents the number of valid votes polled, which were 720. Answer the following questions by reading the pie chart:

(i) Who has won the election?

(ii) What is the minimum number of votes obtained by any candidate?

(iii) By how many votes did the winner defeat the nearest rival?

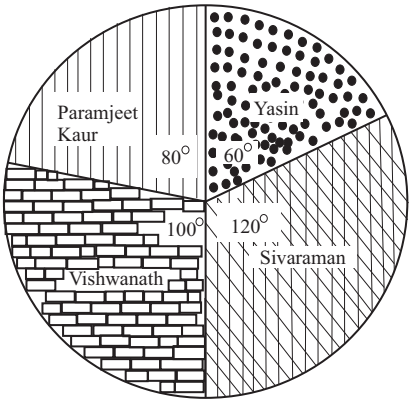


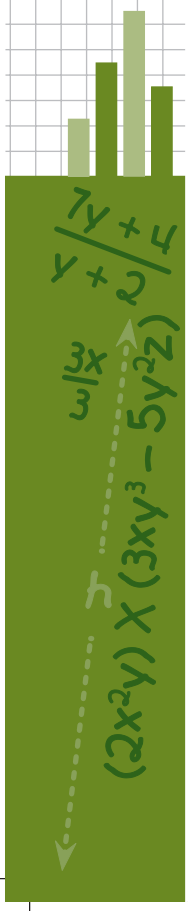
Fig. 8

3 UNIT

Measures of Central Tendency and Probability

Structure

- Introduction
- Main Concepts and Sub-concepts
- Objectives
- Teaching Points
  1. Arithmetic Mean
  2. Mode
  3. Median
  4. Chance and Probability
- Common Errors
- Exercise



Introduction

In Units 1 and 2, the following aspects of Data Handling have been discussed:

(i) Data and collection of data

(ii) Organisation of data

(iii) Pictorial representation of data in the form of pictographs, bar graphs, histograms and pie charts.

Pictorial representation of data helps in interpreting the data at a glance. These representations have certain limitations. So, there is a need to investigate certain measures which summarise the main features of the data. To draw conclusions from data, it is useful to calculate averages. An average indicates the typical value of a data. For example, in a class, let there be two groups A and B. In a mathematics test, the marks (out of 50) obtained by the students of these two groups are given as follows:

Group A: 26, 16, 24, 36, 28, 20, 18, 12, 28, 38, 48, 40, 20, 15, 36,

Group B: 20, 24, 16, 36, 38, 42, 48, 46, 32, 18

Now the question is : How will you compare the performances of the students of two groups? Certainly it will not be meaningful, if we compare the best marks of each group or the worst marks of each group.

So, there is a need to find some way which can summarise the performance of each group. Such ways provide certain measures are called the measures of central tendency. In this unit, some measures of central tendency like mean, mode and median will be discussed for ungrouped data. Finally some, basic concepts of probability will also be discussed.

Main Concepts and Sub-concepts

- Arithmetic mean or mean of ungrouped data
- Mode of ungrouped data

- Median of ungrouped data
- Idea of probability
- Events which are certain to happen, impossible to happen and which may or may not happen.
- Equally likely and not equally likely outcomes and probability of an event.

Objectives

After teaching this unit the students can

- find the arithmetic mean of ungrouped data;
- find mode of ungrouped data;
- find median of ungrouped data;
- identify the situations (events) which are certain to happen impossible to happen and which may or may not happen; and
- understand the notion of probability.

Teaching Points

1. Arithmetic Mean

Teacher may first introduce the term 'average' by taking some specific statements involving the term 'average' from day-to-day life such as "Isha spends an average of about 5 hours daily for her studies". (page 59 – 60 of Class III, Mathematics, NCERT).

Ask the students – Does it mean that Isha spends 5 hours daily on her studies?

No! She may spend less than 5 hours a day, for some days, more than 5 hours a day for some days or exactly 5 hours a day for some days.

Then, the teacher, may explain how this figure of 5 hours has come. She first finds the number of hours actually spent by Isha daily on her studies for each day of a week:

Monday: 4 hours, **Tuesday:** 5 hours,

Wednesday: 4.5 hours **Thursday:** 5.8 hours,

Friday: 4.8 hours, **Saturday:** 5.2 hours

Sunday: 6 hours.

$$\text{Average time spent} = \frac{\text{Sum of hours spent on studies on all seven days}}{\text{Number of days of a week}}$$

$$= \frac{4+5+4.5+5.8+4.8+5.2+6}{7} \text{ hours}$$

$$= \frac{35.3}{7} \text{ hours} = 5.04 \text{ hours}$$

So, Isha spends on an average about 5 hours daily on her studies.

Now teacher may explain that average is a number that represents or shows the central tendency of a group of observations or data. Observe that some values of data are more than, some are less than or some are equal to the average. Observe also that 5 lies between the highest and the lowest value of the data above. Teacher may now explain that average is a number **that lies between the highest and the lowest values of the given data.** This average is commonly known as **arithmetic mean** or simply mean. **Arithmetic mean** (or **mean**) of the observations of data is defined as

$$\text{Arithmetic Mean} = \frac{\text{Sum of all observations}}{\text{Number of observations}}$$

Teacher may now refer back to the example of marks obtained by two groups given in "Introduction".

$$\text{Mean of marks in Group A} = \frac{\text{Sum of all marks}}{\text{Number of students in Group A}} = \frac{405}{15} = 27$$

$$\text{Mean of marks in Group B} = \frac{\text{Sum of all marks}}{\text{Number of students in Group B}} = \frac{320}{10} = 32$$

If we compare mean marks of both the groups, we can say that students of group B have done better than those of Group A.

In the example of marks obtained by two Groups A and B, 27 is the mean marks of Group A which is not one of the observations, whereas in Group B, mean marks is 32 and it is one of the observations. However, both the means lie between the maximum and the minimum value of the data.

Now the students may be asked to do Examples 1, 2 and 3 given on pages 60-62 of Class VII Mathematics, NCERT.

In Example 1, mean = 4 hours a day, which itself is one of the observations.

In Example 2, mean = 47, which is not one of the observations.

In Example 3, mean = 35 which is again one of the observations.

From these examples, teacher has to highlight the fact that

(i) Mean of a data may or may not be one of the observations in the data.

(ii) Mean always lies in between the greatest and the smallest observations in the data.

Teacher may motivate the students to understand the concept of mean through the following activity.

Activity 1: Arrange the students in groups of say 5 and ask them to write names of children, one name on one squared sheet in their family as shown below: (for one group)

Student 1: Raj Mani

Student 2: Ravi Vibha Radha

Student 3: Anita Suman Hari Vinit Abha Saba

Student 4: Anvi

Student 5: Naman Kavita Savita

Now ask each group to redistribute these sheets among themselves so that each student gets the same number of sheets. In the above group after redistribution equally we may have:

Raj	Mani	Saba
Ravi	Vibha	Radha
Anita	Suman	Hari
Anvi	Abha	Vinit
Naman	Kavita	Savita

The number of sheets with each student represents the mean. Thus, the mean here is 3.

Now ask the students to attempt Exercise 3.1 Questions 1 to 9 of Class VII Mathematics. In some questions, range has also been asked to find out. Recollect, range is the difference between the greatest and the smallest observations. This was discussed in Unit 1.

2. Mode

The teacher may introduce the concept of mode through an illustration given on page 64 of Class VII Mathematics, NCERT. One more illustration is given below:

Example: A charity show was planned to raise funds for the Red Cross Society. In a random sample of 100 tickets sold, the following information was recorded:

Cost of a ticket	₹ 100	₹ 50	₹ 25	₹ 10	₹ 5	₹ 2	₹ 1
Number of tickets sold	2	4	6	10	40	20	18

Can you find from the table, which type of tickets should be printed in maximum number to meet the demand?

(Initially, teacher may encourage the students to find mean of the above data as it is the only central tendency known so far.)

If we find mean of the tickets sold, we see that

$$\text{Mean} = \frac{2+4+6+10+40+20+18}{7} = \frac{100}{7} = 14\frac{2}{7} = 14 \text{ approx.}$$

Then, teacher may ask the inference from the mean calculated above as:

Should 14 tickets of each type (denomination) be printed to meet the demand?

Certainly not. Here mean is not the true representative value of the data. From the table, it is clear that tickets of ₹ 5 have to be printed in largest number to meet the demand. Frequency of tickets of ₹ 5 is maximum as compared to the tickets of other denominations.

Such observation with the maximum frequency is called mode of the data. In the above data, ₹ 5 is the mode. In fact, in such an example, table should be clearly explained. Students should know that the same data may also be written as

₹ 100, ₹ 100, ₹ 50 ..... ₹ 50,
(4 times)

₹ 25 ..... ₹ 25, ₹ 10..... ₹ 10,
(6 times) (10 times)

₹ 5, ₹ 5..... ₹ 5, ₹ 2..... ₹ 2,
(40 times) (20 times)

₹ 1, ₹ 1..... ₹ 1, from the above, it should be
(18 times)

made clear that mode is also obtained from data and not only from the frequency.

Mode is a useful measure in those situations where sale or production of readymade garments like shirts, trousers, shoes, etc. is done.

Teacher may now refer to Examples 4 and 5 on Pages 64 and 65 of Class VII Mathematics, NCERT for finding mode of the data.

Also, teacher should discuss the 'Try these' Questions of page 65.

In Example 4, students can find mode just by seeing the data that '2' occurs more frequently than other observations. Here teacher may advise the students that when the number of observations in the data is small then arrange them in ascending/descending order. This will help the students to find the observation with maximum frequency or which occurs the maximum number of times.

If the data has a large number of observations as in Example 5, advise them to form a frequency distribution table so that the observation with maximum frequency (i.e. mode) can quickly be seen from the table itself.

Ask the students to find mode of the following data:

2, 3, 3, 5, 2, 6, 2, 3, 5, 6, 5, 6, 8

Arranging the data in increasing (ascending) order, we get

2, 2, 2, 3, 3, 4, 5, 5, 5, 6, 6, 8

Here, 2 occurs three times,

5 occurs three times,

3 and 6 occur two times each and so on.

So, 2 and 5 both are the modes of the data, because both occur maximum number of times.

So, a data can have more than one mode.

Next situation may be of the following type:

What is the mode of the data 1, 2, 3, 9, 11,?

Here each observation has frequency 1. And, if each data has the same frequency, then each may be said to be the mode.

In such case, we may say that the data does not have any mode or each observation is a mode. If each observation in the data has the same frequency, then in this case also, we may say that there is no mode of the data or each observation is a mode.

Teacher may ask the students to try Questions 1 and 2 on page 65-66 under 'try these'and questions in Exercise 3.2 (in which mode has to be calculated) of Class VII Mathematics, NCERT.

Activity 2: Put about 1 kg of raw peanuts in a bag. Ask each student to take out a handful of peanuts and count them. Prepare a table of the results/observations and then help the students to draw a graph on a graph paper or a squared paper as shown below:

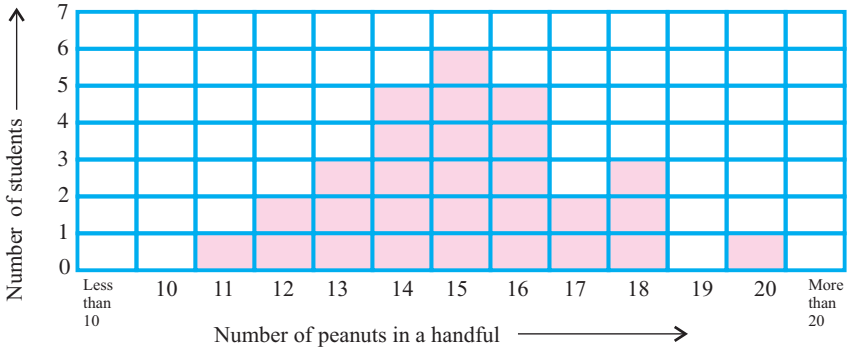


Fig. 1

Encourage the students to find the mode of the number of peanuts in a handful from the graph.

Activity 3: Next, ask the students to open their peanuts and count the number of nuts in each shell. The number of nuts in each peanut may be 0, 1, 2, 3, 4, etc. Ask the students to draw a table and graph of this information as shown below, using bars:

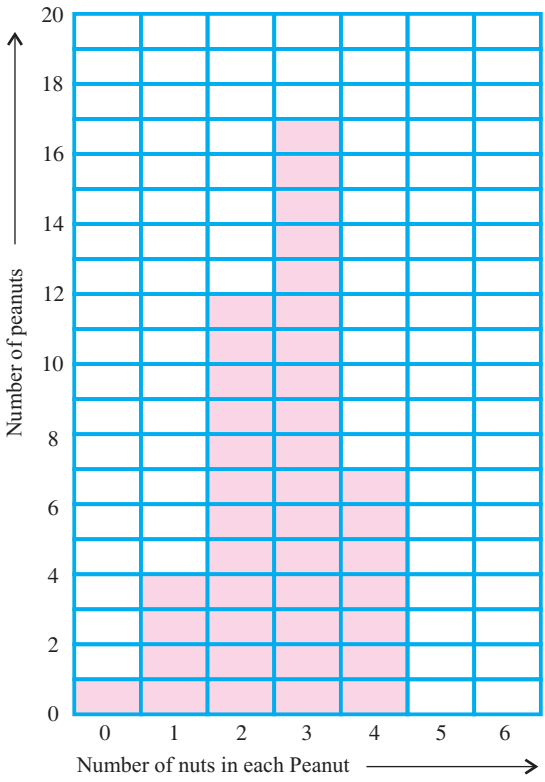


Fig. 2

Help the students to find the mode (the most frequently occurring number of nuts in a shell) from the figure.

3. MEDIAN

Before introducing the concept of median, ask the students to concentrate on the following example.

Example: In an organisation, there are 5 employees consisting of a superviser and 4 workers. Their salaries per month are as follows:

Supervisor : ₹ 20000

Workers : ₹ 5000, ₹ 6500, ₹ 7500, ₹ 8000.

Find the mean salary of the employees.

Here mean $= \frac{20000+5000+6500+7500+8000}{5} = 9400$

So, mean salary is ₹ 9400.

Does mean give the right picture of the salaries of employees?

No! four out of 5 employees are getting their salaries below the mean salary ₹ 9400.

In this example, mean is affected by the extreme value (20000) in the data.

This example drives us to look for another kind of average which is unaffected by the extreme values in the data and give a fair representation of the data.

In such situations, we use another average measure of central tendency called median.

Median refers to the value which lies in the middle most of the data when arranged in increasing (or decreasing) order.

On arranging the data above in the increasing order, we get 5000, 6500, 7500, 8000, 20000.

Since the median is the middle most observation, so 7500 being the middle most observation is the median. So, median salary is ₹ 7500. This means two employees get salary below ₹ 7500 and two of them get salary above ₹ 7500.

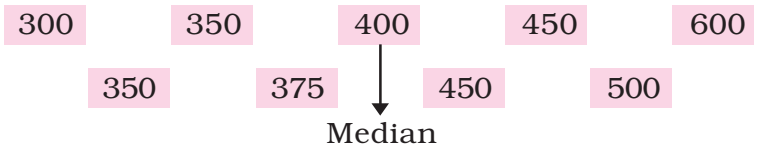
To find median

(i) Let the number of observations be n.

(ii) Arrange the observations in an increasing order/ decreasing order.

(iii) If n is odd, then $\left(\frac{n+1}{2}\right)^{th}$ observation will be the median.

(iv) If n is even, then there will be two middle most terms $\left(\frac{n}{2}\right)^{th}$ and $\left(\frac{n}{2}+1\right)^{th}$ and their mean will be the median.



By now the students must have seen that all the three measures mean, mode, median of the given data generally lie between the maximum and minimum values of the observations in the data. That is why these measures are called measures of central tendency.

Activity 4: Teacher may put some beans in a jar. Give each student a slip of paper on which let her write an estimate of the number of beans in the jar. Ask one of the students to arrange these slips in such a way that the numbers on them are in increasing order.

Let the students point out the approximate location of the slip in the middle. Confirm the location of the middle slip by counting slips on either side.

If there is an odd number of slips, the median will show the value with an equal number of slips on either side. A sample is given as follows, when there were 9 students:

Similar type of activity may be designed for the case when number of observations is even.

Ask each student to write his height (in cm) on a separate slip. Let them arrange these slips in an increasing order and try to find two middle most terms. Finally mean of these two numbers on these slips will be the median.

