CUSTOMISED TEACHER'S TRAINING PACKAGE FOR KGBV TEACHERS

MATHEMATICS

TEACHING OF GEOMETRY

BOOK 2

Department of Women's Studies
राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
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Foreword

The National Curriculum Framework–2005 states that a critical function of education for equality is to enable all learners to claim their rights as well as to contribute to society and the polity. We need to recognise that rights and choices in themselves cannot be exercised until central human capabilities are fulfilled. Thus, in order to make it possible for all learners from different socio-economic backgrounds, especially girls, to claim their rights as well as play an active role in shaping collective life, education must empower them to overcome the disadvantages of unequal socialisation and enable them to develop their capabilities of becoming equal citizens.

Reaching out to the girl child has been central to the efforts of Universalising Elementary Education (UEE). The Sarva Shiksha Abhiyan (SSA), a national flagship programme for UEE, recognises the need for special efforts to bring girls, especially from disadvantaged groups, to schools, and to bridge gender disparities in education at the elementary level. In this regard the Ministry of Human Resource Development instituted the Kasturba Gandhi Balika Vidyalaya (KGBV) scheme, an innovative and promising initiative that attempts to address the social, cultural and economic deprivation faced by girls from deprived and disadvantaged sections of rural society. Introduced as a scheme in 2004, it became a part of SSA in 2007. Currently it is operational in twenty-four states and one Union Territory.

A National Consultation on KGBV was organised by NCERT from 11-12 August 2008 to share experiences generated by the KGBV scheme over the last few years. This consultation brought together scholars in the field. The consultation strongly recommended development of Bridge Course for girls entering in KGBV and Customised Teacher Training Package for upgrading the skills of KGBV teachers. Under this backdrop Department of Women's Studies took initiative for developing Bridge Course and Teacher Training Package based on NCF-2005, in collaboration with other Curricular Departments of NIE, RIEs, University Departments, DIETs of Delhi, NGOs and practising school teachers including teachers of KGBV. This material has been developed in the content areas of science, maths, history, geography, social and political life, languages—English and Hindi—and art education and is based on NCERT textbooks at elementary level.

The success of this effort largely depends on the multiple contextual steps that KGBV school principals and teachers would undertake to encourage girls to reflect on their own learning and to pursue imaginative activities and questions. We must recognise that, given space, time and freedom, girls generate new knowledge by engaging with the information passed onto them by adults. Treating the prescribed textbooks as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. Inculcating creativity and initiative is possible if we perceive and treat girls as participants in learning and not as mere receivers of a fixed body of knowledge. The teacher should encourage girls to build on their own acquired and perceived knowledge and link it with their lived realities.

The present training material attempts to upgrade the skills of teachers during their in-service training in their subject areas of science, maths, history, geography, social and political life, languages—English and Hindi—and art education. Different participatory pedagogical methods have been adopted in all the subject areas to encourage activity based teaching and learning. This material developed by the NCERT can be treated as an initial material that can be supplemented later. It is not exhaustive in nature and it can be adopted or adapted according to the contextual needs of KGBV teachers.

The Department of Women's Studies (DWS) could not have gone ahead with this endeavour without the direction and guidance of Professor Krishna Kumar, former Director, NCERT. He had rightly envisioned the importance of the present Teacher Training Package in meeting the academic challenges of teachers of KGBV scheme.

We also gratefully acknowledge contributions of the Review Committee chaired by Dr. Sharda Jain, Director, Sandhan, Jaipur; and other members – Sister Sabina, former State Project Director, Mahila Samakhya Society, Patna, Bihar; Ms. Seema Bhaskaran, State Project Director, Mahila Samakhya Society, Kerala; Ms. Amukta Mahapatra, Director, School Scape, Chennai for their expert reviews and suggestions. We are thankful to the members of Evaluation Team constituted by MHRD – Ms. Sarita Mittal, Director EE8; Ms. Kiran Dogra, Consultant Gender, Ed.CIL; and Ms. Dipta Bhog, Director, Nirantar for their inputs and suggestions.

As an organisation committed to systemic reform and continuous improvement in the quality of its products, NCERT welcomes comments and suggestions which will enable us to undertake further revision and refinement.

Director
National Council of Educational
Research and Training

New Delhi
September, 2011

Preface

The training materials for the Kasturba Gandhi Balika Vidyalaya (KGBV) teachers have been developed keeping in view the principles of the National Curriculum Framework–2005 of the NCERT. These materials developed in different subject areas, viz. English, Hindi, History, Geography, Social and Political Life, Arts Education, Science, and Mathematics are based on the NCERT upper primary textbooks. All these areas will contribute to the upgradation of professional skills of the KGBV teachers. These materials provide ample avenue to the KGBV teachers for their growth in pedagogy, methodology and approach in dealing with their subject areas. There is a considerable scope for exploration and creativity in the classroom. The use of bilingual technique in English will take teachers ahead in their thinking skills. The flexibility in the approach and suggested activities such as taking the help of worksheets, teacher demonstration, anecdotes, reciting poems, crossword puzzles, experimenting, hands on skills, oral traditions and reading material across various subjects are the highlights of the manual.

Each subject area has picked up key concepts across the upper primary textbooks. Each concept has been dealt through a different kind of activity without bringing any definition and the content for rote learning. The concept or the idea has been floated through activities for the learners to catch and analyse. It is hoped that this material will be of use as a resource and also as reference material. The activities are suggestive. Any alternate activity can also be carried out based on the local-specific contexts. Each activity has the scope of creating similar other local-specific activities not making it necessary to stick to the materials given in this package. Its scope will get enhanced if this creates a space for more such activities.

The motivating material on Legendary Women of Science makes the training package of Science even more interesting and gives an edge for making it very gender sensitive. The women of India and the world, who have achieved heights in this area, will always encourage girls of the KGBVs to even explore these areas which have a masculine image. Kalpana Chawla will motivate girls to reach the heights of space while Florence Nightingale, The Lady of Lamp, will encourage them to think in numbers. Marie Curie, the only woman to receive the Nobel Prize twice, will make them feel proud for being the women themselves. The KGBV teachers thus oriented to take up such challenges will certainly become guides and agents of social change.

In mathematics there has been a conscious effort of demystifying the masculine image of mathematics. The processes underlying everyday mathematics done by women both within the home and outside have been highlighted. In several concepts there is emphasis on the use of mathematic kit by teachers to make the learning of mathematics more concrete and useful for girls.

Women have always been the backbone of several historical and contemporary movements. Recognising their important role in social reform and national movement the teacher training package in history highlights the contributions made by women like Rani Gaidinliu, Pandita Ramabai, Sarojini Naidu, Aruna Asif Ali and many more. Their trials and successes will continue to inspire girls to meet multifaceted challenges in life.

Similarly the motivating material on legendary women like Rani Jhansi and Ila Sachani has been included in languages. The training package for social and political life attempts to make teachers sensitive towards unconventional roles and responsibilities. Examples like Laxmi Lakra and Fatima Bi will touch hearts of common people. Geography equips teachers in spatial phenomenon. It instills human values and appreciations for regional inter dependence and resource conservation. Arts and aesthetics attempt to inspire diversity in expression of art. It is through performing art that all girls through various art forms can become communicative, creative and expressive.

Overall, keeping these variations in mind, the pedagogical approaches needed in the KGBVs will be multilevel and diverse for meeting the needs of KGBVs in different socio-cultural contexts.

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Contents

Block

Teaching of Geometry

Geometry is recognised as not only one of the most important components of the school mathematics curriculum but also, alongside algebra, as one of the most important elements of mathematics itself. The reasons for including geometry in the school mathematics curriculum are providing opportunities for learners not only to develop spatial awareness, geometrical intuition and the ability to visualise, but also to develop knowledge and understanding of, and the ability to use, geometrical properties and theorems.

The primary purpose of teaching geometry must surely be to develop the children's spatial intuition.

Spatial Awareness

Spatial awareness is simply an organised awareness of the objects in the space around us, and also an awareness of our body's position in space. Without this awareness, we would not be able to pick food up from our plates and put it in our mouths. We would have trouble in reading, because we could not see the letters in their correct relation to each other and to the page. Athletes would not have the precise awareness of the position of other players on the field and the movement of the ball, which is necessary to play effectively.

Spatial awareness requires that we have a model of the three-dimensional space around us, and it requires that we can integrate information from all of our senses. Spatial awareness should come naturally to most children, though there is much that parents and teachers can do to promote spatial awareness.

This can be done by talking about the concepts such as direction, distance and location. For example, a child with spatial awareness will understand that as he/she walks towards a football, the football is coming closer to his/her own body.

Studies have suggested a link between a well-developed sense of spatial awareness and artistic creativity, as well as success in mathematics. It is also important in the development of abstract thought. The ability to organise and classify abstract mental concepts is related to the ability to organise and classify objects in space. Visual thinkers, in particular, will tend to use their visual imagination to organise abstract thought.

Spatial intuition or spatial perception is an enormously powerful tool and that is why geometry is actually such a powerful part of mathematics—not only for things that are obviously geometrical, but even for things that are not. We try to put them into geometrical form because that enables us to use our intuition. Our intuition is our most powerful tool.