4. Chance and Probability

Probability means estimating how likely (probable) something is to happen. At the upper primary stage, just a feel for probability has to be given to the students. The description is informal which leads to the notion of an experiment – random experiment, equally likely/not equally likely outcomes, an event, events—less likely, more likely—the events which are certain to happen, impossible to happen, may and may not happen, etc.

Treatment is mainly through examples.

The teacher can give situations/events and ask which of them are certain, which are possible and which are impossible, For example,

- It will get dark tonight
- A dog will fly in the sky
- It will rain tomorrow
- The day will be longer than the night today.

Teacher can also encourage the students to think of some possible and some impossible events.

Experiment : When we talk of an experiment in probability, we generally talk of a random experiment. The operation which can produce some well-defined outcomes is called an experiment.

What is a random experiment?

To introduce the term random experiment, a teacher should not define it first rather she should ask the student different examples, like whether they are certain about the result of an experiment or not?

For example, teacher can bring a bag full of different types of toffees and then ask a particular student to pick and write the name of toffee on the board, likewise she can ask to pick another one and write the name on board, this could be done for at least 5 to 6 times and make all the recordings.

Next, she can give a bag full of same kind of toffees and observe the recordings.

Now she can compare both results saying whether we are certain about a specified result or not?

After enough discussion, it may be concluded that 'An experiment in which all possible outcomes are known but the exact outcomes cannot be predicted in advance, is called a random experiment'. For example, when we throw a coin, we know all the possible outcomes but we cannot predict in advance whether the outcome will be a head or a tail.

Some examples of random experiments are:

(i) Tossing a coin

(ii) Throwing/rolling a dice

(iii) Drawing a card from a well-shuffled deck of playing cards.

(iv) Drawing a ball from a bag which contains, say, 4 red and 2 yellow balls of the same size without looking into the bag.

(v) Spinning a wheel (and observing the outcome where the needle finally stops).

(vi) Observing the number of road accidents on a given day in a city.

(vii) You may score 90% and above in each terminal exam.

Give some examples of experiments which are not random, such as throwing a stone from a roof of a building to the ground.

Explanation

(i) When we toss a coin, there are two possible outcomes – head or tail. When we toss a coin, we cannot say with certainty whether it will show up a head or a tail. We may toss a coin 100 times and know how many times it has shown heads and tails. Still we cannot predict in advance whether it will show up head or tail in the next toss. So, tossing a coin is a random experiment.

(ii) If we throw a stone from a roof of a building on the ground, we are very much sure that it will land on the ground whether we do this experiment one time, two times, three times or even 100 times. So, throwing a stone from the roof of a building is not a random experiment.

Teacher may give some more examples of random experiments and experiments which are not random and help the students to find why they are random and why they are not random.

Outcome of a (random) experiment

(i) In the experiment of tossing a coin, head and tail are two possible outcomes

(ii) In case of throwing a dice, possible outcomes are: 1, 2, 3, 4, 5, 6.

Teacher may ask the students to tell the outcomes of the random experiments listed above.

Event: One or more outcomes of a (random) experiment make an event.

For example, in the experiment of tossing a coin getting a head is an event. Similarly getting a tail is also an event.

In case of throwing a dice, getting each of the outcomes 1, 2, 3, 4, 5 and 6, is an event. Getting an even number, i.e. 2, 4, or 6 is also an event.

Teacher may give some situations to the students and ask which are certain to happen, which are impossible to happen and which may or may not happen. The students can also do experiments in some cases (ii) to (iv) given below:

(i) It is cloudy today. It will rain.

(ii) If a drawing pin is dropped on a board, it will land point down.

(iii) A dice when tossed will land up with a number less than 7.

(iv) When a pair of dice is thrown at a time, the sum of both the numbers appearing on the tops of the dice is more than 12.

(v) Some situations are also given on page 76, Question 1, Exercise 3.4 of Class VII Mathematics, NCERT.

Through some experiments/situations, the idea of less likely, more likely or equally likely may be given to the students.

For example

(i) A marble drawn form a bag containing 8 black marbles and 2 white marbles is more likely to be of black colour and less likely to be of white colour.

(ii) A dice is tossed once and its outcome is recorded.

Appearing of the number 1, 2, 3, 4, 5, 6 on the dice will be equally likely.

Probability of an event

An informal approach to find probability of an event has been discussed in Classes VII and VIII, Mathematics, NCERT. The concept has been given in Class VII itself. (Refer page 75 of Class VII, Mathematics, NCERT) and then the concept has been reinforced in Class VIII.

You may start in the same way. For example, head or tail, both are equally likely outcomes when a coin is tossed.

So, probability of head $= \frac{1}{2}$, probability of tail $= \frac{1}{2}$

Similarly for a dice, probability of getting either of 1, 2, 3, 4, 5, or 6 is $= \frac{1}{6}$.

So, probability of an event

$$= \frac{\text{Number of outcomes favourable to the event}}{\text{Total number of outcomes}},$$

if the outcomes are **equally likely.**

Teacher may ask the students to attempt Exercise 3.4 of Mathematics Textbook for Class VII and Exercise 5.3 of Class VIII, Mathematics, NCERT.

Students should be told to observe that each time the answer they get is less than 1 and greater than 0.

Common Errors

(i) Median of the raw data is calculated without arranging the observations in ascending/descending order.

(ii) While finding the median when number of observations are even (say10), the median is the mean of 5th and 6th observations. But some students

take the mean of 5 and 6, i.e. 5.5 and call it the median, which is not correct. In fact, it should be the mean of the values of 5^{th} and 6^{th} observations.

(iii) Some students may say that mode of 2, 5, 2, 1, 5, 6, 3, 2, is 6 (since 6 is biggest of all the observations), whereas 2 is the mode here (i.e. the observation with maximum frequency).

(iv) Some students may write the maximum frequency as the mode instead of corresponding observation.

You may evaluate the students through following exercise.

Exercise

1. What is the arithmetic mean of the marks (out of 25) of 20 students in a test given as below?

   4, 8, 3, 16, 18, 21, 10, 9, 7, 18, 24, 12, 14, 15, 12, 9, 0, 7, 6, 16.

2. The heights (in cm) of 10 students are as follows:

   158, 154, 162, 160, 165, 152, 156, 164, 168, 163.

   What is the mean height?

3. Find the median height of students in Question 2 above.

4. The number of goals scored by a football team in 12 matches are:

   1, 2, 2, 3, 1, 2, 2, 4, 5, 0, 3, 5.

   What is the modal score?

5. A student says that the median of the data

   3, 14, 18, 20, 5 is 18. What is not understood by the student to find the median?

6. A dice is rolled once.

   (i) What is the probability of getting 6?

   (ii) What is the probability of getting 7?

   (iii) What is the probability of getting a number less than 7?

UNIT 4

Introduction to Graphs

Structure

- Introduction
- Main Concepts and Sub-concepts
- Objectives
- Teaching Points
  1. A line graph
  2. Double line graph
  3. Coordinates of a point
  4. Linear graphs
- Common Errors
- Exercise

Introduction

Students have already studied the following pictorial representations of data:

(i) Pictograph

(ii) Bar graph

(iii) Pie chart or circle graph

(iv) Histogram, etc. in Data Handling.

So, they are already aware of the term "graph".

As a pre-activity, a teacher can encourage student to collect electricity bills of previous five months and try to represent the same using a bar graph. Later on, students can be encouraged to draw a line graph for the same and to visualize and draw inferences from it.

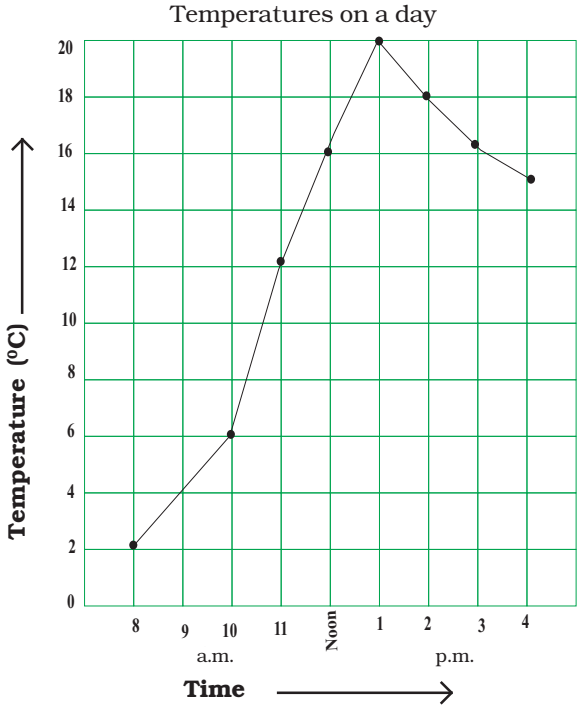


Fig.1

After giving them the idea of double line graph, a student can be asked to share a bill from his friend and then draw double line graph for both the data.

Students might have seen some other types of graphs in the newspapers, television, magazines, etc. which are sometimes not of the type of graphs given in (i) to (iv) above.

Some of them will be of the type as given below:

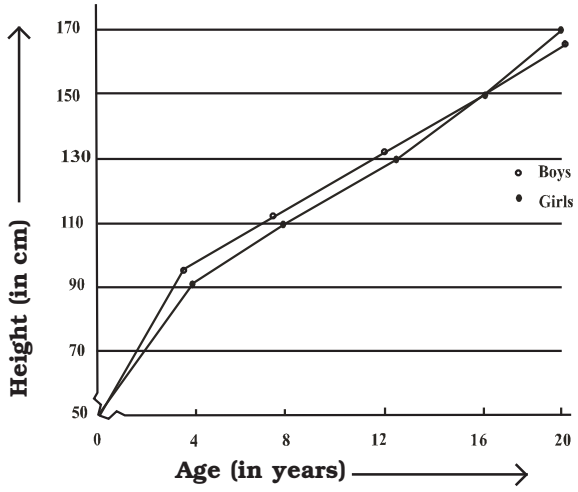


Fig.2

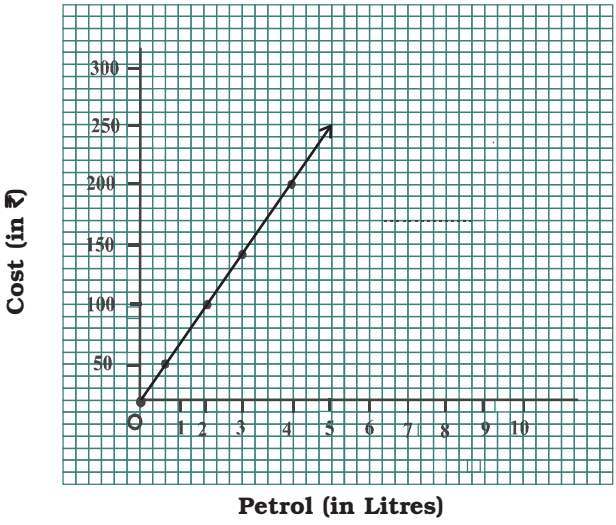


Fig.3

Graphs as given in Fig. 1 and Fig. 2 are called line graphs and in Fig. 3 is called a linear graph. Such graphs are useful in showing trends or variation/comparison in data.

In this unit, these types of graphs will be discussed. For this, there will be a need of some knowledge of coordinates of a point (this idea was also indirectly involved in drawing of bar graphs, line graphs, etc.) which will also be discussed.

Main concepts and Sub-concepts

- Line graphs
- Coordinates of a point in the plane
- Linear graphs

Objectives

After teaching this unit the students can

- study a line graph and answer questions related to it;
- draw a line graph of some specific data;
- plot a point in the plane (1st quadrant only); and
- apply the knowledge of plotting of points in a plane to draw linear graphs.

Teaching Points

Teacher may give a brief idea of the notations of coordinates in the beginning.

For example, Sudha has moved 5 cm right and then goes 1 cm up. So, she has reached the point (5,1), if we consider her initial position as origin. A detailed discussion will be done later on.

1. A Line Graph

Teacher may ask the students to draw a bar graph of the data regarding body temperature of Renu at different times

of a day as given on pages 233–234 of Class VIII, Mathematics, NCERT as shown in Fig. 4.

Time	6 a.m	10 a.m.	2 p.m.	6 p.m.
Temperature (°C)	37	40	38	35

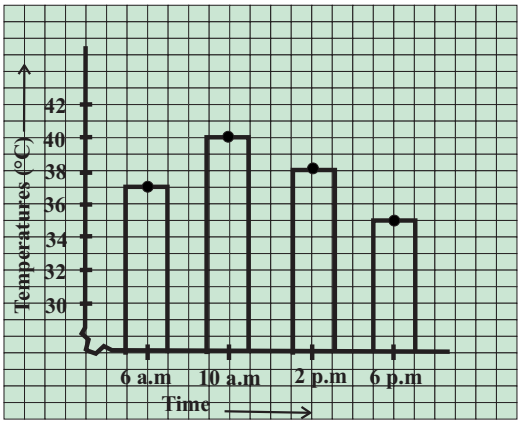


Fig.4

In this bar graph, the height of each bar corresponds to a particular temperature. Explain to the students that height of the first rectangle corresponds to the temperature 37°C at 6 a.m. and so on.

Now, tell the students that a dot can also be put corresponding to each temperature as shown in Fig. 5.

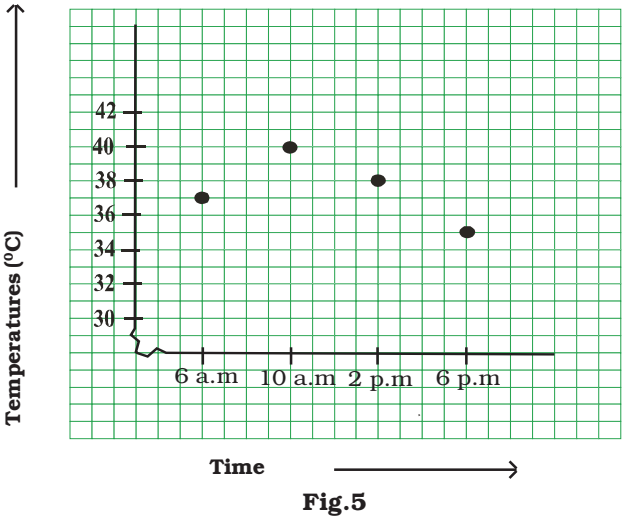


Fig.5

Now ask the students to join these dots by line segments (Fig. 6). The graph so obtained is a line graph of the data given in the table above.

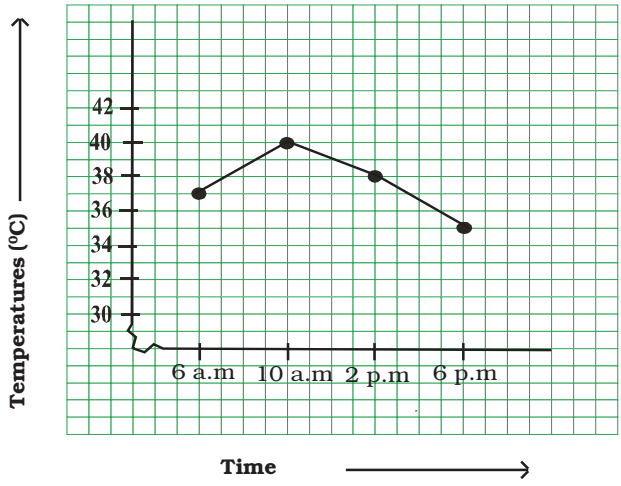


Fig.6

Teacher may now explain the interpretation of this line graph as:

(i) Body temperature of Renu is increasing between 6 a.m. to 10 a.m. and after that decreasing till 6 p.m.

(ii) Body temperature was highest at 10 a.m. and lowest at 6 p.m.

(iii) Temperature increased by 3°C (40°C – 37°C) during the period 6 a.m. to 10 a.m. etc.

No recording of temperature was done at 8 a.m. However, the graph suggests that it was more than 37°C. Help the students to find approximate temperature at 8 a.m. Thus, **a line graph can be used to show trends in the data.**

Before coming to Example 1 on page 234 of Class VIII Mathematics, NCERT teacher may discuss one or two more situations of the above type which involve single line graph only.

Example

The following table gives the attendance for a class of 30 students during one week from Monday to Friday:

Day	Monday	Tuesday	Wednesday	Thursday	Friday
Number of Students	28	26	30	28	24

Help the students to draw the graph in the following steps:

Step 1. Plot the dots as shown in Fig. 7 (i).

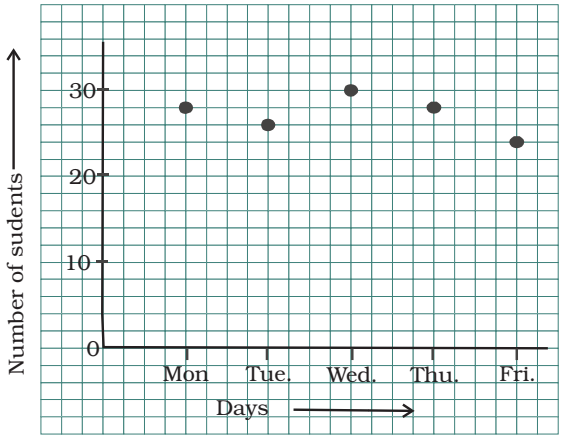


Fig.7(i)

Step 2. Let them join these dots by line segments as shown in Fig. 7 (ii).

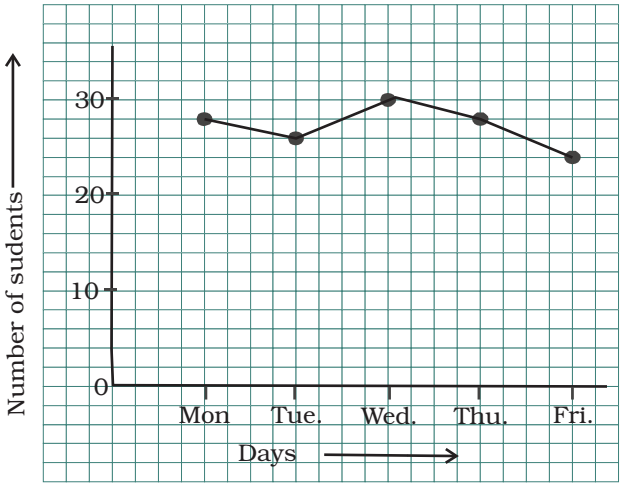


Fig.7(ii)

Reading and Interpreting Line Graphs

Teacher may now ask the students to concentrate on this line graph (Fig.7(ii)) and ask some questions of the type given below:

(i) How many students were at School on Friday?

(ii) On which day the attendance was the highest?

(iii) On which day the attendance was the least?

(iv) On which days the attendance was the same?

Although these questions can directly be answered by looking at the table, still we want that students should learn to read and interpret a line graph. It is not always necessary that the table is provided to a student. A line graph can be given and a student is expected to interpret the graph.

Consider the line graph given in Fig. 1, which shows the temperatures (°C) at various times of a particular day.

As an activity, ask the students to record temperatures at different times of a day (Students may choose their own times from morning till evening) and then draw line graphs of these data. These line graphs may be redistributed among students and then students may be asked to answer questions as listed below:

(i) For which hours during the day, did the students collect data?

(ii) What was the highest temperature of the day?

(iii) Will the temperature at 5 p.m. be higher or lower than 16 °C?

Now discuss Example 2 on page 235 of Class VIII, Mathematics, NCERT. Before coming to Example 1, you may give two or three more examples of reading and interpreting line graphs as given below.

Example

The graph given in Fig. 8 shows a patient's temperature taken at the same time for 7 days of a week. Study the graph and answer the following questions:

(a) What was the highest recorded temperature?
(b) What was the lowest recorded temperature?
(c) What was the temperature on Saturday?

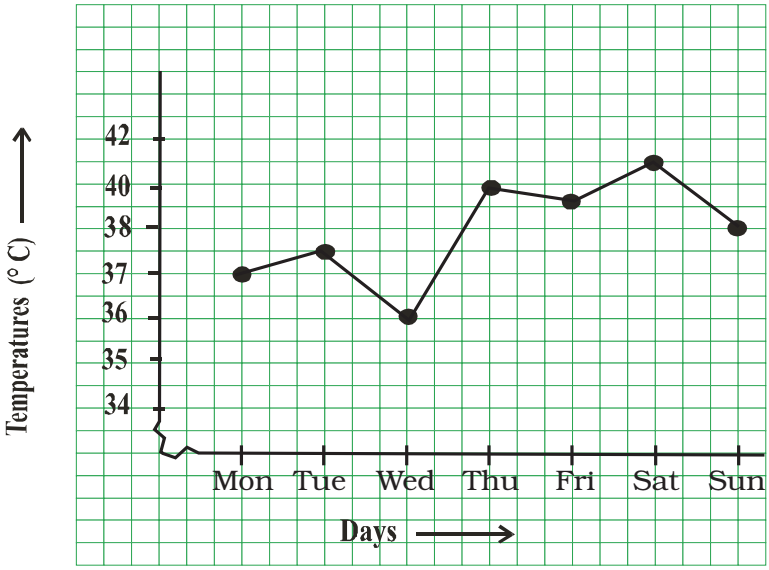


Fig.8

Answers to these questions are as given below:
(a) 41°C (b) 36°C (c) 41°C

Example

Fig. 9 shows the number of visitors to an exhibition during a period of 7 days. (Sunday to Saturday). Study the graph given and answer the following questions:

(i) How many visitors came on the first day?
(ii) How many visitors came on the last day?
(iii) How many visitors came on Tuesday?
(iv) How many people visited the exhibition altogether in 7 days?

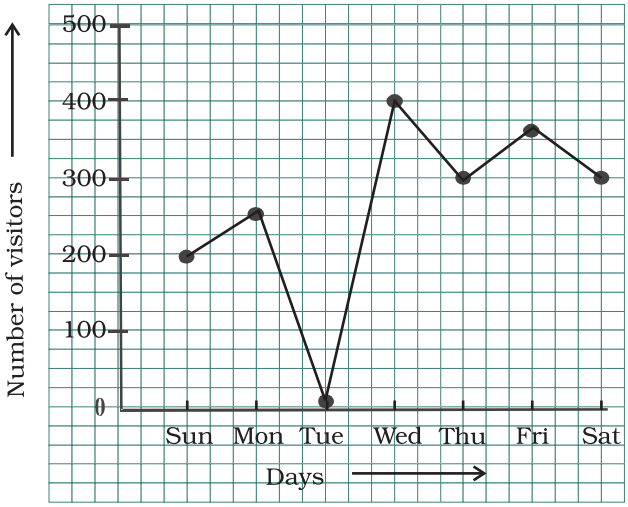


Fig.9

In this line graph, an important point to be discussed with the students is that on Tuesday, no visitor came. Its one of the interpretations could be that the exhibition was not open on that day.

2. DOUBLE LINE GRAPH

Before discussing Example 1 on page 234 class VIII, Mathematics (A graph on performance), teacher may refer to Fig. 2 where a double line graph is shown. This double graph consists of two line graphs, one (with dark line) shows the heights of boys as they grow in age, while the other (with dotted line) shows the heights of girls as they grow in age. (See Science textbook for Class VIII). What can be interpreted from this graph?

From this double graph, it is clear that most of the time, the line graph related to girls is below the line graph that of boys.

This shows that the heights of girls is generally lower than those of boys of the same age. Thus, a double line graph helps in comparing two items at a particular instant. In double line graph given in Example 1 on page 234 of

Class VIII, Mathematics, NCERT again a comparison between the performance of two batsmen is given.

Teacher may now ask the students to do Question 3 and 4 of Exercise 15.1 on pages 237 and 238 of Class VIII Mathematics, NCERT which involve double line graphs.

Construction of a Double Line Graph

For the data in Question 5 (b) on page 238 of Class VIII, Mathematics, NCERT students have been asked to draw a double line graph. How to draw a double line graph?
To draw a double line graph, proceed as follows:

1. In Question 5(b), the first data is related to the number of men in a village in different years. Draw a single line graph of the first data. The line segment may be drawn by using a sketch pen or a dark pencil of some colour.
2. On the same graph paper, draw the line graph of the second data (In Question 5(b), the second data is related to the number of women in a village in different years). This time, the line segment has to be drawn either by dotted line or pen/pencil of different colour.

We have taken first data for men and second data for women. Either of them can be taken as first data and the other as second data. This point has to be highlighted in the class.

Example

The following table shows the number of vehicles parked in Parking A and Parking B at different times:

Time	9 a.m.	10 a.m.	11 a.m.	12 noon	1 p.m.
Number of vehicles at A	600	800	900	750	850
Number of vehicles at B	700	850	900	800	750

Use the data in the above table to draw a double line graph.

As explained above, draw a line graph for the number of vehicles in Parking A and then draw a line graph for the number of vehicles in Parking B (Fig. 10).

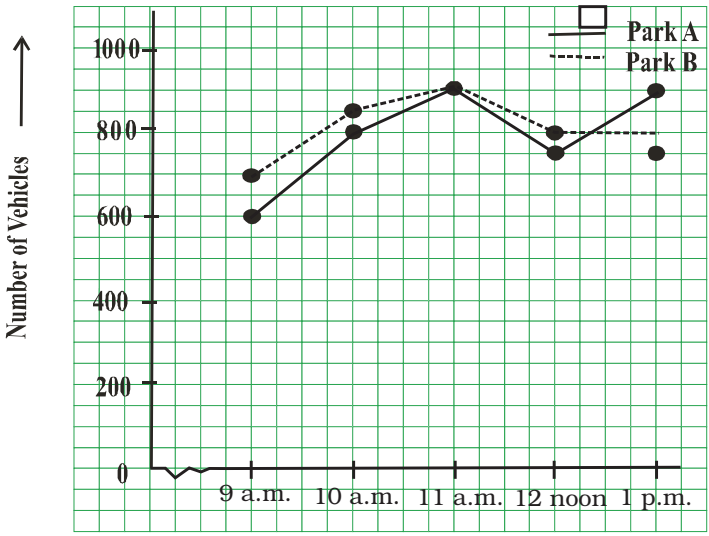


Fig.10

Teacher may help the students in interpreting this double line graph by asking questions such as

(i) At what time, the number of vehicles was the same in both the parkings?

(ii) What trend does the graph show? etc.

3. COORDINATES OF A POINT

In the book of Class VIII, Mathematics, NCERT, on page 240, an activity has been suggested for finding the location of a dot (later on, it will represent a point) on the black board. You may also do this activity in the class. This may be discussed by putting a dot on the black board and ask the students to tell its location.

Let the students give their own responses for finding the location of the dot.

It is possible that some of the students may come up with the right response by measuring the distance of the dot from the left edge and bottom edge of the blackboard. This again depends on the position of the dot. It could be anywhere on the blackboard. So accordingly, the responses from the students will vary.

In the given activity, the position of the dot is written as (90, 160).

Then the teacher may also ask the students whether (90, 160) is same as (160, 90). Certainly they are not the same. Let the students know this fact from the position of the point drawn.

Teacher may narrate the story (given in the box on page 241) of French mathematician Rene Descartes as to how he conceived the idea for fixing a point with the help of two measurements – vertical and horizontal. It later on came to be known as Cartesian System in his honour.

How to plot a point on a graph sheet ?

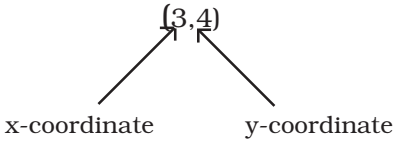
We have been plotting points in case of drawing line graphs without telling their coordinates. What are the coordinates of a point? To explain this, proceed as follows:

Ask the students to draw two perpendicular lines OX and OY on a graph sheet/grid paper as shown in Fig.11.

Students already know how to represent numbers 1,2,3,4,..... on a number line. Ask them to represent the numbers 1, 2, 3,....on the line OX and OY as shown in Fig.11 [taking O as the initial point representing the number 0 (zero)]. OX and OY are called **horizontal axis** or **x-axis** and vertical axis or **y-axis,** respectively.

With the same understanding as in the activity of locating a point on the blackboard, the point (3, 4) denoted by A which is 3 units from left edge (y-axis) and 4 units from the bottom edge (x-axis) is plotted on the graph sheet (See Fig. 15.12 on page 241, Class VIII, Mathematics, NCERT).

Teacher may now say that 3 is called the ***x*-coordinate** of A and 4 is called the ***y*-coordinate** of A and coordinates of the point A are (3,4).



The coordinates of a point may be written along with the point as shown in Fig. 11.

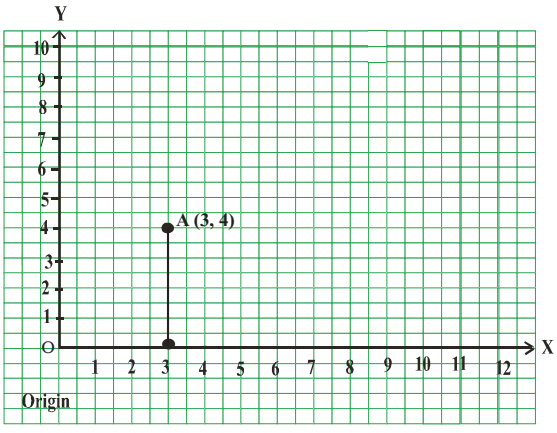


Fig.11

Activity 1: Teacher may divide the class into groups of 3 or 4 students. Give each group the coordinates of the points such as A (3, 4) and B (4, 3); A (1, 2) and B (2, 1); A (5, 2), B (2, 5) and so on. Let them verify that the points A and B are different points by plotting them on the graph sheet.

Activity 2: Teacher may mark different collections of points say three or four on a graph paper with OX and OY as *x*-axis and *y*-axis, respectively for each group and ask them to write.

(i) *x* -coordinate

(ii) *y*-coordinate

of each point.

Note : As far as possible, the coordinates should be in whole numbers and plotting of points has to be restricted to 1st quadrant only. (However, teacher may avoid the use of word quadrant in the very beginning).

Activity 3: Teacher may give two or three collections of points such as (0,2), (0,5), (0,6), (1, 2), (1, 5), (1,6), etc. which lie on a line, to each group. Let them plot the points and verify whether they all lie on a line. (Refer to Example 5 on page 242 of Mathematics Textbook for Class VIII).

Further

(i) Teacher may ask the students to plot points such as (0, 1), (0, 5), (0, 9), etc. and let the students observe that such points whose x-coordinate is 0 lie on y-axis.

(ii) Similarly, teacher may ask the students to plot points such as (1, 0), (2, 0), (5, 0), etc. and let the students find that all these points where y-coordinate is zero lie on x-axis.

(iii) Coordinates of the origin O is (0, 0).

Students may now be asked to do questions of Exercise 15.2 on page 243 of Class VIII, Mathematics, NCERT.

4. LINEAR GRAPHS

Teacher may ask the students to see graph in Fig. 3. Here graph is a straight line. It is called a linear graph. How is it different from line graph?

Before answering this question, let the students plot a graph of the data given in Example 6, page 243 of Class VIII, Mathematies, NCERT regarding quantity of petrol and its cost.

Quantity (in litres) of petrol	10	15	20	25
Cost of petrol (in ₹)	500	750	1000	1250

(i) Ask the students to draw two perpendicular lines OX and OY . Call them as horizontal and vertical axes, respectively and choose a suitable scale on both the axes.

(ii) Let them mark quantity in litres along the horizontal axis and cost of petrol along the vertical axis.

(iii) Help them to plot the points (10, 500), (15, 750), (20, 1000) and (25, 1250)

(iv) Ask them to join these points. (Refer Fig. 15.16 on page 244 of Class VIII, Mathematics, NCERT).

- Let them observe that the graph is a line. That is why it is called a linear graph.

  (Teacher may also tell that for drawing a linear graph atleast three coordinates should be taken in consideration because with two coordinates it will always be a linear graph).

- Let them extend this line and observe that it passed through the origin (0,0)

  Ask the students what does it show.

  (Teacher should ensure students that this may not happen in each case).

  It shows that cost of petrol is ₹ 0 when the quantity in litres of petrol is 0.

- Ask the students "can you find the cost of 5 litres of petrol"from this graph?

  Help them to find out the y-coordinate of the point which is on the graph corresponding to x-coordinate 5. It is 250. So, 5 litres of petrol will cost ₹ 250.

Similarly, you may ask another question "How much petrol can be purchased with ₹ 800" Let the students answer it by looking at the graph.

This problem is other way round. Here, let the students find x-coordinate of the point on the graph corresponding to y-coordinates as 800.

It is 16 (See Fig. 15.16 of Class VIII Mathematics, NCERT). So, 16 litres of petrol can be purchased with

₹ 800. At this stage, highlight the importance of the linear graph stating that one can easily find the quantity of petrol that can be purchased from the given amount.

Teacher should make it clear that a graph is called a linear only if it is giving straight line on joining all the points representing the given coordinates.