—Sir Michael Atiyah

Geometry comprises that branch of mathematics which encourages visual intuition (the most dominant of our sense) to remember theorems, understand proof, perceive reality and give global insight that force a learner to identify the beauty of geometry in surroundings. These are transferable skills that are needed for (but not taught by) all other branches of mathematics. The aims of teaching of geometry content can be summarised as follows:

(a) To develop spatial awareness, geometric intuition and ability to visualise;

(b) To provide an opportunity of geometrical experiences in two and three dimensions;

(c) To develop knowledge and understanding of and the ability to use geometrical properties and theorems;

(d) To encourage the development and use of conjecture, deductive reasoning and proof;

(e) To develop skills of applying geometry through problem solving in real world context; and

(f) To encourage and inculcate a positive attitude towards mathematics.

During the upper primary stage, students are making the transition from inductive methods of reasoning (conclusions based on several past observations) to a more formal method of deductive reasoning (proving statements from accepted postulates, definitions, theorems and given information). However, students at this age may be functioning at many different levels of reasoning ability. Therefore, before beginning instructions, it is important to assess students' reasoning level. This allows teachers to differentiate instructions based on student's readiness.

There are five developmental levels of geometric reasoning based on a study by Dina van Hiele-Geldof and her husband, Pierre Marie van Hiele. They are:

- **Level 0 (Basic Level): Visualisation**

  At this level students view objects as entire entities, not noticing individual components or properties. The focus is on the whole object, not its parts.

- **Level 1: Analysis**

  Students begin to recognise that geometric shapes have parts and special properties. However, neither are they able to describe how properties of different shapes are related, nor are they able to understand definitions.

- **Level 2: Informal Deduction**

  At this level students comprehend the connection between properties within geometric figures and between one set of figures to another. Students are able to follow proofs, but are not able to construct one themselves.

- **Level 3: Deduction**

  At this level students can construct a geometric proof and understand the connection between postulates, theorems, and undefined terms.

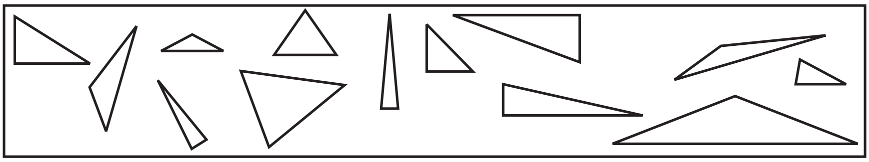
- **Level 4: Rigour**

  At this level students see geometry as abstract. Students can move between different geometric systems and can compare and differentiate them.

Students at upper primary stage are usually moving from Level 2 towards Level 3. Students taught at their instructional level master skills necessary for them to progress in geometric reasoning. If a student is at one developmental level and the teacher instructs concepts at a different developmental level, it is very likely that the student will not grasp and retain the information.

For teachers in order to identify the developmental level of geometric reasoning of each of their students, assessment is required. The following is an example of an activity which serves as a tool to determine each student's geometric reasoning level by creating a handout that consists of at least 20 triangles of varying sizes and classifications.

Let the students sort the triangles into as many sets as possible. Then ask students to write a paragraph describing



why they placed each triangle into a particular set. Using this information, the teacher should be able to determine students' developmental level based on the following divisions:

- Level 0: Students divide the triangles into sets based on size (i.e. small, medium, and large).
- Level 1: Students divide the triangles according to one characteristic, most likely focusing on either the length of the sides or the size of the angles.
- Level 2: Students observe more than one characteristic of the triangles. For example, they will see that there are isosceles right triangles and scalene right triangles, or that an isosceles triangle can be right, acute, or obtuse.
- Level 3: Students use definitions, postulates, or theorems to make connections and give reasons for the connections.
- Level 4: Students grasp abstract concepts and apply them through more than one geometric system.

This block contains the following seven units:

Unit 1: Basic Geometrical Ideas

Unit 2: Triangles and their Properties

Unit 3: Congruence of Triangles

Unit 4: Understanding Quadrilaterals

Unit 5: Symmetry

Unit 6: Visualising Solid Shapes

Unit 7: Practical Geometry

Unit 1 is devoted to the development of basic geometrical ideas such as point, line, line segment, ray, different types of curve (simple, open, closed, simple closed, etc.), angles and their classifications. Different pairs of angles (adjacent, linear pair, vertically opposite angles, etc.) have also been dealt along with the pairs of angles formed by the intersection of two lines with a transversal. The properties of these pairs of angles relating to two parallel lines have also been dealt with. Circle as a special type of simple closed figure along with its parts has also been discussed.

Units 2 and 3 are devoted to 'triangles'. In Unit 2, different types of triangles have been discussed along with their properties related to their sides and angles. Pythagoras Theorem has also been discussed in this unit.

Unit 3 deals with different criteria of congruency of two triangles along with their applications.

Unit 4 deals with quadrilaterals and special types of quadrilaterals along with their properties.

Unit 5 discusses symmetrical objects or figures. It deals with figures having line (reflection) symmetry and rotation symmetry.

Unit 6 discusses various ways of visualising solid shapes (3D shapes) in two-dimensional (2D) shapes.

Unit 7 deals with the constructions of different geometric figures. It starts with some basic constructions and goes up to the construction of quadrilaterals for a given data.

UNIT 1

Basic Geometrical Ideas

Structure

- Introduction
- Main Concepts and Sub-concepts
- Objectives
- Teaching Points
  1. Point, line segment, line and ray
  2. Angles and their measurement
  3. Classification of angles
  4. Different types of curves
  5. Circle and its parts
  6. Pairs of angles
  7. Angles formed by a transversal with two lines
- Common Errors
- Exercise

Introduction

The word 'geometry' has been derived from the two Greek words—'geo' meaning the earth and 'metron' meaning to measure. Thus, the study of geometry began with the idea of measurement of earth (i.e. land). Later on, this study led to the study of some geometric figures made up of points, lines, line segments, rays and so on. Teacher may help the students to realise the importance of geometry by exposing them to various ancient monuments as well as some modern structures. Teacher may ask students to recognise and differentiate different shapes in their surroundings. They may be asked to draw different shapes which they usually see in their surroundings like the shape of blackboard or the shape of fan, regulator, etc.

Through interaction, teacher may point out the basic geometrical figures with the help of these different shapes drawn by students.

Further, teacher may also point out that for a better study of geometry, understanding of some elementary geometric figures is essential. In this unit, some elementary geometric figures shall be discussed along with their properties, starting with the basic concepts such as point, line, line segment, angle and so on.

Strategies to Teach Plane Figures

The following strategies may be used effectively to teach plane figures to upper primary students. Research has shown that when teachers incorporate these four strategies in their instruction, retention is increased. These strategies include:

- Using Mathematics Kit (supplied with the package).
- Cooperative learning.
- Similarities and differences by using diagrams.
- Vocabulary enhancement by group activity.

(a) Using Mathematics Kit

When parallelograms are first introduced to the class, it is helpful for the students to have a few teaching aids to explore, e.g. Mathematics kit, sheet of paper, etc. Geo board may help students to discover the properties of parallelograms.

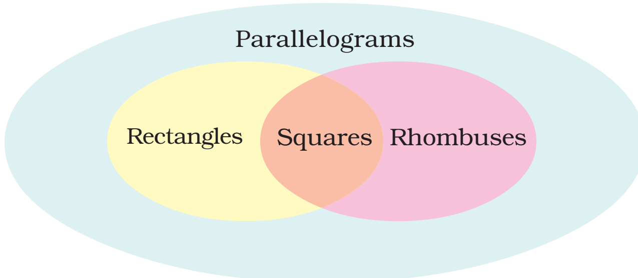
(b) Cooperative learning

Once students have been introduced to parallelograms and their basic properties, this method may be used to further explore special types of quadrilaterals. Teachers can follow these steps:

1. Divide the class into groups of four. Within each group assign different students to be a rectangle, a square, a rhombus, or a trapezium.
2. The students of a particular name, say 'Rectangle', from each group will leave their respective group and join each other to form a new group. This new group will stand in a corner of the classroom. Similarly the students with name, say 'Square', will stand in another corner and so on.
3. Provide each new group with a guided activity that will allow its members to explore the shape to which they belong and learn its properties. The group members must come to a consensus on their properties and feel confident that they can teach these properties to their own groups.
4. The groups for each figure should prepare examples, diagrams, properties, and three quiz questions to share with the members of their own groups.
5. After the allotted time, students return to their original groups to share their knowledge with their respective group members.

(c) Using Diagrams

As students further study the properties of different types of parallelograms, they need to learn to compare and contrast the properties of these shapes. Use of diagrams is an excellent method for displaying the shared as well as unique properties of each type of parallelogram.



Using Diagram for Comparing Parallelograms

(d) Vocabulary Enhancement

Finally, to reinforce new vocabulary explored in the unit, students can participate in a group game that focuses on the properties of each quadrilateral. The teacher can do the following:

1. Divide the class into groups of four students.
2. Groups will use Mathematics Kit.
3. Provide each student with a card that contains the description of one of the quadrilaterals studied.
4. Each student must use the items of kit to construct the named quadrilateral on her card.
5. Help the students justify the construction of the figures.

Main Concepts and Sub-concepts

Point, Line segment, Line, Ray, Intersecting and Parallel lines, Angle, Acute angle, Right angle, Obtuse angle, Straight angle, Reflex angle, Complete angle, Zero angle, Perpendicular lines.

Different types of curves—simple, open, closed and simple closed polygons as special case of simple closed curves.

Adjacent angles, complementary and supplementary angles, linear pair, vertically opposite angles, circle, its centre, radius, diameter, chord, arc, sector and segment.

Angles formed by a transversal with two lines–corresponding angles, alternate interior angles and interior angles on the same side of the transversal.

Objectives

After learning these concepts/sub-concepts, the student can

- give examples of points, line segments, rays and lines from her environment;
- name point, line segment, ray and line using letters and symbols;
- identify different types of curves from a given collection;
- understand the meaning of a circle, its centre, radius and diameter;
- identify different terms in a circle such as chord, arc, sector and segment;
- give examples of angles from her environment;
- measure given angles using a protractor;
- classify given angles as acute, right, obtuse, straight, reflex, complete and zero;
- identify intersecting, parallel and perpendicular lines;
- identify the following pairs of angles:
  - (i) adjacent angles
  - (ii) vertically opposite angles
  - (iii) complementary angles
  - (iv) supplementary angles
  - (v) linear pair;
- verify and prove the property: vertically opposite angles are equal;
- identify various pairs of angles formed by a transversal with two lines such as
  - (i) corresponding angles
  - (ii) alternate interior/exterior angles
  - (iii) interior angles on the same side of the transversal;
- verify the following properties and the converse of these:

If two parallel lines are intersected by a transversal then

(i) corresponding angles are equal

(ii) alternate interior angles are equal

(iii) interior angles on the same side of the transversal are supplementary;

- apply all these concepts/properties in solving problems.

Teaching Points

1. Point, Line Segment, Line and Ray

Point, line segment, line and ray may be explained through concrete objects from the student's environment (Refer to pages 69 to 73 of Class VI, Mathematics Textbook, NCERT). Students may be encouraged to give some more examples of their own. It may be pointed out to the students that howsoever finer point, we mark on the notebook with a sharpened pencil. In reality it will not be a point, it just gives an idea of a point. In fact point is a relative term. The twinkling stars at night seem to be points but in reality they are much bigger than our earth. If we see persons roaming on a road from a multistorey building, all of them look as small as a point. So, an object can be considered as a point depending on the frame of reference we are talking about.

In our context, a point has no length, breadth or thickness. It just indicates a location. Same is the case with a line segment. A line segment has length but no breadth or thickness.

Students should be told about all possible ways of denoting the following as:

Ray	Line segment	Line
O A $\overrightarrow{OA} \neq \overrightarrow{AO}$ It should be told that $\overrightarrow{OA}$ does not have a fixed length, rather A is a point on its path	A B $\overline{AB} = \overline{BA}$ $\overline{AB}$ or $\overline{BA}$ has a fixed length	A B Again, A and B are points on path of the line. $\overleftrightarrow{AB}$. It has no fixed length or line m

A line can be imagined to be a line segment which is extended indefinitely in both the directions. Thus, a line has no end points, while a line segment has two end points. A ray extends indefinitely in only one direction and has only one end point. It is usually called the **end point** or **the initial point** or **the starting point of the ray.**

Collinear and Non-collinear points

Students may be asked to mark the points A, B, C, and D on their notebooks as given in Fig. 1.1 They may be asked to answer the following questions through paper folding.

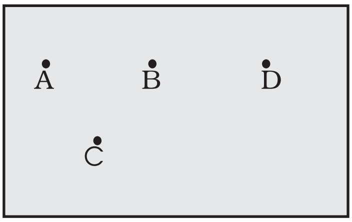


Fig. 1.1

(i) How many lines can be drawn to pass through the point A or B or C or D?

(ii) How many lines can be drawn passing through (a) A and B, (b) B and C, (c) C and D, (d) A and D and (e) A and C?

(iii) How many lines pass through A, B and C?

(iv) How many lines pass through A, B and D?

(v) How many lines pass through A, B, C and D?

Through such activities, the following facts may be explained to the students:

(i) Through a given point, infinitely many lines can be drawn.

(ii) Through two given points, one and only one line can be drawn. Therefore, a line can be denoted by taking any two points; say A and B on it.

(iii) Points A, B and D lie on a line and therefore points A, B and D are called **collinear points**. Here, points A, B and C do not lie on a line and are therefore called **non-collinear points**. It may be noted that two points will always be collinear and hence collinearity of points can be seen only with reference to three or more than three points.

Intersecting and Parallel lines

This can be explained through an activity. Some students may be asked to draw several lines on the blackboard as shown in the adjoining figure. Now, students may be asked to note the number of points of intersection of any two lines. Through the answers of the students, it may be observed that the two lines either intersect or do not intersect. For example, lines *l* and *m* intersect at the point A, lines *m* and *n* intersect at the point B, lines *n* and *r* will intersect if extended further, lines *l* and *n* do not intersect each other even after extension, lines *p* and *n* intersect at D, lines *q* and *n* intersect at B, lines *p* and *q* do not intersect each other and so on. Through these observations, it may now be explained that:

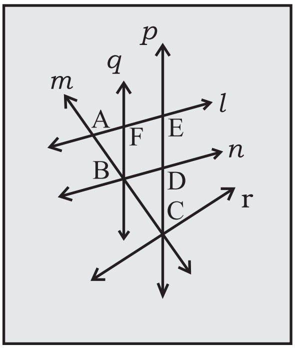


Fig. 1.2

Two lines that do not intersect (even after extension) are called **parallel lines** and the two lines that intersect are called **intersecting lines**. The symbol '||' is used to indicate parallel lines. Here, the students may be asked to observe that if the two lines intersect, they intersect at a unique point (single point). Teacher may be asked to observe the points through which more than two lines are passing, for example, lines *m*, *n* and *q* pass through a single point B. As a special case, the teacher may point out that such lines are called **concurrent lines** and point of intersection is called their **point of concurrence**. Teacher may give example of a wheel or Ashok Chakra.

Here, it may also be emphasised that all of our discussions at this stage are in a plane only. Teacher may also explain these concepts through paper folding.

2. Angles and their Measurement

These ideas have been discussed in Chapters 4 and 5 of Class VI, *Mathematics*, NCERT. Students are already familiar with angles in primary classes or in Bridge Course. It may be recalled by giving the example that angles are made when corners are formed and then formally it may be stated that:

An angle is a figure formed by two rays with a common initial point.

The initial point is called its **vertex** and the two rays are called its **arms**.

For example, in Figure 1.3, Q is the vertex of the angle, QP and QR are its two arms. It is named as $\angle PQR$ or $\angle RQP$. Note that while naming the angle, its vertex is always written in the middle. Sometimes, when there is no ambiguity, an angle can be denoted by a single letter also. For example, in Fig. 1.3, we can denote the angle PQR as $\angle Q$ but in Fig. 1.4, we cannot denote the angle at the point Q as $\angle Q$, because there are more than one angles at Q as Q is a common vertex of three different angles, i.e. $\angle PQO$, $\angle OQR$ and $\angle PQR$.

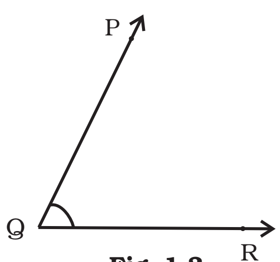


Fig. 1.3

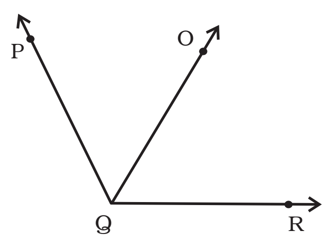


Fig. 1.4

In the above definition of an angle, the teacher may point out that some idea of the rotation of a ray is also involved. Enough practice should be provided to the students for measuring different types of angles by using a protractor.

It may be pointed out that there are two scales— 0°–180° from the right to left and 0° – 180° from left to right along the circular edge of the protractor (see Figure 1.5). When the angle is drawn in such a way that its vertex is on left of the initial arm, then this angle should be measured by the scale 0° – 180° from right to left and when the vertex is on the right of initial arm, then the angle should be measured by the scale 0° to 180° from left to right. The teacher may help the students to understand that angle given in Fig. 1.6 is measured by the scale 0^o to 180^o from right to left and angle given in Fig. 1.7 is measured by the scale 0^o to 180^o from left to right. Teacher must point out that the centre and base line of the protractor must coincide with the vertex and one arm of the angle respectively and the scale with 0^o at that arm must be taken into account while measuring the angle.

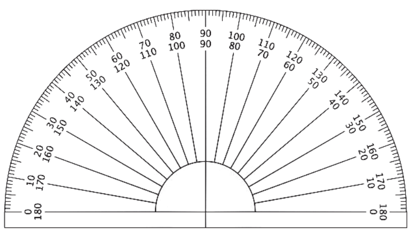


Fig. 1.5

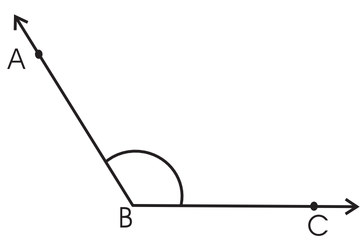


Fig. 1.6

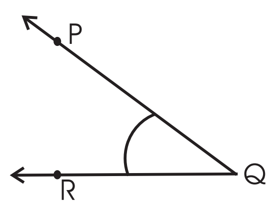


Fig. 1.7

Angle formed by two line segments

During the discussion on angles, it may also be explained that generally we come across angles between two line segments. In such a case, this type of angles forms an outline of the angle formed by the two rays (Fig. 1.8(i)).