Independent Variable and Dependent Variable

Through this example, the teacher may explain that the cost of petrol depends on the quantity of petrol. Thus, the quantity of petrol is an **independent variable** and the cost of petrol is a **dependent variable**.

Now the teacher may discuss the other examples such as Example 7 (Principal and Simple interest), Example 8 (Time and Distance), etc. In both the examples, discuss which is the **dependent variable** and which is **independent variable**.

Help the students to observe that the graphs of both the examples are linear graphs.

Some more situations of linear graphs.

(i) Relationship between the side and perimeter of a square.

(ii) Relationship between the side and perimeter of an equilateral triangle

Discuss at least one situation where graph is not linear. One such example is: Relationship between the side and area of a square (Refer to Question 2 (ii) of Exercise 15.3 of Class VIII, Mathematics, NCERT).

Teacher can now discuss the cases of direct variation too.

Common Errors

(i) Sometimes students do not label the horizontal and vertical axes.

(ii) Sometimes, the students do not mark different values on the axes according to the chosen scale for line graphs.

(iii) In case of linear graphs, the markings on both the axes are not done properly.

(iv) While drawing a double line graph, sometimes, students first plot the points of both the line graphs and then join them. In this process, they mix up the joining of points resulting in an incorrect graph.

(v) Students get confused about whether to choose same scale for both the axes if the numbers are similar. So they need to be told that there is no relation between both scales.

You may evaluate your students through the following exercise.

Exercise

1. The speeds of a car at different times of a day are given below:

Time	7 a.m.	7.30 a.m.	8 a.m.	8.30 a.m.	9 a.m.
Speed (km/hour)	45	50	60	40	60

Draw a line graph for this data.

2. Following line graph (Fig.12) shows monthly rainfall in a city for the months, February to September, of a particular year:

Study the graph and answer the following questions:

(i) Which was the wettest month?

(ii) Which was the driest month?

(iii) What is the total rainfall from February to September?

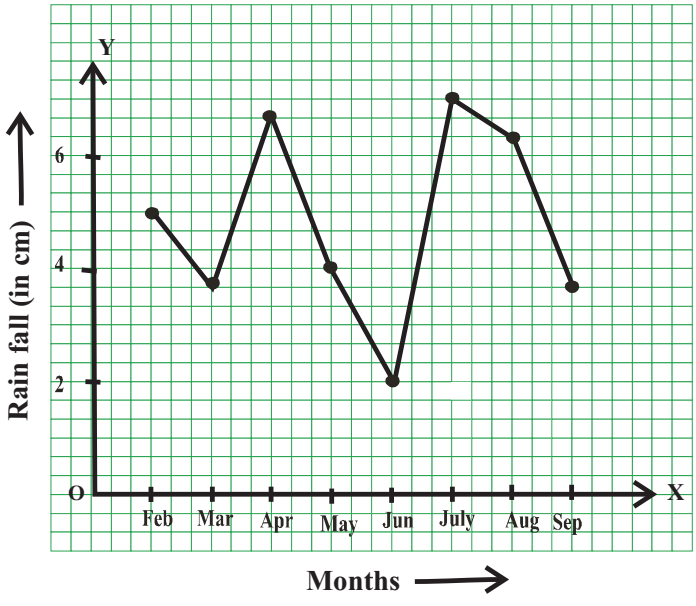


Fig.12

3. The graph given below represents the total number of licensed dogs in two cities—A and B from 2001 to 2006. Study the graph and answer the following questions:

   (i) What was the total number of licensed dogs from 2001 to 2006 in city A?

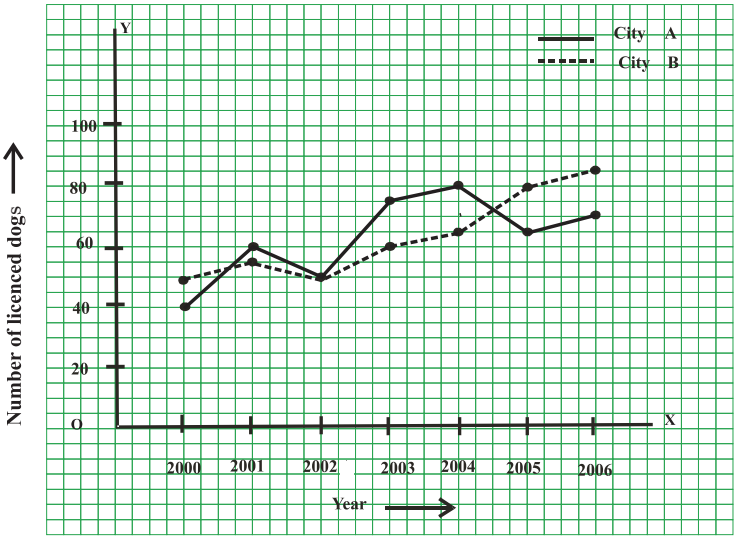


Fig.13

(ii) What was the total number of licensed dogs from 2001 to 2006 in city B?

(iii) In which year was the number of licensed dogs same for both the cities?

(iv) In which year was the number of licensed dogs maximum in city A and in city B?

(v) In which year was the number of licensed dogs minimum in city A and in city B?

4. Plot the points on a graph sheet. Verify that they lie on a line.

(i) A (3, 0), B (3, 2), C (3, 6), D (3, 8)

(ii) A (1, 1) B (3, 3), C (5, 5), D (7, 7)

5. Which of the following points lie on x-axis and on y-axis?

(i) A (0, 2) (ii) B (1, 0) (iii) C (9, 0)

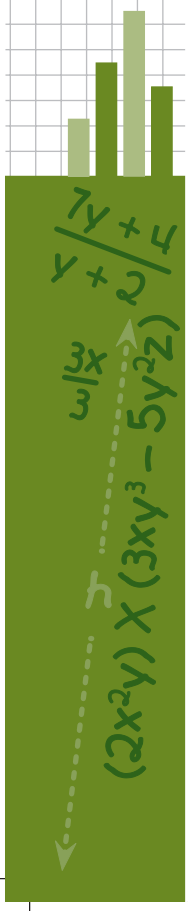
(iv) D (6, 0) (v) E (2.5, 0) (vi) (0, 3)

6. Draw the graph for the following data:

Number of rungs	10	12	14	16	18	20
Price of a ladder (in Rs)	70	80	90	100	110	120

Is the graph linear?

Using the graph, estimate the price of a ladder having 15 rungs.



ANNEXURE 1

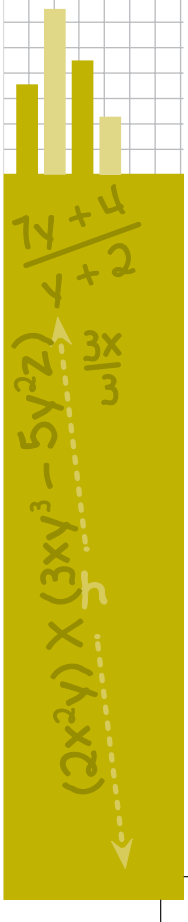
Nature of Mathematics

"If people do not believe that Mathematics is simple, it is only because they do not realize how complicated life is."

— John Von Newmann

Mathematics drives on the wheels of logic and creativity both, and these skills are pursued for a variety of practical purposes. The essence of Mathematics does not lie in its beauty and its intellectual challenge only, for many professionals the main value of Mathematics is how it applies to their own work. To achieve its place in Science and in practical arts from informational and computational standpoints, as well as its cultural significance, students need to perceive Mathematics as part of the scientific endeavour, comprehend the nature of mathematical thinking, and become familiar with key mathematical ideas and skills.

To help children learn Mathematics, a teacher must have a thorough understanding of what mathematics is because a teacher's reflections upon the subject will determine to a great extent his or her approach of dealing the subject with children. It is said that God scripted the language of Mathematics in the universe. People call Mathematics an intellectual activity. Some feel refreshed after solving a puzzle and some feel tired after solving a problem. The different feelings depend on different perceptions about Mathematics. Mathematics is probably



the only branch of knowledge that has a formal proof for a number of propositions. The strength of Mathematics is its ability to prove its propositions. Perceptions may change with regard to certain ideas in Science but it is not so in Mathematics. Scientific observation depends upon the condition under which the observations were recorded. Over the last few decades, perceptions of a number of ideas in Science have changed, e.g. today Pluto is not considered, a planet anymore. Mathematics does not depend on observations alone. Once proved mathematically, a proposition holds good for all times to come. Therefore, Mathematics is not a Science alone.

Mathematics is such a beautiful game of numbers and notions that even nature behaves mathematically. Nature provides us beautiful patterns in bubbles, waves, branches, bodies, breakdowns, fluids, grains and communities, etc. Patterns are a result of naturally occurring processes.

Scientists do not study nature just because it is useful; they study nature because it is beautiful. If nature were not beautiful, it would not be worth knowing. And if nature were not worth knowing, life would not be worth living. We cannot understand nature if we do not first learn its language and grasp the symbols in which it is written. Mathematics does not only describe nature, it is nature's language. There is nothing in our lives, in our world, in our universe, that cannot be expressed with mathematical theories, numbers and formulae. Mathematics is the key to understanding the world around us. It is perhaps the purest of the pure mental endeavour of human kind. Mathematics has been called the mother of all sciences; it works as a backbone of all systems of knowledge.

Mathematics is abstract in nature. Abstraction can be arrived through concrete examples. It should not be rushed through. The approach to teaching Mathematics is concrete-semi concrete-abstract.

Historically speaking Mathematics began in the days of pre recorded history when man needed some way to quantify his surroundings. Man initiated his study with a

matching one-to-one relationship of pebbles and sheep or tally marks and days in order to introduce some regularity into his life. Later he found fingers a handy set of counters.

Thus, Mathematics is a man-made venture. As Mathematics is abstract in nature and has a language dimension, mathematical language is limited to certain kinds of ideas, which are related to form, size and quantity. The functions of mathematical language are twofold: (1) to record ideas and communicate them to others (2) to aid the individual in his efforts to organise and clarify his own thoughts. Symbols have a one-one correspondence with terms. Every symbol has a unique meaning (represents a unique concept or term or idea). The need to use symbols needs to be stressed at the background of precision and universal acceptance. Symbols for operations of addition, subtraction, multiplication and division have special significance at the primary level. These symbols should be introduced gradually and as per the need, should not be introduced in one go. The role of symbols in expressions must be appreciated in the light of space and language. Concepts of line, line segment and ray, etc. are visualised through their corresponding symbols. The specific symbols like <, > require some time for the child to appreciate and use. It should not be rushed through. The use of brackets and their role in the operations should be brought home through a good number of activities and experiences of the child. Clarity should always be maintained. For example, generally 3×4 is confused with 4×3. 3×4 is to be read as 3 times 4 and 4×3 as 4 times 3, i.e. $3 \times 4 = 4+4+4$, $4 \times 3=3+3+3+3$.

Mathematics at Upper Primary Stage

Mathematics at Upper Primary Stage is a major challenge and has to perform a dual role of living both close to the experience and environment of the child and being abstract by its own nature. Children at this stage often are not able to work in terms of ideas alone. They need the comfort of contexts and/or models linked to their experiences to find meaning.

This stage presents before us the challenge of engaging the children while using the contexts but gradually moving them away from such dependence.

Characteristics of Mathematics

Following are five basic characteristics of Mathematics and also known as the inviolable scientific principles in mathematics.

Precision: Mathematical statements are clear and unambiguous. At any moment, it is clear what is known and what is not known. Results are either right or wrong. There is no midway possible between right and wrong. For example, sum of all the angles of a triangle is 180°. Sum is neither less than or greater than 180°.

Definitions: Bedrock of the mathematical structure (no definitions, no Mathematics).

At upper primary stage, children should be reached to definitions in a precise manner, i.e. with proper experiences and inferences of activities.

Reasoning: Lifeblood of mathematics; core of problem solving.

Coherence: Every concept and skill builds on previous knowledge and is a part of an unfolding story. The learning of mathematics is different from learning of other subjects. For example one cannot understand multiplication and division unless he/she has learnt addition and subtraction. Simple Interest and Compound Interest, Profit and loss cannot be understood by a student unless he/she has learnt the concept of percentage.

Purposefulness: Mathematics is goal-oriented. It solves specific problems.

Besides these, following are also important characteristics of Mathematics:

Mathematics has its own language and symbols: Peculiar language and symbolism of Mathematics is the most important characteristic which distinguishes it from the

other subjects. Mathematical language and symbolism cut short the lengthy statements and help the expression of ideas or things in exact and precise form. For example, area of a rectangle is expressed as $A = l \times b$. Mathematical language is free from verbosity and helps to make the sense clear. For example, the statement 'the age of father is three times that of a son' can be expressed as $y = 3x$, where y is the age of father and x is the age of his son.

Mathematics is Abstract

Mathematics is abstract in the sense that it does not deal with actual objects, as in the case of Physics.

In Mathematics, idea of numbers is abstract. The concept of infinity is something that we can never experience and yet it is a central concept of Mathematics. In Geometry, a triangle drawn on a paper is not a triangle. The concept of triangle is abstract.

Mathematics is Full of Patterns and Relationships

Mathematics is known as science of patterns. In fact, in queries about the attributes of things, children apply reasoning to answer 'what is next?' not with a number but with a description. Basically, it may always be a nice tool to initiate mind action in an interesting way. Through visual and written forms of expressions, children connect language and Mathematics to develop skills for thinking clearly, strategically, critically and creatively.

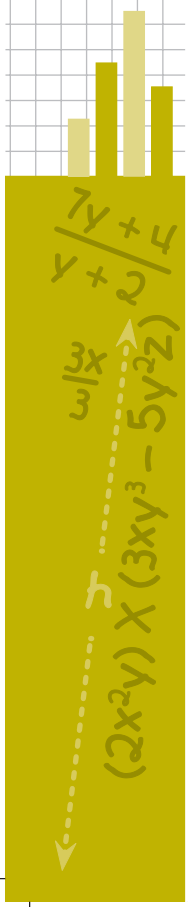
Mathematics has Rigour and Logic

Logic is an important fact in Mathematics. It governs the pattern of deductive proof through which Mathematics is developed. The presentation of Mathematics in rigorous form is not advisable at this stage. Mathematics must be understood at this stage in visual or geometrical terms.

2 ANNEXURE

Mathematics Anxiety
a Caution Needed

Fear is the most common and the most natural emotion we experience. Fear can be constructive, but can also be dangerous to the point where it leads to decay of personality. Sometimes, fear develops into forms of uncontrollable psychological disorders and becomes phobia. According to the basic theory, the occurrence of phobia and fear are psychological traumas, experienced in one's childhood, forgotten, but stamped on one's subconscious mind. Further fear also causes anxiety.



How Does a Student Develop Mathematics Anxiety?

Mathematics anxiety has been defined as feelings of tension and anxiety that interfere with the manipulation of numbers and solving of mathematical problems in various situations. Mathematics anxiety can cause one to loose one's self-confidence.

Sometimes this anxiety starts from very early years of schooling mainly from Class I and Class II. This could in part be due to social learning from parents and teachers having themselves Mathematics anxiety or negative perceptions of Mathematics. Parents or teachers might give children mixed messages about Mathematics. They might give an impression that Mathematics is a difficult subject and at the same time tell them how mathematical

skills are of importance for their future achievements. Mathematics anxiety often develops due to student's negative experience while learning Mathematics in the classroom or at home. Every negative experience affects the thought processes of any future Mathematics work, ultimately causing a lack of understanding of Mathematics.

Students are being taught to memorise mathematical concepts without actually working through problems and comprehending the reason behind the mathematical skill. This along with the lack of preparation greatly contributes to a child's increased level of Mathematics anxiety.

Negative school experiences might also contribute to the development of Mathematics anxiety. For example, teachers' threatening and authoritarian attitudes could lead to fearsome classroom climate in which student might hesitate to ask questions or answer the teachers' questions. Furthermore, students fearing their Mathematics teacher might have a conditioned reaction to Mathematics as well. Observing teachers giving Mathematics homework as punishment could also cause students to perceive Mathematics as an unpleasant subject. As punishment is inherently negative, extra Mathematics assignments as punishment could play a role in cultivating negative attitude towards Mathematics in students who receive the punishment as well as those who witness it. Low grades or failure in Mathematics could also lead to Mathematics anxiety. Failure in Mathematics, fear and anxiety about it could cause extreme feelings of dislike about Mathematics. Indeed, it has been seen that persons with Mathematics anxiety make more mistakes in dealing with Mathematics problems than who do not suffer from it. Such mistakes lead to lower grades in Mathematics which in turn increases anxiety about Mathematics. As such, the vicious cycle of anxiety, failure and more anxiety is perpetuated.