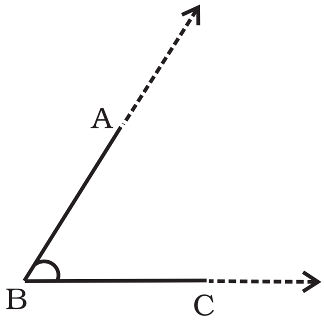


Fig. 1.8 (i)

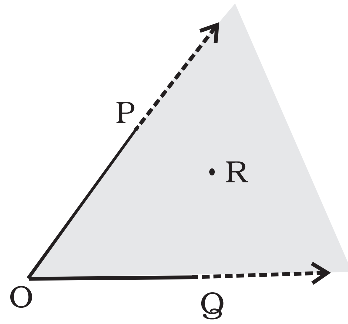


Fig. 1.8 (ii)

Strictly speaking, we say that an angle ABC is determined by the rays BA and BC.

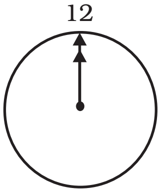
Students may be given clarity about interior of the angle. For example in Fig. 1.8 (ii), if they interpret that point R is in the exterior of ∠POQ, then they are incorrect as OP and OQ are rays and so their interior region includes point R as shown by shaded portion.

3. Classification of Angles

Students are already familiar from primary classes with the terms like 'a right angle', 'more than a right angle' and 'less than a right angle'. The teacher may reinforce these ideas through activities of "turning about" in some direction as suggested on pages 89 and 90, Chapter 5, Class VI, Mathematics Textbook, NCERT. In the beginning, idea of a complete angle, (one complete turn), a straight angle (a half turn) and a right angle ($\frac{1}{4}$ of a turn) may be discussed. It may also be explained that if there is no turn or revolution, then it is said that the angle formed is a zero angle.

Teacher may use a model of a clock as a teaching aid

(1) Teacher may demonstrate that when time is 12 O' clock both hands are at the same position, i.e. they have not moved, so the



angle between the two hands is 0° and is a zero angle.

(2) At 1 O'clock or 2 O'clock, angle between the hands is an acute angle.

(3) At 3 O'clock, angle between the hands is 90° and is a right angle. Thus from above we see that an acute angle is more than 0° but less than 90°.

(4) At 4 O' clock or 5 O' clock, angle between the hands is an obtuse angle.

(5) At 6 O'clock the hands of the clock are in a straight line and they form a straight angle with measure 180°.
Thus, we see that obtuse angle is more than 90° or a right angle but less than a straight angle.

(6) At 7 O'clock, 8 O'clock, 9 O' clock, 10 or 11 O'clock, angle between the hands is a reflex angle.

(7) After one full round the hands overlap with each other and are back at 12 O'clock the angle formed is a complete angle. Measure of a complete angle is 360°.

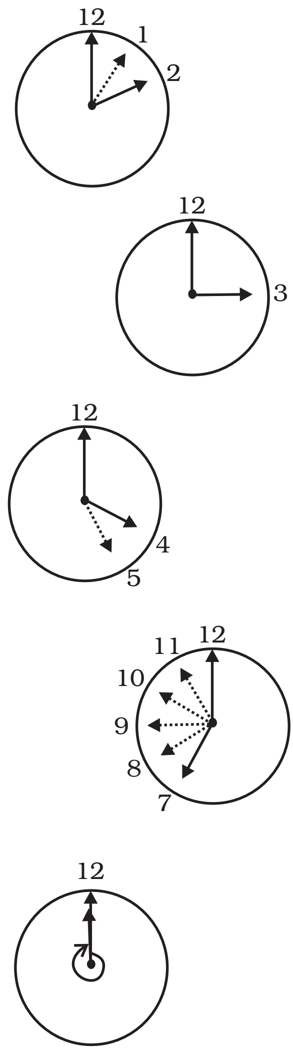


Fig. 1.8 (iii)

Thus, we see that measure of a reflex angle is more than a straight angle, i.e. 180° and less than a complete angle, i.e. 360°.

Thus, it can be explored through activities that

(i) two right angles = a straight angle (180°)

(ii) four right angles = a complete angle (360°)

Again, teacher may give variety of angles on a worksheet to students. Students will divide them into three groups, i.e. angles which are less than right angle, which are right angles, and which are greater than right angle.

Further teacher may ask them to measure each angle and record their observations.

Students are expected to come out with the observations that the angles which are less than right angle measure between 0° and 90°. Similarly students will discover that angles which are right angles are of 90° and angles greater than right angles measure between 90° and 180°.

This activity may be extended further by giving straight angles, reflex angles and complete angles.

At this stage, students may be asked to attempt questions in Exercise 5.2 of Class VI, Mathematics, NCERT. After this, the teacher may take angles from (a) zero angle to right angle (b) right angle to straight angle and (c) straight angle to complete angle and then let students classify the angles in the following categories:

(i) Angle of 0° $\longrightarrow$ zero angle

(ii) Angle between 0° and 90° $\longrightarrow$ acute angle

(iii) Angle of 90° $\longrightarrow$ right angle

(iv) Angle between 90° and 180° $\longrightarrow$ obtuse angle

(v) Angle of 180° $\longrightarrow$ straight angle

(vi) Angle between 180° and 360° $\longrightarrow$ reflex angle

(vii) Angle of 360° $\longrightarrow$ complete angle

Difference between zero and complete angle should be made clear to the students. In zero angle, terminal and final ray overlap each other and there is no movement of any of the arms while in complete angle, the terminal ray overlap on initial ray after completing a circle.

Zero angle

Complete angle

At this stage, students may be asked to attempt questions in Exercises 5.3 and 5.4 of Class VI, Mathematics, NCERT. It may be emphasised that angle of 1^o is an acute angle but angle of 0^o is not an acute angle. Similarly, angle of 179^o is an obtuse angle but angle of 180^o is not an obtuse angle. Further, an angle of 181^o is a reflex angle but an angle of 360^o is not a reflex angle.

While forming an angle by rotation of two rays OA and OB (Fig. 1.8 (iv)), it may be pointed out that at the point O, two angles have been formed. Generally, one angle is larger than the other. Both of them can be named as ∠AOB. To distinguish them, you may write 1 as ∠AOB and 2 as Reflex ∠AOB (Reflex angle is larger than its counter part).

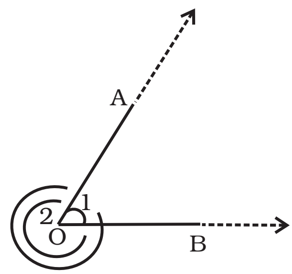


Fig. 1.8 (iv)

Using these ideas, while solving Question 6 of Exercise 5.4 (Class VI, Mathematics, NCERT), students may be acquainted with the fact that here they have to find the smaller of the two angles formed by the hands of a clock. For hands-on experience of this concept, students may be encouraged to perform Activity 4, 'Measurement of Angles' of Mathematics kit.

Perpendicular Lines

After discussion of a right angle, students may be introduced to perpendicular lines as the lines intersecting at right angles to each other (Fig. 1.9). It may also be explained that symbol for perpendicularity is '⊥'. So, in Fig. 1.9, $l \perp m$.

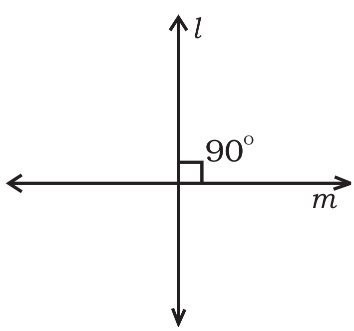
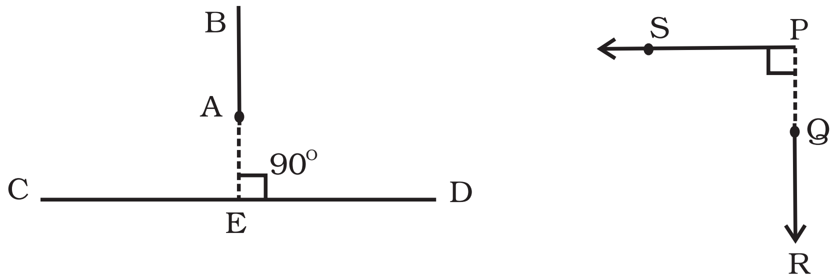


Fig. 1.9

The teacher may also consider some figures such as two line segments AB and CD [Fig. 1.10 (i)], and so on. In (i), line segment AB is produced to meet CD at point E making $\angle E = 90^o$; in (ii), rays PS and QR meet at P on production and ∠P = 90°. Through such examples, the teacher may bring home the idea before the students that two line segments (or two rays) are said to be perpendicular, if the two lines formed by these are perpendicular to each other. The same is the



(i) (ii)

Fig. 1.10

case with one line segment and one ray. **Thus, for the perpendicularity of two line segments or two rays or one line segment and one ray, they need not be intersecting**.

Students may be asked to identify objects whose edges are perpendicular to each other, e.g. adjacent edges of a textbook are perpendicular to each other.

Teacher may further ask to identify such edges which are not perpendicular.

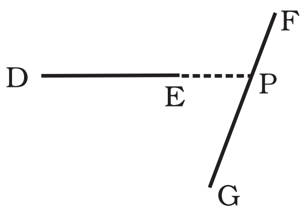


Fig. 1.11

A caution: Consider line segments DE and FG of Fig. 1.11. These two line segments do not intersect each other. However, on production, they intersect at P. But in such a case, we never say that line segment DE and FG are intersecting. However, the lines formed by them are intersecting.

4. Different Types of Curves

These ideas have been discussed in Class VI (Chapter 4) as well as in Class VIII (Chapter 3). The discussion may be started, through some examples:

(i) **Open and closed curves:** If we put our pencil on any point of given curve and start moving the pencil along the curve without lifting the pencil and without retracing the path and arrive at the starting point, then the curve is called a closed curve, otherwise it is called an open curve. Thus, in Fig. 1.12, curve (i) is a closed curve and curve (ii) is an open curve.

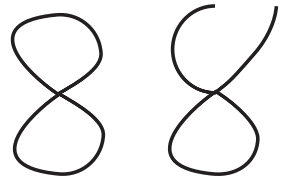

(i) (ii)

Fig. 1.12

(ii) **Curves that are simple and curves that are not simple:** Curves that do not cross themselves are called simple curves, otherwise they are not simple curves. In Fig. 1.13 (i) is a simple curve, while (ii) is not a simple curve.

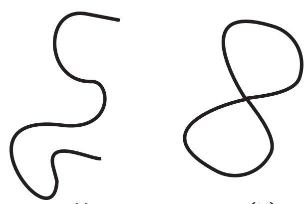
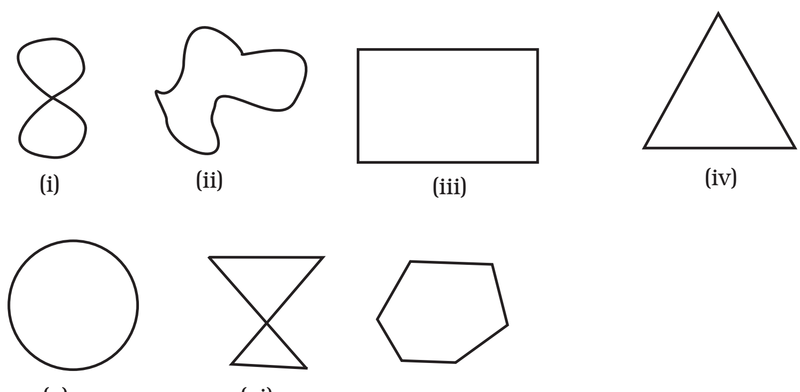

(i) (ii)

Fig. 1.13

(iii) **Simple closed curves:** For this again the teacher may ask the students to consider the following curves given in Fig. 1.14:


(i) (ii) (iii) (iv)

(v) (vi) (vii)

Fig. 1.14

It may be pointed out that all the above curves are closed curves. Further, none of the curves (ii), (iii), (iv), (v) and (vii) cross itself anywhere. So, each of the curves (ii), (iii), (iv), (v) and (vii) is a simple closed curve.

Students may be encouraged to observe further that out of these simple closed curves (iii), (iv), and (vii) are made up of line segments only. Such simple closed curves are called **polygons**. Thus, a polygon is a simple closed curve made up of line segments only.

Here, it may be pointed out that curve (vi) is also a closed curve made up of line segments only. But clearly, it is not simple. Therefore, it cannot be categorised as a polygon. After this, the teacher may give the idea of sides and diagonals of a polygon as given in Chapter 4, Class VI, Mathematics Textbook, NCERT.

It is nice if students are asked to draw different types of curves on the blackboard and other students can judge drawings and rectify or modify if they find that these drawings are not correct.

5. Circle and its Parts

Students have already learnt something about a circle, its centre and radius in primary classes. They have drawn a number of circles using bangles, bottle caps as well as compasses. This practice may be continued in Class VI also. By drawing different circles, the following may be emphasised.

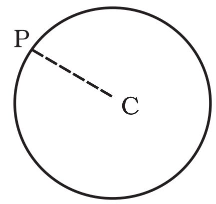


Fig. 1.15

A circle is a path of a point moving in a plane at the same (fixed) distance from a fixed point of that. The fixed point is called the **centre** and the same (fixed) distance is called the **radius** of the circle. In Fig. 1.15, **C is the centre** and **CP is the radius of the circle.**

It may also be pointed out to the students that every point P lying on the circle has the same distance CP from the centre C and every point not lying on the circle is not at a distance of CP from the centre C. A teacher may show a circular disc or circular cut-out of the sheet of paper. She can point out that the edge of disc or sheet represents a circle. Teacher can also locate the centre by folding this paper cut-out of the circle. It can easily be verified using a straw that any point on the edge of disc/sheet is equidistant from the centre, whose length is same as the radius of the disc/sheet.

Students can easily observe that any point inside the disc/sheet is at a lesser distance than the radius of the circle. Similarly any point outside the disc/sheet is at a greater distance than the radius of the circle.

Here, it may also be explained that the term radius of a circle is used in two senses—(i) as a line segment and (ii) as a length.

It may also be pointed out that as a line segment, there are infinitely many radii of a circle and as a length, a circle has a unique radius.

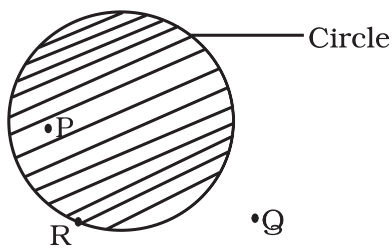


Fig. 1.16

It may also be explained that a circle is a simple closed curve and every simple closed curve divides the plane into three parts namely (i) its interior, (ii) its exterior, and (iii) the curve itself. For example, in Fig. 1.16, point P is in the interior of the circle, point Q is in the exterior of the circle and point R is on the circle itself. The circle along with its interior is called the circular region corresponding to that circle.

Chord and Diameter of a Circle

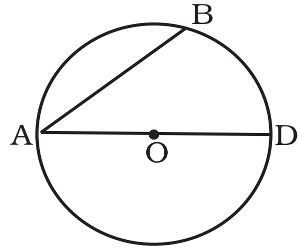


Fig. 1.17 (i)

Teacher may ask the students to take any two points, say A and B on a circle with centre O and ask them to join these two points (Fig. 1.17(i)). The line segment AB so obtained is called a **chord** of the circle. Teacher should make the students familiar with line segments which are not chords, e.g.

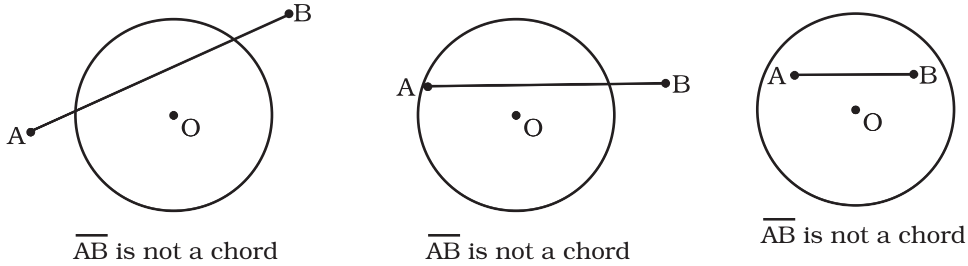


Fig. 1.17 (ii)

It may be pointed out that line segment AD in Fig. 1.17(i) is also a chord of the circle. But it passes through the centre O also. So, it has a special name called **diameter** of the circle. In fact,

"A diameter is the longest chord of the circle."

(Teacher may verify this statement with a ruler. She can fix one end of the ruler on a point of the circle and the ruler can be moved along the boundary on both sides of the diameter.)

Here again like radius, diameter is also used in two senses, namely (i) as a line segment, and (ii) as a fixed distance. In the sense of a line segment, there are infinitely many diameters of a circle and in the sense of fixed distance a circle has a unique diameter.

Teacher should ensure that students are able to visualise that any two diameters necessarily intersect at the centre. Further to ensure this she can give each student a paper cut of circle and ask them to find the centre of any such unknown circle.

They can do paper folding once vertically to get vertical diameter and then horizontally. The point of intersection of vertical and horizontal diameter is the centre.

With the help of figures, students may be encouraged to visualise that Diameter = 2 Radius. Here, it may also be stated that the length or perimeter of a circle is called its **circumference.**

Arcs, Sectors and Segments of a Circle

As before, let the students take any two points say A and B on a circle. The students may observe that the circle has been divided into two parts by these points A and B. Each part is called an arc of a circle. To distinguish between the two arcs, mark points P and Q as shown in Fig. 1.18(i). So, we shall have two arcs, namely arc $\overparen{APB}$ and arc $\overparen{AQB}$. They are also denoted as APB and AQB, respectively. In general, one arc is smaller than the other. The smaller arc is called the **minor arc** and the bigger arc is called the **major arc**. In case, the two arcs so obtained are equal then each is called a **semicircle.**

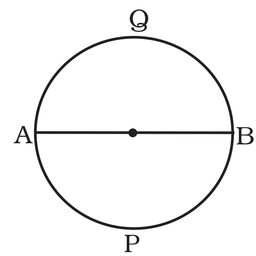


Fig. 1.18 (i)

Teacher may take a circular cut-out from a sheet of paper. A crease made by folding it such that one part overlaps the other will represent a diameter of the circle.