Poor performance in Mathematics has been linked to an increase in Mathematics anxiety. **Belief** and **expectation** to perform **poorly** in Mathematics problem solving could also lead to Mathematics anxiety or intensify

students' existing anxiety. Mathematics anxiety can be seen to be experienced to such a degree that children might perceive their performance in Mathematics as a measure of their self-worth and a reason for losing value in the eyes of parents and teachers. Thus, students with these excessive worries develop negative attitudes toward Mathematics which are expressed as 'I can't do Mathematics' or 'I hate Mathematics.' However, since they do have to deal with Mathematics, these beliefs lead to a great deal of distress and unease.

A mild case of Mathematics avoidance can quickly turn into a severe case of Mathematics anxiety. Cycle begins with an anxious Mathematics student, convinced of is inability to do Mathematics, might avoid the subject or put forth little effort, leaving significant gaps in his/her Mathematics development. Difficulty mounts as the student confronts more advanced skills, causing further anxiety and avoidance. Mathematics anxiety can turn into a permanent block.

Mathematics anxiety can greatly affect children's success throughout their education and their adult life. Since Mathematics is connected to so many professional and personal practices, it is important that we as educators and parents help children to overcome their Mathematics anxiety so that they can learn the mathematical skills that they need to succeed. Mathematics anxiety is real and it can happen to anyone at any age regardless of his/her mathematical ability.

Role of Language

Language also plays a crucial role in developing Mathematics anxiety among the students. "How can a student possibly begin to solve a mathematical word problem if she lacks the reading comprehension or grammar skills to really understand what the problem means?" We believe that the difference in word usage between commonly spoken and written standard language can seriously impair students ability to translate word problems into mathematical equations. Teacher here can play a crucial

part by providing the question in simplest possible language over using complicated terms at the time of initial learning.

WHAT YOU CAN DO

Be aware of the messages about Mathematics you convey to students. Just as parents can help in shaping their children's attitudes towards Mathematics, teachers can also have a similar impact on their students. If you have any doubt about the subject or performance of student, make sure not to convey your feelings to students. Express confidence in their abilities, telling them to continue to take interest. Then definitely they will eventually succeed and perform well.

Be calm and patient. This is especially important for the Mathematics-anxious student. The slightest sign of teacher's impatience in tolerance might demoralise a student to such an extent that he/she will not have the courage to speak in the class at all. Create a climate in which students have no fear of asking a question or offering a wrong answer. Present instructions in a clear, calm manner and give the student time to process the instructions and formulate a response. If you find that the student continues to be afraid of Mathematics, talk to her individually and try to remove her fear by assessing her actual difficulties.

Encourage the student to ask questions. Students with Mathematics anxiety often are reluctant to ask questions in class for fear of appearing dumb or being taken to task by the teacher for not listening. Make it clear to your students that you want them to ask questions, and prove it by leaving time at the end of every class for that purpose. Tell students that their questions help the teacher as well as other students by indicating where you might not have been clear enough. Even with your encouragement, however, some students will still feel uncomfortable asking questions in class, so make yourself available after class or at the end of the day. Respond positively to a student's question, describing it as a 'good question' or an 'important point.' Make sure not to allow students to ridicule classmate's questions.

Promote the student's confidence. Generally the students with Mathematics anxiety might assume that they will not understand a mathematical concept or not be able to do a problem. That lack of confidence might impair their concentration and hamper their performance. Help them in reshaping negative views towards Mathematics by praising their successes and highlighting what they have done well. In presenting Mathematics work, start with problems they can complete easily and as they master the easier problems, move onto more difficult ones. Allow students to find alternative methods for solving problems so that they may feel that there is not just one method to solve a particular problem.

Help the students to understand mathematical concepts properly, rather than giving the results on the basis of rote memorisations. Many Mathematics-anxious students approach Mathematics as a series of procedures to be memorised, without understanding. When their memory fails, or when a problem falls outside the rules they have memorised, they will be nervous and will not perform satisfactorily. Therefore, always teach them 'why' and 'how' of the concepts involved in the discussions, as far as possible. The better they understand a concept, the more effectively they will retain and apply it. Here, role of language is also important. It must be simple and within the comprehension level of the students. Although using difficult words might increase their vocabulary but while explaining, these should not be used in questions.

Use concrete objects to foster understanding. Many students find mathematical concepts abstract and thus find difficult to understand. Use of concrete objects can help the students to grasp and to visualise concepts in a way that words alone cannot. Objects might include anything, that can be counted, or that conveys quantity or amount — such as blocks, beads, coins, matchsticks or Mathematics Kit. Sand and water can also be used to convey amount. As an example, cutting an apple in parts can help students grasp the notion of fractions in a way that worksheets can't. Of course, as the student's understanding grows, she can move from the

concrete to the conceptual. Similarly, they should be encouraged to observe three dimensional objects around them before moving into deeper concepts in 3D.

Make Mathematics relevant. 'Why do we need to study a particular topic in Mathematics?' is a common query in student's mind. Introduce the topic with demonstration that how the skills they are learning, is used in everyday life. Students will be more comfortable with mathematical concepts if they understand their practical value and learn to apply them. Give them problems that relate to their interests and age level that they might encounter outside the school.

Mathematics for fun. Teachers can use a wide variety of mathematical games to reinforce skills and promote a positive attitude towards Mathematics. These can be board games, card games, or games that you or the students create themselves. Typically easy to play and requiring little time, games help break up classroom routine. Keep games tension-free and relatively non-competitive. Also use the computer to stimulate enthusiasm for Mathematics learning. Many good software programmes are available for students of all ages.

Make a special effort to encourage girls in Mathematics. Generally it is said that the girls are more vulnerable to Mathematics anxiety than boys, especially at the upper primary stage. Part of their insecurity might stem from the messages they receive from both parents and teachers. Monitor the messages which are being conveyed to the girls carefully and make sure you don't tell that girls are weaker than boys in a Mathematics class or telling that girls avoid challenging Mathematics courses. Rather show confidence in their ability to do Mathematics, encourage them to take risks, and give them a chance to enhance their capabilities.

Effect of cooperative learning. There should be an approach of encouraging students to work on Mathematics problems in a group. Working in groups is a powerful tool against Mathematics anxiety. First, it solves almost all of the psychological issues identified by feminists: It provides

peer role models and social support and debunks the idea that being good at Mathematics is unfeminine. It also allows corrections without grading, showing alternative ways of solving problems and emphasising staying with a problem until it is solved, rather than until time runs out.

Involvement of Students in Solving a Question

By involving as many students as possible in solving a question, we encourage their interest and class participation. It also develops their level of confidence. At the same time, a teacher should try to enquire reasons behind the incorrect response.

Stepwise Evaluation of Exam Paper

As a teacher, we should encourage a student by giving marks for each correct step and should not consider only answer as the basis of marks.

In order to reduce Mathematics anxiety in the classroom, teachers should also focus on the importance of classroom management. They should strive to create a space that encourages each individual's strength and success. Mathematics lessons should be prepared to address a variety of learning styles. Students today have a need for practical Mathematics. Therefore, Mathematics needs to be relevant to their everyday lives. Students enjoy experimenting. To learn Mathematics, students must be engaged in exploring, conjecturing, and thinking rather than engaged in only rote learning of rules and procedures. The teachers may incorporate technology, cooperative learning, and make Mathematics manipulative i.e. put concrete objects into their Mathematics lessons. Teachers are encouraged to use cooperative learning by placing students in pairs or larger groups where all of the students maintain an equal role. The students will learn to share in the group's success and accomplishment. These successes will contribute to the students overall (positive) feeling about the work. During group activities, it is the teacher's responsibility

to promote the group's success by praising even the smallest accomplishment.

Conclusion

Teaching methods in Mathematics must be so designed that there should be more emphasis placed on the specific methods which include less lecture, more student directed classes, and more discussions. Teacher should be a facilitator and should encourage the students to derive results on their own. All this will help in bringing transaction in Mathematics.

Indeed, teachers and parents must work together to help children to overcome their Mathematics anxiety. The message must be clear: Mathematics instruction must remain positive, relevant, and concrete. Teachers must provide a safe and encouraging environment for their students. In addition, parents should support their children's confidence by making them connect Mathematics to their every day lives.

3 ANNEXURE

Mathematics as Communication

The belief that we learn and refine our learning through the act of writing has long been accepted. However, by its own nature, Mathematics is communicated through the use of symbols and terminology unique to the field. In fact, every subject knowledge when becomes a discipline develops its particular **jargon**, which is specific to the discipline. It helps in developing common understanding and sharing ideas universally. But, we need to question how we shall introduce with the discipline specific **jargon.** At upper primary stage, students are moving towards abstractions from concretes, so it will be better that students should experience specific symbols, terminology or jargon in mathematics with the concrete activities at the beginning.

Language is not meant just to communicate with others but it is also used in communicating with one's own life. While doing any activity, student makes an attempt to understand her own actions and this can happen when one is able to communicate with oneself. This communication occurs in one's own language. Thus, while transacting knowledge, teacher herself should ensure that the child's language is not only accepted but is used by her as well to develop a common understanding.

When students communicate mathematical information, they remember it, understand it, and use it to uncover and find even more information. Teachers need

to know how to help students grow into accomplished communicators of mathematics so that the students can describe their thinking process clearly. Teachers must help students to make their thinking visible to others by encouraging them to talk and write about the process they use to solve the problem.

Through wide range of activities, the teacher can promote communication in the language of Mathematics.

Encourage Students to Write Story Problems

Students can be asked to

- write a story problem using their own imagination or the information in a picture, newspaper advertisement, poster or short story.
- interchange their problems with each other and solve; and
- see the solutions of their problems and have a discussion about the problems and their solutions.

Students Can Play Act

- Student can be asked to sit on a chair that has been designated as the 'teacher's chair' and share original problems that she has written or share solutions to a problem written by someone else.
- Teacher may give useful and usable feedback to author student:
  - What did you like about the problem?
  - Do you agree or disagree with the solution?
  - How could the author improve the problem or solution?
  - How could the author change the problem to create a new problem or change the solution to arrive at a new way to solve the problem?

Cooperative Learning

We all observe individual differences (variability) among students in our classrooms. Some students are very shy,

some very active and some are average in terms of performing any task, like communicating their thoughts, reading, writing and various other co-curricular activities. Some are good at solving problems while some are good at reading and writing.

Sometimes situations arise in the classroom that some students feel a given task to be too boring as they have already mastered the task and at the same time some feel it to be very challenging as they could not understand the concept.

It is difficult for a teacher in a class of 40 or 50 students to help and facilitate learning of each student at her own pace by giving suitable activity and feedback to each student individually. The question arises in every teacher's mind that how he/she can satisfy student's physical, mental and emotional needs for their all around optimum development.

One of the ways to sort out this problem could be forming groups in the classroom, as working in groups is likely to enable teachers to address the need of individual student and enable learning among them. Communication among peers, trust building and peer tutoring help teacher and student both in improving learning environment in the classroom. The mutual influence, approval and support from teachers and peers encourage shy and average student's academic involvement in the group. Active students on one hand can help others in the class in learning the concept and on the other side they gain in-depth understanding of the concept and improve their logical reasoning and communication skills.

Group work also reduces the indiscipline problems and classroom management issues as it provides the students an opportunity to take responsibility by forming their own group norms and caters to both the extremes of the students in the class. It gives safer space to shy and isolated children in the group to perform without any hesitation.

INITIATE DISCUSSION WITH MATHEMATICAL COMMUNICATION

- **Present an already solved problem with a significant error.**

  Directions for the student:

  — Read the problem and look at how this student has solved the problem.

  — Answer each question of the students as a follow up of the problem.

  Sample Problem

  Vinay and Priya have to be home by 9:00 p.m. It is now 7:00 pm. How many hours may they play before they have to go home?

  Abhay solved the problem in this way:

$$\begin{array}{r} 9 \\ +\ 7 \\ \hline 16 \end{array}$$

Explain whether Abhay's reasoning was correct or incorrect.

- **Present a problem and a partial solution.**

  Directions for the student:

  — Complete the solution to this problem.

  — Describe another way to solve this problem.

  Sample Problem

$$\frac{2^3 \times 2^5}{2^2} = \frac{2^8}{2^2} = 2^{8-2} = 2^6 = 64$$

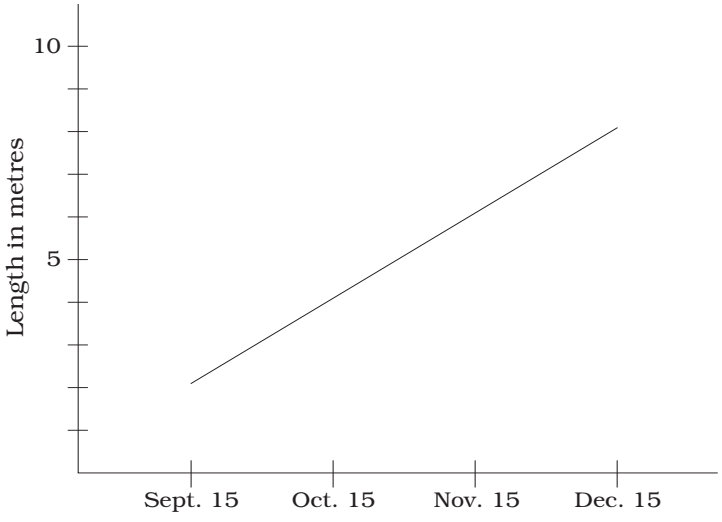
  Alternative approach

$$\frac{2^3 \times 2^5}{2^2} = 2^{3+5-\ldots\ldots} = 2^6 = 64$$

- **Present the students with a graph or table and make them write a story that represents the data on the graph or table, etc.**

Directions for the student:

Sajid's shadow at the same time each day



Write a story as per the data given in the graph above.

In this manner, you can adopt several ways through which communication skills of students in Mathematics can be enhanced.

CONCLUSION

1. Teacher needs to understand the language of the students. Teachers need to understand that students need time to observe, to work together, and to construct an understanding of the language of Mathematics and to make it their own. So they should be engaged with activities for some duration.

2. As soon as students use words, they make their understanding of Mathematics more precise and at the same time more general.

3. When students write or talk about Mathematics problems, they test, expand and extend their understanding of Mathematics.

ANNEXURE 4

Number Sense an Important Skill: Why and How?

Numbers are taught to the students from very early stage. Some important skills which build-up with the learning of numbers are 'number sense', 'operation sense', 'finding patterns' and 'intuitive understanding'. Let us discuss about these phrases and focus on their importance while teaching and learning numbers.

The idea of number sense is not new. In 1930, William Brownell, in 1989 educationist Howden and in 1991, McIntosh, Reys and several others have worked on the skill what we call as 'number sense'.

Number sense enables us to know various uses and interpretation of numbers as mentioned below:

- Detection of mathematical errors while computation.
- Enrichment of our common sense in using numbers.

Power and richness of number sense can be visualised through the following examples:

A student with number sense will:

- *Look at a problem holistically before going in details.*

  For example, suppose we have to find the sum $1\frac{2}{3}+\frac{3}{4}+\frac{1}{3}$.

  A student might mentally reorder it as $1\frac{2}{3}+\frac{1}{3}+\frac{3}{4}$, so

that fractions having same denominator can be added easily, i.e. she can take the advantage of the compatible like fractions.

- *Look for relationship among numbers and operations in the context of question.*

  For example, in buying 4 notebooks priced at ₹ 4 each, the student with ₹ 20 might reason that she has enough money to buy notebooks. She could realize that the total cost of notebooks is less than the money she has.

- *Choose or invent a method of solution according to her own understanding of relationship between numbers and operations and seek the most efficient way to deal with.*

  For example, suppose that at least 75% of the class of 30 students needs to agree on a plan for a programme of school festival before it can be finalised. A student might reason that '75% = 50% + 25%, or half + half of that. So 15 + 8 or 23 students must agree.'

- *Use benchmark to judge number magnitude.*

  For example, $\frac{2}{5}$ of 49 is less than half of 49 (see Fig.4.1).