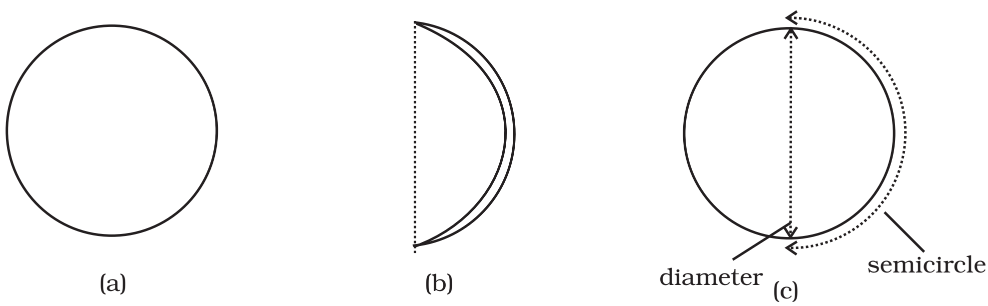
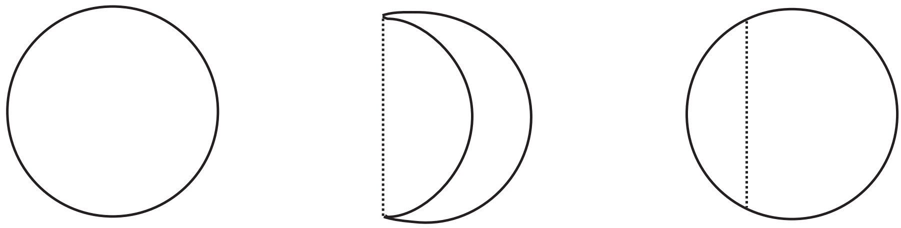


Fig. 1.18(ii)

This activity also gives an idea that diameter divides a circle into two equal halves. Teacher may point out that edges of two halves of this cut-out are of the same length as they completely overlap each other and are known as semicircles.

Similarly, a crease made by folding it (no need to overlap one part on other) represents a chord.



(a) (b) (c)

Fig. 1.18 (iii)

This gives an idea that a chord divides a circle into two parts which may or may not be equal. Teacher may point out that edges of both parts are known as arcs. The smaller arc is the minor arc and larger one is major arc.

Teacher may take few more circular cut-outs and by folding she can make chords of different lengths. By comparing the length of chords (crease of each folded cut-outs) it can be shown that diameter is the longest chord.

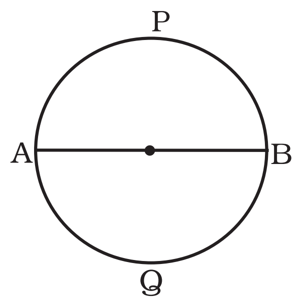


Fig. 1.19

It may be clarified at this stage that in a semicircle, diameter is not included.

Thus, in Fig 19, $\overset{\frown}{APB}$ and $\overset{\frown}{AQB}$ are semicircles but diameter AB is not included in any of these semicircles.

The teacher may now ask the students to observe Fig. 1.20 and Fig. 1.21, where O is the centre of each circle. In Fig. 1.20, shaded portion is enclosed between the two radii OA, OB and arc APB of the circle. It is called a **sector** of the circle. Thus, the part of a circular region enclosed between two radii of a circle is called a **sector** of the circle. Students may be asked to observe that in fact there are two sectors, one is greater than the other. The shaded region in Fig. 1.20 is called the **minor sector**, while the unshaded region is called the **major sector**. It may also be pointed out that ∠AOB is called the **central angle** or simply the angle of the minor sector AOB. Similarly, the corresponding reflex ∠AOB is the angle of the major sector AOB.

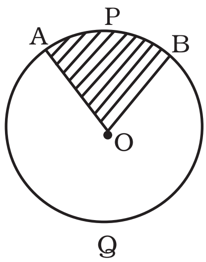


Fig. 1.20

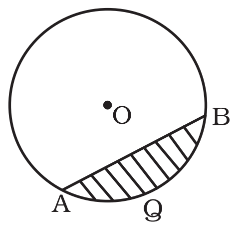


Fig. 1.21

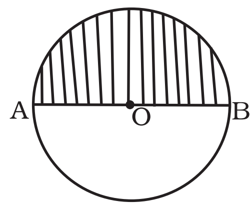


Fig. 1.22

Students may also be asked to observe that if in Fig. 1.22, ∠AOB = 180^{o}, then the two sectors will be equal. Each will be called a **semicircular region** (Fig. 1.22). While observing Fig 1.21, the teacher may point out that here chord AB of the circle has divided the circular region into two parts. Each part is called a **segment** of the circle. The shaded region is called the **minor segment** and the unshaded region is called the **major segment.** Here, again the teacher may encourage the students to look at the situation when chord AB becomes the diameter of the circle. In such a case, the two segments of a circle will be equal (Fig. 1.23). Here, each of the segment is called a semicircular region.

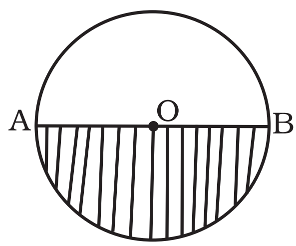


Fig. 1.23

Through such an activity, students may be encouraged to visualise that as a special case, a semicircular region can be considered as a sector with central angle 180^0 as well as a segment divided by the chord AB (diameter) of the circle. They may also be asked to note that in a semicircular region, diameter is also included, while in a semicircle, it is not included.

The teacher may also make it clear to the students that unless stated otherwise, by an arc (or a sector or a segment) we always mean the minor arc (or minor sector or minor segment) of a circle.

After this discussion, the teacher may ask the students to attempt questions of Exercise 4.6 of Class VI, Mathematics, NCERT.

6. Pairs of Angles

Concepts of complementary angles, supplementary angles, adjacent angles and a linear pair may be brought home to the students through activities as discussed in Sections 5.2.1, 5.2.2, 5.2.3 and 5.2.4 of Chapter 5 of Class VII, Mathematics, NCERT. It must be clearly pointed out that all pairs of complementary angles need not be adjacent (pair of angles, which share one common vertex and one common side but do not share interior region). Similarly, all the pairs of supplementary angles need not be adjacent. In case, they are adjacent, they form a linear pair (Fig. 1.24). So, it must be made amply clear to the students that the **two angles of a linear pair will always be supplementary but two supplementary angles need not always form a linear pair.** To explain this idea more precisely, the teacher may include the types of angles in the 'Try These' (Fig. 5.13) given on page 99 of Class VII, Mathematics, NCERT.

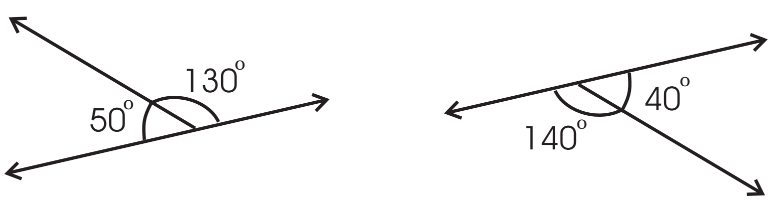


Fig. 1.24

Here, it may also be explained to the students that sometimes a single term is used in two or more senses, depending on the context in which it has been used. For example, when we discuss 'adjacent angles' of a quadrilateral, then the meaning of adjacent angles is different from what we consider in the present case of linear pairs. Note that ∠A and ∠B are adjacent angles of quadrilateral ABCD (Fig. 1.25). Here, there is no common vertex as these angles are formed at the end points of the same segment. Further, interior regions of both the angles are common. So, it is always advisable to see in what context a term has been used. The pair of vertically opposite angles and the property that vertically opposite angles are equal should be explained through activities as suggested in Section 5.2.5 of Chapter 5 of Class VII Mathematics Textbook, NCERT. It must be emphasised that for the formation of vertically opposite angles, **it is essential that the two lines must be intersecting.** Without the intersection of two lines, vertically opposite angles will not be formed. For example, it may be explained that ∠AOB and ∠COD of Fig. 1.26 are not vertically opposite angles. Similarly, ∠AOD and ∠BOC are also not vertically opposite angles. After these discussions, students may be asked to attempt questions of Exercise 5.1 of Class VII, Mathematics, NCERT.

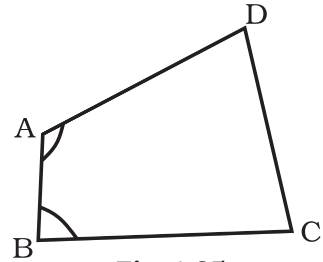


Fig. 1.25

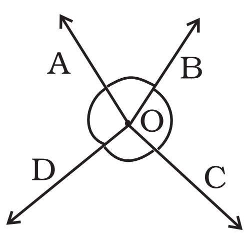


Fig. 1.26

7. Angles Formed by a Transversal with Two Lines

These concepts have been discussed in Section 5.3 of Chapter 5 of Class VII, Mathematics. To start with, the idea of a 'transversal' may be explained as below:

If a line intersects two or more lines in distinct points, then it is called a transversal to these lines. For this, it is necessary that if a line intersects two lines, then it must intersect them at two distinct points; if it intersects three lines, then it must intersect them at three points (not at two or one), and so on. Thus, in Fig. 1.27, l is a transversal to lines m and n in (i) and (ii), but in (iii) l is not a transversal to m and n because it intersects m and n at one point only. Similarly, in Fig. 1.28, l is a transversal in (i) for m, n and p

but it is not a transversal in (ii) for m,n and p. However, l is a transversal for m and n as well *as* m and p.