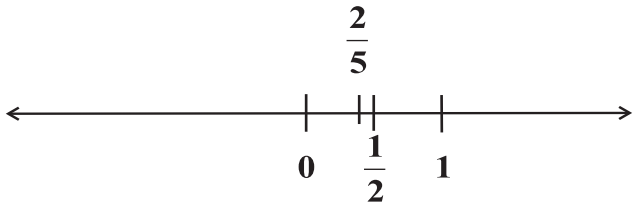


Fig. 4.1

- *Recognise unreasonable results for calculations in the normal process of reflecting on answers.*

  For example, 3.2 × 4.8 cannot possibly be 153.6, since answer must be about 3 × 5, or 15 so an error in decimal-point placement must have been made.

Thus, number sense is:

- the ability of the student to make logical connections between new information and previously acquired knowledge; and
- an attitude build up within the student to form these connections on **priority**.

The Teacher's role in developing Number sense

Number sense will be valued among the students **only if teachers believe** that it is more important for students to **make sense or understanding of Mathematics** they learn than to master rules and algorithms.

USE PROCESS QUESTIONS

Let us see the following conversation:
Teacher: I am thinking of two fractions. Their sum is between 0 and 1. What can you tell me about the fractions?
Ajay: Both fractions are small, may be less than 1.

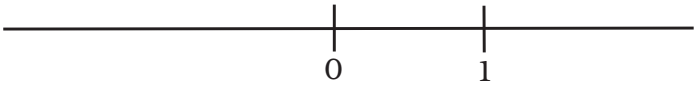


Fig. 4.2

Teacher: Ok, what else?

Ajay: Are they both less than $\frac{1}{2}$?

Teacher: Good question, can any one answer that?

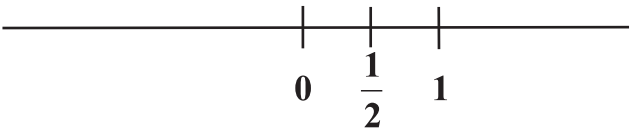


Fig. 4.3

Anita: Not always! One can be $\frac{3}{4}$ and the other very small, like $\frac{1}{10}$.

Ajay: Yes, but if one is bigger than $\frac{1}{2}$, then other must be smaller than $\frac{1}{2}$.

Teacher: Good point. Can anyone tell me anything about my fractions?

Vinay: If their sum is less than 1, their product is also less than 1.

Teacher: What do the rest of you think about the idea?

The teacher's questions may prompt the students to participate in the discussion and examine various examples and prove or disprove given statements and their preconceived notions. Thus process of questioning-

- **requires more than a simple factual response like yes or no only; and**
- **can stimulate discussion of an idea, which can lead to further exploration and use of oral language to explain and justify a thought.**

What is valued most is reflection on answers so that they can be proved either right or wrong. The teacher in this example assumed the critical role of posing questions, which caused students to begin reflective thought, and encouraged other students to get involved in the process.

Teaching for number sense involves a quality-over-quantity attitude toward problem completion. That is, the focus is on understanding a given problem by looking at it from **multiple perspectives** rather than on attempting to work out as many problems as possible in a given period.

Encourage to Write

Students can be asked to write about the results of group activities or their ideas generated by specific tasks. As they write, they may formulate new ideas or questions which may serve as a reference for further question posing and discussion.

Encourage Invented Methods

Teacher should encourage the students to solve a given problem by the methods they know. Different methods to solve a problem should be encouraged. The goal of **'one right answer'** derived from **'one preferred algorithm'** is replaced by **the goal of multiple solution strategies.** Moreover, there should be more emphasis on process rather than on final answer.

Use Appropriate Calculation Tools

Students may solve the problem in many ways: mentally, by estimation or by approximation.

For example

In calculating 5 × 96, one student might change the problem to

- $\frac{10\times96}{2}$ and another might think as 5 × 8 × 12;
- whereas another might use the distributive property and compute
  (a) (5 × 90) + (5 × 6) or
  (b) (5 × 100) – (5 × 4)
- Probing questions and concrete analogies can be used to initiate the exploration of alternative methods of mental calculations. For example:

- For the problem 25 × 49, the teacher might ask, 'can anyone create a problem from this calculation by substituting the word quarter for the number 25?' or
- 'Does it help to find the product of these two numbers, if we think about 25 as a quarter? How?'

HELP STUDENTS ESTABLISH BENCHMARKS

Approximate computation or estimation is another important tool for encouraging students to use what they already know about numbers to make sense of new numerical situations. For example,

- A student using a standard protractor to measure a 30 degree angle is not likely to read the wrong scale and report 150 degrees as the measure if a 90 degree angle has been established as a benchmark (Fig. 4.4).

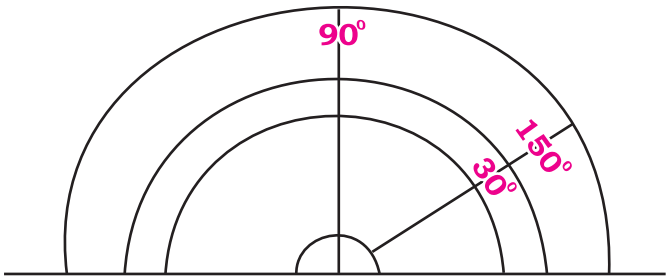


Fig. 4.4

- In the same way, a student who has been encouraged to estimate fractions near $0, \frac{1}{2}$ and 1 will understand that the sum $\left(\frac{2}{5}+\frac{4}{9}\right)$ must be less than 1, since both fractions are less than a half.

Encouraging students to consider these benchmarks is a way of helping them develop better conceptual understanding of fractions, decimals and percents.

The intuitive understanding is a priority and should precede the study of operations on fractions, decimals and percents.

Promote Internal Questioning

An important role for teachers in the development of number sense is helping the students learn to ask themselves key questions before, during and after the solution process. For example,

- What type of number I expect for an answer to this problem?
- About how large will the answer be?
- What is the biggest or smallest value I expect?
- After completing a calculation, students then determine whether the answer is consistent with what they expect.

This process of self-examination may help sensitise students to find the order-of-magnitude of errors as well as prevent them from checking their answer by repeating the same computational error a second time.

Select Activities

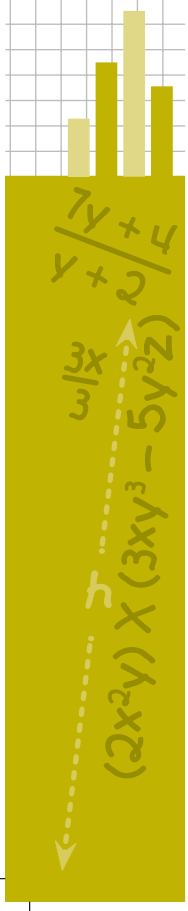
- Activities encourage students to think about what they are doing and to share their thoughts with others.
- Activities promote creativity and investigations and allow for many answers and solutions strategies.
- They help students when it is appropriate to estimate or to produce an exact answer and to see the regularity of Mathematics and the connections between Mathematics and the real world.
- Process-oriented activity also conveys the idea of Mathematics as an exciting, dynamic discovery of ideas and relationships.

Thus, in a classroom where number sense is a priority we see that

- students are active participants who share their hypothesis, reasonings and conclusions; and
- the classroom environment encourages students' exploration, questioning, verification and sense making.

CONCLUSION

Number sense is not a new topic for teachers to include in an already prescribed Mathematics course. By establishing an appropriate classroom atmosphere that encourages exploration, thinking and discussion and by selecting appropriate problems and activities, the teacher can cultivate number sense during all mathematical experiences.



ANNEXURE 5

Fraction
a Conceptual Understanding

At upper primary stage, students have to learn a precise mathematical concept of a fraction and make use of it in logical conclusions. For the study of a particular phenomenon, a scientist first collects the data related to it. Similarly, till primary stage, students just gather the information about fraction (e.g. A part of pizza, one fourth of an apple etc.)

After collection of data, the scientist analyses it to arrive at a certain theoretical result. In the same way in upper primary classes, students are supposed to put the easily acquired isolated bits of information in making a vocabulary of fractions. That is, the earlier acquired isolated bits of information that forms the vocabulary of fractions have to be put in a mathematical framework at this stage. **A conceptual understanding** is essential before involving students in operations with fractions. We will see and discuss the following two models of fractions:

- Area model
- Set model

Through an activity, we will see the important perspectives of these models.

The most basic way of visualising a fraction is part of a whole; this interpretation also is the typical way of introducing fractions to young children.

For example:

- One person may **'see'** the fraction **'one - half' as a picture of a circle with half part shaded (Fig. 5.1).**

1. Here, the circle has been divided into two equal parts having equal area.
2. This is an example of a **model of a fraction based on area.**
3. The **area model** for fractions seems to be the easiest embodiment for students to understand.

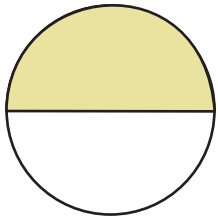


Fig. 5.1

4. Illustrations of the area model for fractions include a geometric figure with part shaded, graph paper with part shaded, a clock with the hands designating a part of a circle and paper folded to give fractional parts (Fig. 5.2).
5. A critical feature of the area model is that **all the parts into which the whole is divided must have an equal area.**

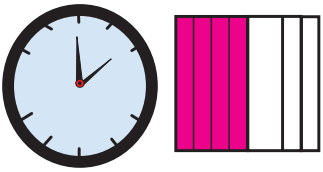


Fig. 5.2

- Another individual may **'see'** the fraction **'one-half' as a bag in which half the pieces are chocolates and half toffees.**

1. In this case, number of elements (toffees) of a set (bag) has been considered and out of these half of them are chocolates. So this fraction is based on set model.
2. Through experience it has been seen that conceptually the set model for fractions is more difficult than the area model.
3. It requires identifying the units and eliminates the requirement that the pieces be of the same size. Accordingly, it is generally introduced in later

classes. This model identifies what fraction of the elements of a set has a specific characteristic, such as same colour. For example,

(a) What fraction of the plants has red flowers?

(b) What fraction of the people in the room wear glasses?

4. In this model, the pieces or members of the set are not required to share any attribute other than membership in the set; they do not need the same shape or same area.

It is important to recognise that fractions may have different meanings in different contexts.

Let us consider an activity to understand different meanings of fractions in different contexts.

Look at the following collection (Fig.5.3) and answer a question, 'What frction is blue (B)?'

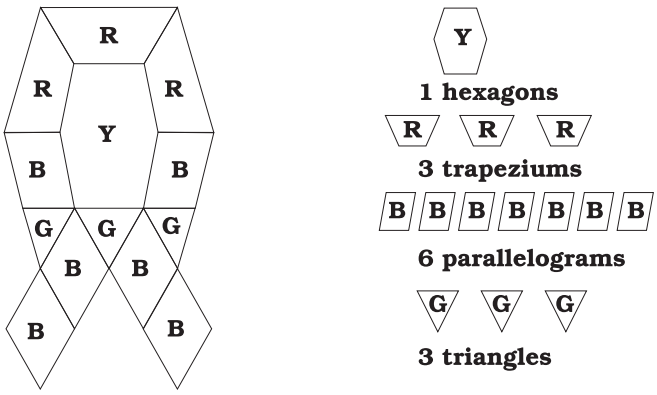


Fig. 5.3

Let us see responses of several students and teachers as follows:

RESPONSES

1. I think it is $\frac{6}{13}$. I think $\frac{6}{13}$ is right because there are 13 pieces and 6 of them are blue (B).

(a) This may be the most common response by students.

(b) They seem to think of a fraction as being some part of the number of elements of a set. So they count the number of pieces and find that six of the thirteen pieces are blue. Here student is thinking in terms of a set model.

2. I think it is $\frac{1}{3}$.

(a) This response may be very rare and given by those students seeking additional possible interpretations of the question.

(b) They had simply found the largest piece, the yellow hexagon block and then decided what fraction of the biggest piece is the blue parallelogram piece (Fig. 5.4).

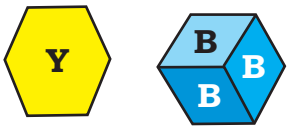


Fig. 5.4

(c) Since three blue parallelogram pieces make up one yellow hexagon block, the blue block must be $\frac{1}{3}$ of the largest piece.

(d) This person is using an area model for fractions but is not considering the entire design.

3. It is $\frac{1}{6}$.

(a) This response is also relatively uncommon but tends to appear more frequently than the previous one.

(b) Here the person has thought that 'Design has six blue pieces in all, so one blue piece is one-sixth of the blue pieces'.

(c) This student is answering the question 'What fraction *of the blue pieces* is one blue piece?'

(d) Without further explanation, it is unclear whether the student has used area model or set model.

4. I think that $\frac{6}{15}$ or $\frac{2}{5}$ is blue.

Here, the student has considered the whole design consisting of 15 blue pieces, 6 of these were really supposed to be blue. (Note that Trapezium + Triangle = 2 Parallelograms) (Fig. 5.5).

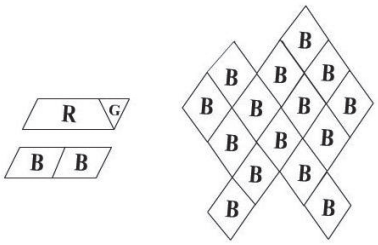


Fig. 5.5

(a) Here student explains that in working with fractions, all the pieces must be of the same size.

(b) This student is using an area model and thinking of the fraction of the total area that is blue.

5. I think that the fraction of this design is $\frac{2}{5}$.

(a) Because there is

i. 1 hexagon (Y)

ii. 3 trapeziums (R)

iii. 6 parallelograms (B)

iv. 3 triangles (G)

(b) All the pieces have been moved around so that like colours are together and then the number of

hexagons have been counted

i. One yellow hexagon

ii. Two of the red trapezium pieces make a hexagon

iii. One red trapezium and three green triangles make one hexagon

iv. One hexagon will be made by three blue parallelograms. So, there will be two hexagons from six blue parallelograms (Fig. 5.6).

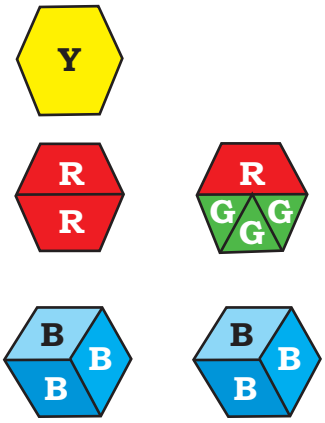


Fig. 5.6

(c) So, there are five hexagons and two of them are blue.

(d) Thus, fraction of the design that is blue is $\frac{2}{5}$.

(e) For many students/teachers, the answer occurs only after they have been encouraged to explore the situation further.

6. So, from the above, it can be observed that students/teachers can interpret the language of the problem on fractions in their own way and obtain different answers. These answers may or may not be all correct.

Conclusion

1. Investigating these or similar types of situations can involve students in activity/discussion for quite some time.
2. These type of activities are quite helpful to have students write about the activity at two points, namely
   (a) writing before discussion; and
   (b) writing after discussion, describing what they have learned from the discussion.
3. For each exploration, students should again have opportunities to communicate about the situation, about their strategies in solving the problem and about their answers, so that students become more aware of their own thinking.
4. The communication aspect of these activities helps teacher to assess each student's reasoning and understanding about fractions.

6 ANNEXURE

Division of Fractions

Division of fractions is not easily understood by students more often. Indeed, the concept of division is similar for both whole numbers and fractions, and this must be so because whole numbers and fractions are both numbers, but the similarity is not along the **line of repeated subtraction.**

The idea, that division among whole numbers is repeated subtraction, is based on the following procedure:

To divide 23 by 4 one can first perform division-with-remainder to obtain 23 = (4×5) + 3, and then with the quotient 5 and remainder 3, one can write it as: $5+\frac{3}{4}$ which is of course $5\frac{3}{4}$. To obtain the answer for (say) $\frac{5}{4}$, divided by $\frac{2}{3}$, one may like to perform a similar division to obtain $\frac{5}{4}=\left(\frac{2}{3}\times 1\right)+\frac{7}{12}$. Then, with the 'quotient' 1 and 'remainder' $\frac{7}{12}$, one may have the answer $1+\frac{\frac{7}{12}}{\frac{2}{3}}$ for the

division of $\frac{5}{4}$ by $\frac{2}{3}$. So goes the reasoning behind the theory of repeated subtraction. Unfortunately, there is no way to make sense of $\frac{\frac{7}{12}}{\frac{2}{3}}$ in the purported answer, because such a division (of $\frac{7}{12}$ by $\frac{2}{3}$) is exactly what we set out to define in the first place.

Students need to be made aware of the connection between division involving fractions and the repeated-subtraction model often used in the interpretation of whole-number division.

Finding examples of practical problems that illustrate the usefulness of division with fractions and mixed numbers is not so easy. During interactions, we have seen that many teachers and prospective teachers have difficulty in constructing examples and concrete models for the operation of division with fractions.

Following activity may be helpful in establishing a conceptual basis for dividing fractions. We suggest that it should be used before the introduction of invert-and-multiply rule for division of fractions.

- Here is a question for you
  - Write a word problem to illustrate the following:

    $1\frac{3}{4} \div \frac{1}{2} =$

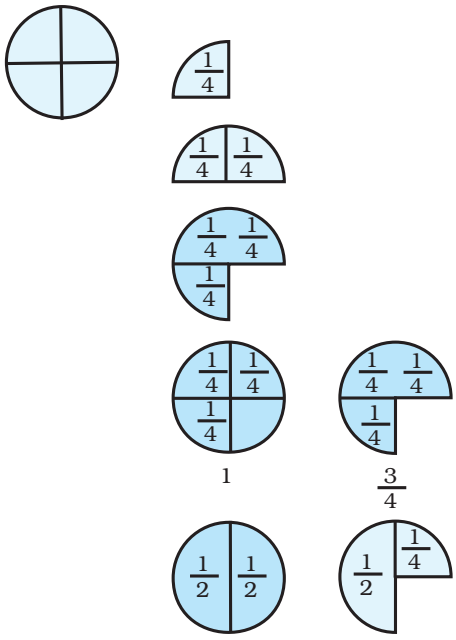
  - The most common responses involve some distribution of bread , like the following:

    Sunita and Ganesh want to share equally $1\frac{3}{4}$ breads. How much part will each get?

  - The error lies in interpreting $1\frac{3}{4} \div \frac{1}{2}$ as $1\frac{3}{4}$ divided

in two parts, i.e. $1\frac{3}{4} \div 2$. In fact, it should be read as how many half $\left(\frac{1}{2}\right)$ breads are there in $1\frac{3}{4}$ breads?

- So, the question $1\frac{3}{4} \div \frac{1}{2}$ = ? is really asking 'How many $\frac{1}{2}$'s are there in $1\frac{3}{4}$?' Let us see the solution in the following two models. Visuals of both models of solutions are as follows:



We have 3 halves and 1 half of half

$$1\frac{3}{4} \div \frac{1}{2} = \frac{1}{2} + \frac{1}{2} + \frac{1}{2} + \frac{1}{4} = 3 \times \frac{1}{2} + \frac{1}{4}$$

Fig. 6.1 (Model 1)

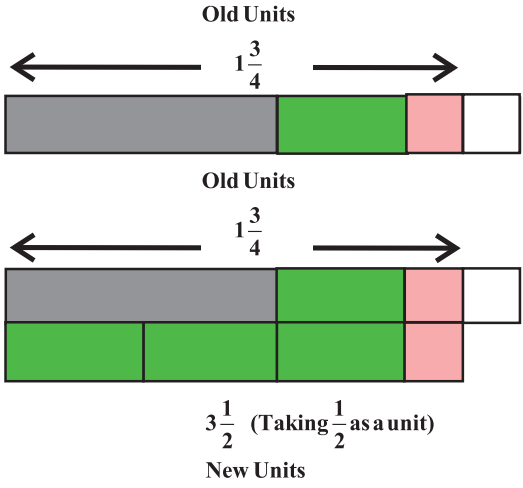


Fig. 6.2 (Model 2)

Students can be asked to use these models to explain why '$\div \frac{1}{2}$' is like '× 2'.

The real similarity between whole number division and fraction division takes a little longer to explain.

- First, consider 48 ÷ 6 = 8. It means that there are 8 groups of 6 in 48, so that 48 = 6 + 6 + · · · 6 (8 times) = 6 × 8.
- So if we know that 48 ÷ 6 = 8, we also know 48 = 6 × 8. Conversely, if we know 48 = 6 × 8, then we also know that 48 contains 8 groups of 6 each and, therefore, by the usual understanding of what division means, we also have 48 ÷ 6 = 8.
- Thus, the division 48 ÷ 6 = 8 and the multiplication 48 = 6 × 8, exactly are the same thing.
- As a matter of fact, we know that in mathematics, division has no independent existence: it is an alternate but equivalent way of writing certain multiplication statements.
- We say 'certain' statements because we cannot rewrite 0 = 5 × 0 as a division statement.

- In general, for any three whole numbers a, b, and c with $c \neq 0$, the meaning of $\frac{a}{c}=b$ is the same as $a = bc$. (We have replaced the division symbol by the fraction bar, and we have also omitted the multiplication symbol between letters, as usual.)

  We know that division of whole numbers (except 0) has been thoroughly discussed in earlier units. We have not yet given meaning to the division of two fractions. But we know that the latter cannot differ from the former, conceptually, because they are both numbers. Thus, using the conceptual framework of whole number division as a guide, we are led to define, for fractions a, b, c with $c \neq 0$, the division $\frac{a}{c}=b$ to mean exactly that $a = bc$.

- We emphasise that, by definition, the only way to understand a division of fractions is to understand the corresponding multiplication statement. And this leads to the 'invert and multiply' rule.

CONCLUSION

Before the formal introduction of invert and multiply rule for fractions, conceptual understanding of division of fractions should be introduced with specific word problems and visual models.

ANNEXURE 7

Playing with Numbers
Making Problem

Number system is an area of interest to many students and can serve as a motivation for learning Mathematics. It is important for students to understand the relationships among whole numbers, integers, rational numbers, etc., and their various properties.

Let us look at the following problem:

In a certain school, there were 1000 students and 1000 boxes. All the students stood in a queue and performed the following activity:

The first student opened every box. The second student went to every second box and closed it. The third student went to every third box and performed the reverse process (i.e. if the box was open, he closed it; and if it was closed, he opened it). In a similar manner, the fourth, fifth, sixth... student reversed every fourth, fifth, sixth... box, respectively. If all the students have participated in this activity, which boxes remained open finally?

This is an open-ended problem for which we do not have any readymade algorithm. This type of problem will require a variety of problem-solving strategies which involve thinking too.

While discussion on this problem, usually students immediately begin to guess the following:

- All of them
- None of them
- All the evens
- Only the first and so on.

While going through the problem carefully, one may think of considering the case for a few students instead of taking all the 1000 students, as follows:

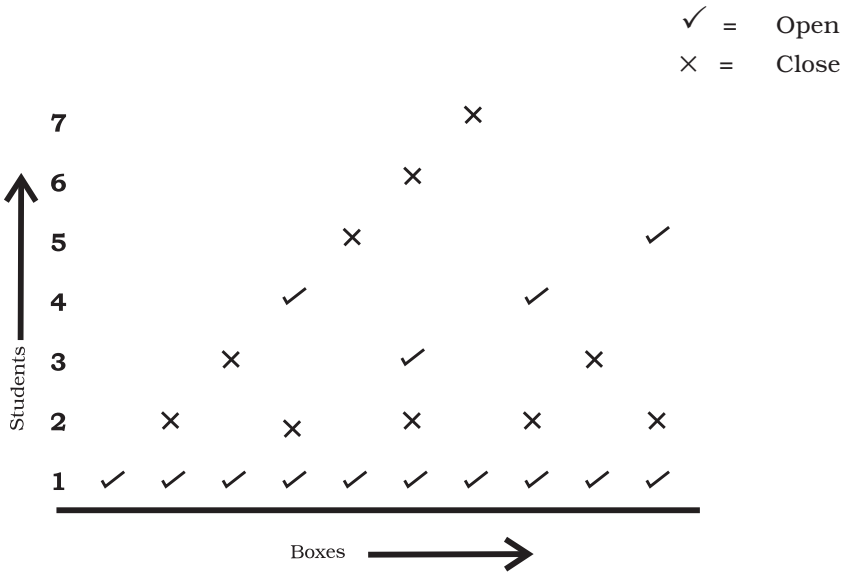


Fig. 7.1

By observing Fig. 7.1, one can easily recognise that the first and the fourth boxes will remain open. Often they guess a pattern of 'one open, two closed', and hence 7th box is open. But it is not so.

After understanding this, students may generalise as follows:

- One open, two closed; one open, four closed; one open, six closed and so on.

- So, the students at this point may get some insight of the solution.

UNDERSTANDING THE SOLUTION

Experience with teachers/students solving this problem has led to two observations:

(i) They usually do not recognize the numbers of the open boxes (B_1, B_4, B_9, B_{16}, B_{25}... where B_n is the n^{th} box) as the sequences of squares.

(ii) They do not distinguish between seeing the pattern and determining the necessary conditions for a box to be open.

- A student will visit the box only if the student's serial number is a divisor of the serial number of the box.
- From this, it can be concluded that for a box to remain open, it must be visited by an odd number of students (i.e. the box number must have an odd number of divisors).

24	36
1×24	1×36
2×12	2×18
3×8	3×12
4×6	4×9
	6×6

- B_{24} is visited by: 1, 2, 3, 4, 6, 8, 12, 24
- B_{36} is visited by: 1, 2, 3, 4, 6, 9, 12, 18, 36

Students serial numbers in the queue

- We see that divisors come in pairs [i.e., (1, 24), (2,12), (3,8), (4,6)], except for the special case of the square number [i.e., (1,36), (2,18), (3,12), (4,9), (6, 6): 6 is not paired with different number].

- This enables us to explain why only open boxes are the ones for which the box number is a perfect square.
- All boxes with prime serial numbers will be closed because these boxes will be opened by 1st student and closed by the students whose serial number matches the serial number of the box. (For example B_7) No other student will touch these boxes.
- Here the teacher might pose the following questions:
  - Are there any other boxes which you know are closed? Why is this so?
  - Which student did visit B_9? B_{12}?
  - Why did the 5^{th} student not go to either B_9 or B_{12}?

RELATING THE PROBLEM WITH OTHER CONCEPTS

- From the above observations, we can raise new questions that lead to other topics of interest:
  - — Can we locate the boxes which are visited by only two students?
    - This leads to Sieve of Eratosthenes for locating 168 prime numbers less than 1000.
  - — Do the open boxes get farther apart? Closer together? Stay the same?
    - Get farther apart as the differences between the terms in the sequence 1,4,9,16,25,..... increase by successive odd numbers:

      1 + 3 = 4
      4 + 5 = 9
      9 + 7 = 16
      16 + 9 = 25

  - — Why do the square numbers increase in this manner?
  - — Each successive square requires an L shape array (called a gnomon) containing (2s+1) points, where s is the side of the previous square. (See, x x x marked shapes in Fig. 7.1)

- The next sequence of questions develops some more topics such as factors (divisors), multiples, common factors and common multiples.
  - — How can you determine if B_{10} was visited by 3rd student? By 5th student? By 6th student? and so on.
    - That is, how do you determine if the student number is a factor of the box number? (Factor)
  - — Which boxes were visited by 5th student and 7th student? By 6th student and 8th student? By 4th student and 8th student? and so on.
    - Common multiples
  - — What is the number of visits to the first box by 5th and 7th students? By 6th and 8th students? By 4th and 7th students? and so on.
    - Least common multiple
  - — Is there any student who visited every box that was visited by both 6th student and 8th student?
    - Are 6 and 8 co-prime? (co-prime or relatively prime)
  - — Which students visited B_{24} and B_{36}? B_{45} and B_{49}? B_{10}, B_{25} and B_{45}? and so on.
    - Common factors
  - — Who was the last student to visit B_{36} and B_{48}? B_{15} and B_{32}? and so on.
    - Greatest common factor (highest common factor).

CONCLUSION

We just saw how a simple problem can be related to so many concepts in Mathematics. This problem may still have several questions which could help to explore much deeper ideas about different concepts related to numbers. Hence, it should be a good way to make a problem of upper primary stage which makes the learners conceptually strong.

8 ANNEXURE

Concept of Educational Evaluation

Educational evaluation is an activity which is conducted by the educators/teachers in order to continuously and systematically review and then enhance the teaching-learning process that they are endeavouring to facilitate. It is of immense importance in the total span of educational process due to its potential of creating impact on the society as a whole. The merit of any educational system depends on the standards of attainment shown by its products in terms of competence and excellence.

Concept of Educational Evaluation

Generally speaking, evaluation is a very comprehensive term which includes evaluating any object, individual, institution, position of an office, event, trend, etc. However, educational evaluation deals with students' evaluation which includes the assessment of the performance of the students in the areas of their personality development in terms of intellectual, social and emotional development after they have been provided learning experiences through classroom processes.

Evaluation is often confused with the term measurement and both the terms are used synonymously. But both are not the same. The term measurement stands for measuring the performance of the student on a

particular scale. The pattern of measurement which is mostly followed in our assessment system relates to marking on a scale of 0-100 marks. This also includes pass-fail system where in all those who secure 33 percent marks and above are declared pass and below this are tagged fail. This scale is a metre for classifying the students on the basis of the marks they obtain in a test or examination. Therefore, measurement provides a quantitative description of pupils' performance based on artificial classification. It does not include value judgement and thus it gives a fragmented picture of student's performance. Moreover, all these aspects are related only to intellectual growth.

On the other hand, evaluation is a more broader term as compared to measurement and it includes both quantitative and qualitative description of the performance and value judgement. Regarding quantitative description as discussed earlier, measurement on a scale is applied and marks are allotted. For qualitative description, interpretation of the marks secured by the student are made in reference to him/herself, his/her group and certain criteria. It also includes value judgement regarding the desirability of behaviour related to all the domains of personality development.

Following example will help in understanding the difference between evaluation and measurement:

Consider the following question given in an examination paper:

Question: *l* and m are two perpendicular plane mirrors. A ray AB strikes at mirror *l*, gets reflected as ray BC to

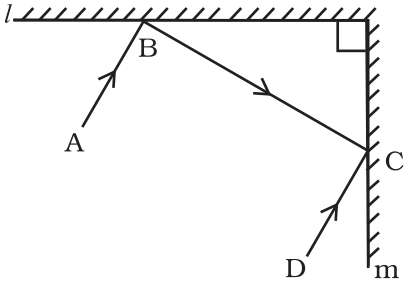


Fig. 8.1

strike again at mirror m and CD is the reflected ray from mirror m as shown in Fig. 8.1. Prove that AB || CD.

In **Measurement**, the teacher will check the solution and award marks accordingly.

But **Evaluation** is a broader term. To understand the students learning level, the teacher can use the following evaluation tool (known as rubric) for the above question.

Topic: Geometry

Name of Students	Able to write down 'given' by understanding the question	Able to judge what is to be proved	Able to analyse the situation to do required construction	Able to relate concept of other subject (laws of reflection here)	Able to apply knowledge of laws of reflection in the question	Able to organise her thoughts in a sequential manner and able to put it on paper	Able to conclude the result to be proved	Remarks
Neha	✓	✓	✓	×	×	×	×	She doesn't have the knowledge of laws of physics hence is not able to attempt the question further
Shikha								
Reemu								
Avantika								
Kusum								

The above rubrics help teacher, student and parents to judge the achievement of instructional objectives.

For the purpose of evaluation, the teacher can also maintain a class response register in which she can note

down general and specific behaviour of students on regular basis.

Conducting surveys also form a part of evaluation. For example,

- Survey of favourite news channel in your locality.
- Survey of effects of advertisement on children.

(The above surveys demand the use of data handling by the students).

Students can be given projects in groups like study on the famous buildings/monuments of world and geometrical need of structure so formed (Finding out answers to questions like, why most of the old monuments have hemispherical domes.)

Thus, evaluation is a process of collecting evidence about student's achievement or development in terms of educational objectives. Judgements are formed and decisions are taken on the basis of evidences. Therefore, evaluation has the following four components:

(i) information gathering,
(ii) interpretation of information,
(iii) judgement forming, and
(iv) decision making

Information gathering pertains to the collection of evidences regarding pupil's performance in subject specifics through giving tests and scoring the answer scripts. Regarding social and personal qualities evidences can be collected through observing the behaviour indicators related to the identified qualities.

Information gathering is followed by analysis of the evidences and forming the judgement regarding the pace of learning as well as level of the learning of the pupils. Analysis of evidences is done in terms of three reference points. The first point is related to the previous performance of the child. Whether he has improved or deteriorated in terms of his/her achievement? The second point is his

performance in peer group constituting the whole class, that is his/her rank in the class on the basis of his/her performance. The third point is concerned with the criteria determined by the teacher whether the child could attempt successfully all the given questions or he could do only a part of them. The analysis leads to decide whether the learning is improved or retarded and the students need help or remediation.