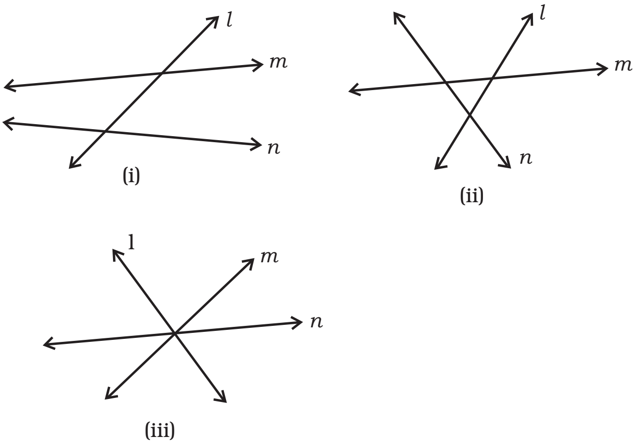


Fig. 1.27

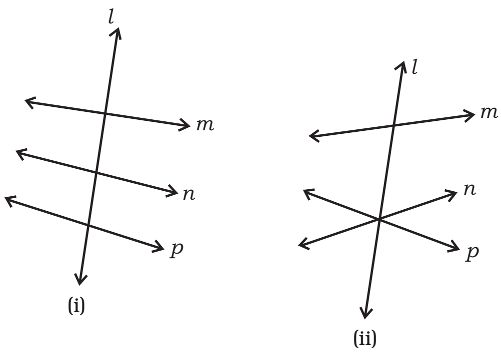


Fig. 1.28

After explaining the concept of a transversal, it may be explained that if a transversal intersects two lines, then eight angles are formed (Fig. 1.29). Out of these, four angles are interior angles (3, 4, 5 and 6) and four are exterior angles (1,2,7 and 8).

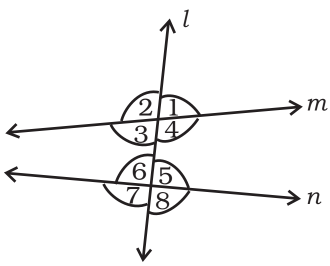


Fig. 1.29(i)

Teacher should ask the students to draw two lines *m* and *n* and a transversal *l* to these lines on a sheet of paper. Ask them to number all the angles as shown in Fig. 1.29(i).

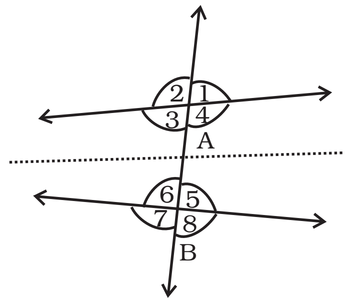


Fig. 1.29(ii)

Now ask them to cut the paper along the dotted line and put the vertex B on the vertex A. By doing so, let the students observe that ∠5 falls on ∠1, ∠8 falls on ∠4, ∠6 falls of ∠2 and ∠7 falls on ∠3, though they wouldn't exactly overlap. The pair of angles which fall on each other are called corresponding angles.

Their classification as (i) corresponding angles, (ii) alternate interior angles and (iii) interior angles on the same side of the transversal may further be explained as given in Class VII Mathematics, NCERT (Section 5.3). Here it may also be emphasised that the word 'corresponding angles' has been used in a different sense than in the case of corresponding angles of congruent triangles.

Here for hands-on experiences the students may be encouraged to perform Activity-5, 'Two parallel Lines and a Transversal' of Mathematics kit supplied with the package.

After hands-on experiences, the identification of the above pairs of angles may also be done by drawing two parallel lines using the edges of a ruler on notebooks or on a notebook having ruled lines as discussed on page 107 of Class VII, Mathematics, NCERT. After drawing two such parallel lines, they may be asked to draw a transversal and note down the measures of all the eight angles so formed, using a

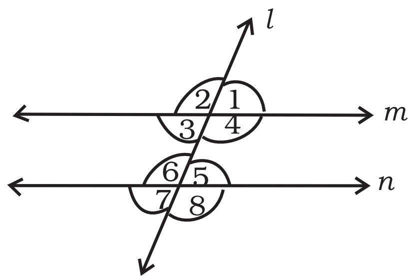


Fig. 1.30

protractor (Fig. 1.30). Let them observe that

(i) $\angle 1 = \angle 5$, $\angle 4 = \angle 8$, $\angle 2 = \angle 6$ and $\angle 3 = \angle 7$. That is, when $m || n$, then **corresponding angles are equal.**

(ii) $\angle 3 = \angle 5$ and $\angle 4 = \angle 6$. That is, if $m || n$, then **alternate interior angles formed are equal**.

(iii) $\angle 4 + \angle 5 = 180°$ and $\angle 3 + \angle 6 = 180°$. That is, if $m || n$, then **interior angles on the same side of the transversal are supplementary.**

They may be asked to repeat this activity by drawing several pairs of parallel lines and a transversal to each pair. Let them arrive at the following properties:

If two parallel lines are intersected by a transversal, then

(i) corresponding angles are equal;

(ii) alternate interior/exterior angles are equal; and

(iii) interior angles on the same side of the transversal are supplementary.

To examine the converse of the above, students may be asked to draw several pairs of non-parallel lines and a transversal. Then ask them to measure all the eight angles so formed (Fig. 1.31), using a protractor. Let them observe that in this case,

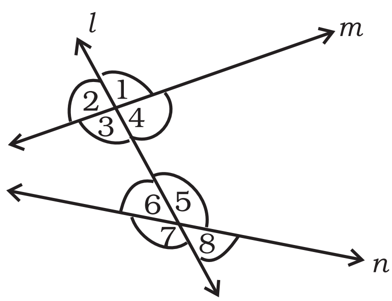


Fig. 1.31

(i) $\angle 1 \neq \angle 5$, $\angle 4 \neq \angle 8$, $\angle 2 \neq \angle 6$ and $\angle 3 \neq \angle 7$. That is, corresponding angles are not equal, when m is not parallel to n.

(ii) $\angle 3 \neq \angle 5$ and $\angle 4 \neq \angle 6$. That is, alternate interior angles are not equal, when m is not parallel to n.

(iii) $\angle 4 + \angle 5 \neq 180°$ and $\angle 3 + \angle 6 \neq 180°$. That is interior angles on the same side of the transversal are not supplementary, when m is not parallel to n.

Through such activities, students may be helped to arrive at the following converse:

If two lines are intersected by a transversal such that

(i) Corresponding angles in a pair are equal

or

(ii) Alternate interior angles in a pair are equal

or

(iii) Interior angles in the same side of the transversal are supplementary,

then the two lines are parallel.

Using these facts, students may be asked to construct parallel lines as given on page 109 of Class VII Mathematics Textbook. After clear understanding of these properties, students may attempt questions in Exercise 5.2 of Class VII, Mathematics, NCERT.

Use of Mathematics Kit: To understand various concepts related to angles, pairs of angles and angles formed by transversal with two lines, teacher may encourage the students to use various items given in Mathematics Kit.

Common Errors

(i) **Segment, Ray and Line:** Some students are not able to make a difference between a ray and a line or a line segment and a ray. They are unable to make a difference between ray AB and ray BA (Fig. 1.32).

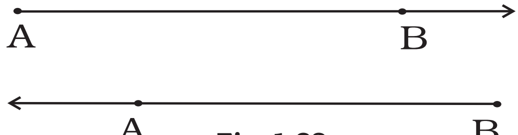


Fig. 1.32

(ii) **Angle and its Measurement:** Some students are not aware of the proper use of protractor. For example, they are not aware of particular scale on the protractor.

(iii) **Classification of Angles:** Sometimes, students think that 0^0 angle is an acute angle, which is not correct. Similarly, sometimes, they think that 181^0 angle is an obtuse angle, which is not correct.

(iv) **Different types of curves:** Some students have a confusion that every simple curve is open and vice versa. Every curve made up of line segments is a polygon, which is not correct.

(v) **Circle and its Parts:** Some students think that diameter is included in a semicircle, which is not correct. Further many a time the word circumference is used for a circle.

(vi) **Pairs of Angles:** Many times the students think that a linear pair and supplementary angles are the same. Sometimes students also think that vertically opposite angles can be formed without the intersection of two lines.

(vii) **Pairs of angles formed by a Transversal:** There is a common feeling that corresponding angles or alternate interior angles, etc. occur only when the transversal intersects two or more parallel lines, which is not correct. Sometimes students are not able to identify corresponding angles or alternate interior angles, etc. correctly.

You may evaluate students through the following Exercise.