Decision-making is the third step of evaluation process. On the basis of the judgement, decision can be taken in the form of allotting marks or grades. The other decision may be whether a pupil is to be promoted to next class or to be retained in the same class. Decisions may be communicated to the pupils and parents through report card or certificates, etc.

There are some related terms associated to evaluation which needs to be understood, otherwise misunderstandings are carried on. Following related terms are explained as under:

Examination

It is a process of collecting evidences about pupil's achievement. Therefore, an examination includes developing a number of tests, conducting them and then marking answer scripts or awarding grades for reporting the achievement of the students.

Tests

Test is a tool consisting of a number of questions for finding out the knowledge, understanding, aptitude and interest, etc. of the students. The test is based on a pre-determined set of objectives. It can be written or oral.

Assessment

Assessment is the process of estimating the status of pupils' development in different aspects of learning. It can be done both in terms of quality or quantity. In British literature,

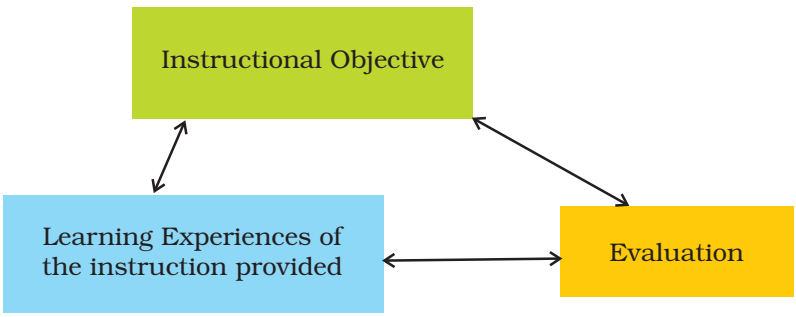
assessment is occasionally used as a synonym to American term evaluation. Through assessments, teacher can help the students to learn and gauge their performances.

Assessments are often categorised as either objective and subjective or formal and informal.

Evaluation in Teaching and Learning

Evaluation is an integral part of any teaching and learning programme. Whenever a question is asked in a class and answered by a student and the answer is judged by the teacher, evaluation takes place. Thus, both teaching and evaluation go hand in hand with each other. In fact, it is not possible to have teaching and learning without evaluation.

Both teaching and evaluation are based on the instructional objectives which provide direction to them. Instructional objectives are those desirable behaviour which are to be developed in students through the learning experiences. These are reflected in the form of syllabus, instructional material and information given by the teacher. Instructions are given for achieving the objectives and evaluation is done to see whether the instructional objectives have been achieved and to what extent. The interrelationship of objectives, instructional process or the learning experiences and evaluation in a programme of teaching can be expressed more clearly through the following diagram:



The above diagram illustrates that the three components teaching, learning and evaluation constitute

an integrated network in which each component depends on the other. Thus, through evaluation, the teacher not only assesses as to how far the student has achieved the objectives but also examines the effectiveness of the teaching strategy such as methodologies, means and the materials used for achieving those objectives.

Purpose of Evaluation

Evaluation has varied purposes in the field of education. Few of the following are given below:

- It judges effectiveness of instructions.
- It motivates students for better learning.
- It diagnoses strengths and weaknesses of students, i.e. it helps to understand which skills they have mastered and which ones are left out to target further instruction.
- It helps in determining as to how far the learning objectives could be achieved.
- It determines the rate of progress of students.
- It assists in planning future instructions for the students. Thus, it enhances instructions.
- It gives all the students a chance to demonstrate what they have learnt and how much?
- It provides a base for certifying the students.
- It predicts the success of students in future.

Evaluation is not just a process where a set of various techniques of testing are used. In fact, it is a process to determine the extent to which the instructional objectives have been achieved by the students.

In teaching-learning process, evaluation should be done with a purpose, and not for the sake of evaluation only. Administering a test, marking the solution scripts and collecting the data without making any use of this

information for the pupils is a wastage of efforts. In fact, evaluation should be used for taking decision regarding further teaching, adoption of new materials and methods, necessity of remedial teaching, guidance to the pupils, etc.

Kinds of Evaluation

Evaluation is mainly of four types: Placement, Formative, Diagnostic and Summative.

Placement Evaluation/Entry Behaviour

Placement evaluation is meant for finding out the position of the child in the initial stage of learning. When a teacher tries to introduce some instruction to the children in a class, it is imperative to know where a child can be placed in terms of his previous knowledge so as to enable him to be ready for further learning. This placement is a very important stage of evaluation as it gives the teacher an idea of the weakness and strength of the child's learning. If it is not ascertained and the instruction is imparted to the child, probably he will not be able to understand the forthcoming concepts because he does not have the sound background of previous related learning. The child will be able to attain the desired abilities and competence developed through new instruction only with a strong background otherwise his weakness will continue to persist and his attainment will continue to be low. Therefore, to avoid this situation and to enable the child to be strong in the concept and competence, placement evaluation is very essential. This could be done before introducing any topic.

The placement evaluation is necessary for KGBV students, because many girls who take admission in Class VI may be drop-out students at different stages of primary schooling. Through placement evaluation, you may find out the level of students and the concepts which are necessary to understand the concepts required at this stage. The Bridge Course developed for this purpose will be useful to bridge the gap.

Formative Evaluation

Formative evaluation is an in-built process of teaching and learning. It is desirable to know whether a student has developed a certain ability stated in the objective and it is also required to know about his progress during the course of teaching and learning. If there is any deficiency in his learning, it can be removed by re-teaching. This system of evaluation is known as formative evaluation. Formative evaluation can happen during the course or unit, to check upon the students' learning process. If students are not performing upto the mark, then back-up or re-teaching can be planned at that very instant. Home work assignments and class discussions can also be beneficial for formative evaluation. The main purpose of this evaluation is to find out the extent to which the child is following the instructional process. It provides feed back to both the teacher and the student regarding the progress of the student and the efficiency of the teaching methods so that teaching-learning process may be improved. This type of evaluation can be done by means of oral tests, unit tests, informal class tests, assignments and other classroom activities. This evaluation is continuous in nature.

During classroom teaching, you may pose questions to check whether students are able to understand the instructions or not. If required, the re-teaching may be done. To check the level of attainment after each concept/ sub-concept, a small test may be given and the responses may be analysed. This will help in modifying the teaching strategies, if required. Some exercises are given at the end of each unit. You may supplement them by giving some more relevant exercises, keeping in mind the comprehension level of the child. It is important to recapitulate all the concepts imparted on a particular day for stronger retention.

Diagnostic Evaluation

Diagnostic evaluation as the name itself suggests is meant for diagnostic purposes. It enables in finding out the

learning difficulties of a child in a particular subject with reference to conceptual understanding, process of learning, language deficiency, etc. Sometimes formal testing helps in diagnosing the hard spots of learning but sometimes specific tests are prepared with a definite purpose of diagnosing the learning problems. The learning problems in Mathematics may be due to lack of understanding in computing and recognition of symbols where the children generally commits mistake. The teacher is supposed to go deep into the problem through these tests and find out the specific difficulty of the child in learning a concept or a particular step in doing a sum. While carrying out formative evaluation, the diagnostic test supplements the process of evaluation. If diagnosis of hard spots of learning is properly done and suitable remedial measures are taken, the learning attainment as well as learning pace of the weak and low achievers will certainly improve.

Summative Evaluation

Yet another type of evaluation is summative evaluation. It comes at the end of the course or the term. It involves a formal testing of the pupil's achievement and is used for grading, ranking, promoting and certifying the achievement of the students. It does not provide any scope for diagnosis and remediation. Summative evaluations can also take up many forms like quizzes, projects, presentations, etc. rather than only tests or examinations.

Following is an example of an evaluation sheet showing parameters where a teacher who wants to check the entry level of Class VI Mathematics (Chapter 1) of her students.

(1) Read four/five digit numbers and write it in words.

(2) Read four/five digit numbers written in words and write it as numerals.

(3) Identify the greatest and the smallest number from a group of given numbers.

(4) Arrange given numbers in ascending/descending order.

(5) Know the place value of all the digits of a given number.

(6) Write the given number in expanded form.

For every parameter, a teacher may ask 5 to 10 questions depending on the requirement. Outcome of evaluation will help teacher to understand her students and their requirement which further helps teacher to plan her next lesson.

For example, for parameter (1) teacher may ask to read 5009 and write it in words. Students may write 'five thousand nine' or 'five hundred nine' or any other answers.

Now, teacher may evaluate the performance by giving grades for every parameter. It is mandatory for a teacher to avoid giving negative remarks to enhance confidence and generate interest in the subject. Hence, three grade system can be used.

(a) Satisfactory (almost all answers given by student are right).

(b) Needs practice (some of the answers given are right).

(c) Needs improvements (very few or none of the answers are right).

Evaluation sheet marked with grades not only gives an insight to the teacher, but it also helps students and parents to understand the level of learning.

CHARACTERISTICS OF A GOOD EVALUATION PROGRAMME

Following are some of the characteristics of good evaluation programme:

1. Evaluation should be an objective-based process

The objective of evaluation is to measure educational achievement, which is reflected in terms of intended learning outcomes or the instructional objectives. As such all evaluations must be geared to the instructional objectives because these objectives represent the product

of learning. Evaluation must be based on instructional objectives for valid evaluation.

2. Evaluation should be continuous process

Since growth is a continuous process, the teacher must remain cognizant and vigilant of the changes that are taking place in the child's learning from time to time. Frequent or rather continued evaluation is, therefore, essential for getting reliable evidence about pupils' growth and development. Unless evaluation is made part and parcel of teaching-learning process, it cannot help in diagnosing pupils' difficulties and provide opportunities for remedial teaching. Continuous evaluation augments improvement in learning. Therefore, it need not be an end of the activities to be undertaken for completion of the course.

3. Evaluation should be an integral part of instruction

It is obvious that evaluation is part and parcel of instructional process. It should not be an end of the course activity rather, it should be a built-in component of the whole process as it is integral in nature. Therefore, both instruction and evaluation must go hand in hand.

4. Evaluation should be a dynamic process

The dynamic process of evaluation signifies the changes in the level of objectives, instructions, and evaluation procedures. Evaluation is based on objectives of instruction but at the same time it helps us to judge how far those objectives are attainable for a particular group of students. Once the objectives related to a subject specific content for a class during instructional span are attained then new objectives are to be specified for further instruction. Simultaneously, new evaluation techniques have to be designed to meet the requirement of new objectives. This ensures dynamism in evaluation process.

5. Evaluation should be a judgement-making process

At every step of the teaching-learning process, appraisal is essential. Before the instruction, it is necessary to

determine the entry behaviour of student to decide for remedial or developmental teaching. During instruction, it is necessary to use the feedback of the results of instructional efforts and students' learning so as to ensure the success of further instruction.

If the schools keep in mind the above aspects of evaluation, there is no doubt that the evaluation system in our schools can be improved which in turn will help in improving the learning achievement of the students. The schools which carry out improved evaluation practices may really prove to be effective schools. The effective schools need to be strengthened to bring about improvement in the quality of education. It is certain that if evaluation is employed sincerely as a tool for quality improvement, undoubtedly, the excellence in performance of students would be captured systematically. If there are any weaknesses in the learning they can be plugged immediately by providing interventions at the right time to improve the learning deficiency. Thus, evaluation will be touchstone in the educational system.

6. Evaluation should meet the intended accuracy and reliability standards.

There should be no bias and all the students should be treated with equality in outcomes and opportunity to learn. The evaluations should properly assess the syllabus upon which the examination/assessment is based. The evaluation should be a process of looking at what is being assessed to make sure that right areas are being considered.

ANNEXURE 9

You Can also Be a Mathematician

Motivating Material on Four Legendary Women of Mathematics

This motivating material aims to introduce the students of Kasturba Gandhi Balika Vidyalayas to the women of India and the world who dared to enter the unconventional domain of Mathematics and Statistics.

In this module biographies of four legendary women of Mathematics have been presented describing in brief about their contribution, specialisation and various achievements in the field of their work. It is hoped that this material will demystify the fears and inspire the girls to develop aptitude for Mathematics.

The biographies of the following legendary women of Mathematics have been discussed:

Raman Parimala, Rajinder Jeet Hans-Gill, Bhama Srinivasan and Florence Nightingale.

Professor Raman Parimala

Professor Raman Parimala was awarded Shanti Swarup Bhatnagar Prize in 1987 for her landmark contribution in Algebra for the quaternary quadratic modules in mathematical research. Her scientific contribution in the counter example to the quadratic analogue gave rise to a new field of mathematical research. Professor Raman Parmila was born on 21 November 1948 in Mayuram of Thanjavur district, Tamil Nadu. She studied in Madras (now Chennai) at Sarada Vidyalaya for her schooling and at Stella Maris College for her B.Sc. and M.Sc. She carried out her Ph.D work in Bombay University under the supervision of Professor R. Sridharan; and her postdoctoral Research at Ramanjun Institute, Madras. Professor Parimala gained her professional experience mainly at Tata Institute of Fundamental Research, Bombay. Besides, S.S Bhatnagar award, she is the Fellow of Indian Academy of Sciences and Indian National Science Academy (INSA). She has authored many important publications in the area of her work.

Professor Rajinder Jeet Hans-Gill

Professor Hans-Gill has made substantial contributions to the field of geometry of numbers. Her main interest is in problems of packings and coverings, diophantine approximations, inhomogeneous quadratic forms and discrete geometry. In 1967, Professor Hans-Gill gave a simple proof of Rogers' theorem. The method was also used to prove various analogous results for

maximal and minimal sets for packings and coverings in different classes.

Professor Hans-Gill has been the Fellow of Academy of Sciences, National Academy of Sciences and Indian National Science Academy (INSA). She will continue to motivate the young girls in the field of Geometry.

Bhama Srinivasan

Bhama Srinivasan was born in 1935 in Madras (now Chennai), India. She received her B.A. and M.Sc. degrees from the University of Madras and went to England for further studies. She received her Ph.D. in 1960 under the direction of J.A. Green at the University of Manchester. She taught in England at the University of Keele, held a postdoctoral fellowship at the University of British Columbia, and also taught at the Ramanujam Institute of Mathematics, University of Madras. She came to the United States in 1970 and became a U.S. citizen in 1977. She taught at Clark University until 1979, and since then has been a professor at the University of Illinois at Chicago.

Srinivasan served as the President of the Association for Women in Mathematics during 1981-83. She was a member at the Institute for Advanced Study in 1977 and at the Mathematical Sciences Research Institute in 1990. She held visiting professorships at the Universities in France, Germany, Australia and Japan. In January 1979, she presented an AMS invited address at the Joint Mathematics Meetings in Biloxi, Mississippi. She has also served as an editor of the *Proceedings of the AMS* (1983-87), *Communications in Algebra* (1978-84), *Mathematics Surveys and Monographs* (1991-93), and has been a member of the Editorial Boards Committee of the AMS (1991-94).

FLORENCE NIGHTINGALE

Florence Nightingale is most remembered as a pioneer of nursing and a reformer of hospital sanitation methods. However, during the Crimean War, she recorded the incidence of preventable deaths in the military by using new techniques of statistical and mathematical analysis. With this, Florence Nightingale revolutionised the idea that social phenomena could be objectively measured. She was an innovator in the collection, tabulation, interpretation and graphical display of descriptive statistics.

Florence Nightingale's two greatest life achievements were amazing considering that most Victorian women of her age group did not attend universities or pursue professional careers. It was her father, William Nightingale, who believed women, especially his children, should get an education. She in particular received excellent early preparation in Mathematics from her father and aunt, and a tutor by the name James Sylvester.

At Scutari during the Crimean war, she collected data and systematized record-keeping practices. Nightingale was able to use the data as a tool for improving city and military hospitals. When Nightingale's sanitary reform was implemented, the mortality rate declined. Nightingale took her statistical data and represented them graphically. She invented polar-area charts, what we know as pic diagrams. Her mathematical approach saved the British army at Scutari during the Crimean war and led to hospital reforms.

Nightingale demonstrated that statistics provided an organised way of learning which led to improvements in medical and surgical practices. She also developed a Model Hospital Statistical Form for hospitals to collect and generate consistent data and statistics. She became a

Fellow of the Royal Statistical Society in 1858 and an honorary member of the American Statistical Association in 1874. Karl Pearson acknowledged Nightingale as a "prophetess" in the development of applied statistics.

Nightingale was a feminist and also cautioned the women against extremism. She was a true mathematician. She loved reasoning, always questioned assumptions and took great care in the process of reaching at conclusions.

Notes