Exercise

1. In Fig. 1.33, name
   (i) four points
   (ii) a line
   (iii) three rays
   (iv) four line segments

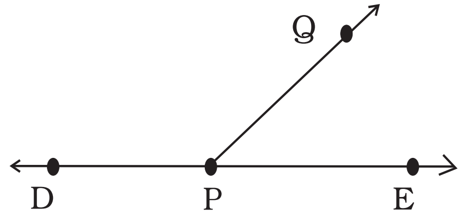


Fig. 1.33

2. Identify the following in Fig. 1.34:

(i) simple curve (ii) closed curve

(iii) open curve (iv) polygon

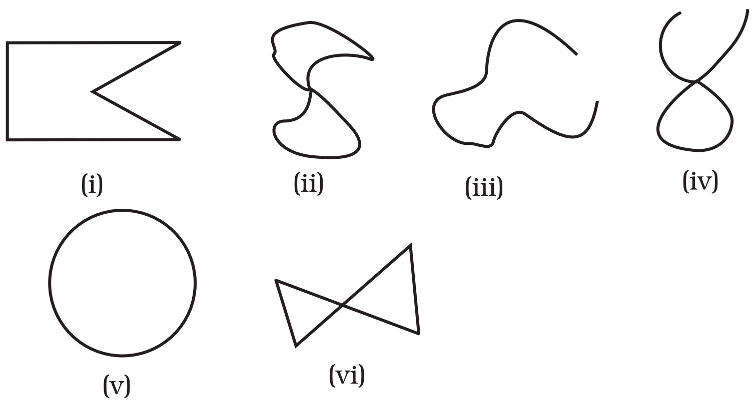


Fig. 1.34

3. Classify the angles whose measures are given below. (acute, obtuse, right, etc.)

85°, 90°, 79°, 182°, 0°, 360°, 271°, 176°, 180°.

4. Using a protractor, measure the angles drawn in Fig. 1.35.

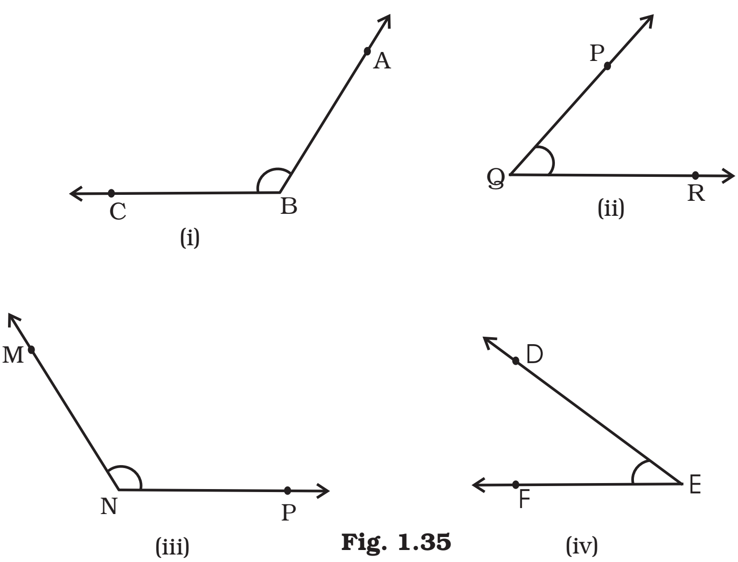


Fig. 1.35

5. In Fig. 1.36, O is the centre of the circle. Write
   (i) two radii
   (ii) a chord

Also, shade minor sector and major segment of the circle.

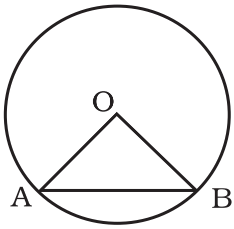


Fig. 1.36

6. Look at Fig. 1.37 and write the following pairs of angles:
   (i) Complementary angles
   (ii) Supplementary angles
   (iii) Vertically opposite angles
   (iv) Linear pair
   (v) Adjacent angles

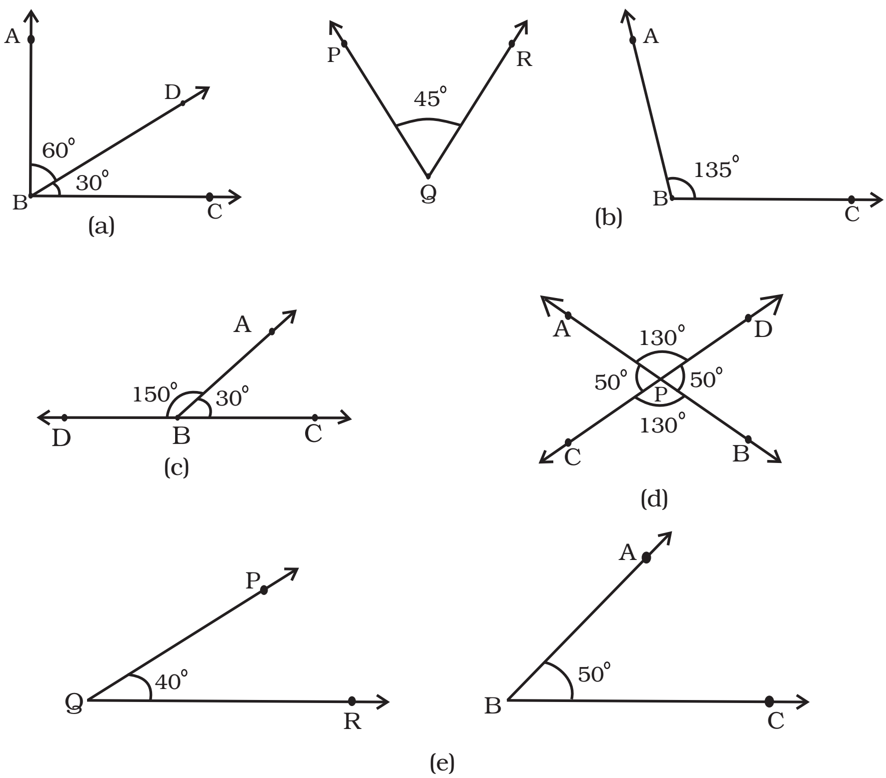


Fig. 1.37

7. Name the following pairs of angles in Fig. 1.38
   (i) Alternate interior angles
   (ii) Interior angles on the same side of the transversal
   (iii) Corresponding angles

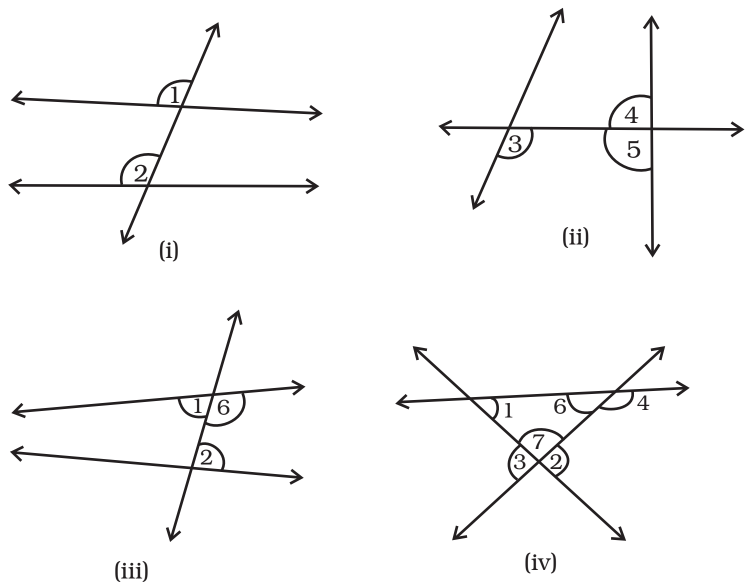


Fig. 1.38

8. In Fig. 1.30, n || *m*. If ∠1 = 60°, find the angles 2, 3, 4, 5, 6, 7 and 8.

9. If two angles in a pair of alternate interior angles are 89° and 91°, can you say that the two lines forming these angles with a transversal are parallel? Why or why not?

UNIT 2

Triangles and their Properties

Structure

- Introduction
- Main Concepts and Sub-concepts
- Objectives
- Teaching points
  1. Polygons
  2. Diagonals
  3. Convex and concave polygons
  4. Regular and irregular polygons
  5. Triangle
  6. Altitudes, medians, angle bisectors and perpendicular bisectors in a triangle
  7. Classification of triangles
  8. Exterior angle property of a triangle
  9. Angle sum property of a triangle
  10. Equilateral and isosceles triangles
  11. Property of lengths of sides of a triangle
  12. Pythagoras property
- Common Errors
- Exercise

Introduction

In Unit 1, the idea of basic geometrical figures such as point, line segment, ray, line, angle, etc. has been given. From the knowledge of earlier classes and Unit 1, students understand what a simple closed curve is. In this unit, the teacher may introduce various geometrical shapes in the form of a simple closed curve made of line segments only.

Teacher can pick up 2 pens kept in front of children and then tell them to visualise them as line segments. She can also ask the children to pick up 2 pens and try to join them. The possible ways could be

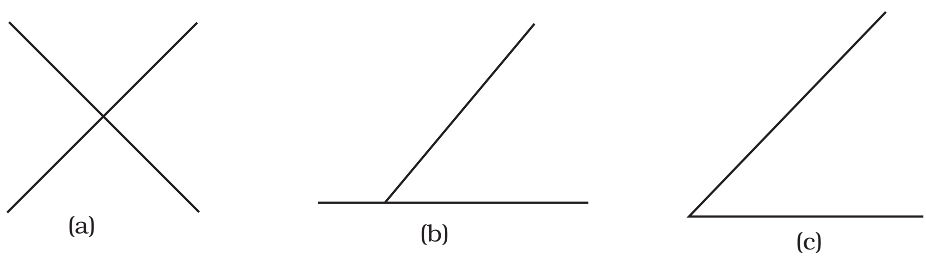


Fig. 2.1(i)

Teacher can then reinforce the concept of simple figure to be considered and reject the others. They can easily reject (a) and (b) and consider (c) of Fig. 2.1(i) as the simple figure entirely made up of line segments. They can further be told to see if it is possible to achieve a closed simple figure with either 1 or only 2 line segments. Possible answer would be 'no'.

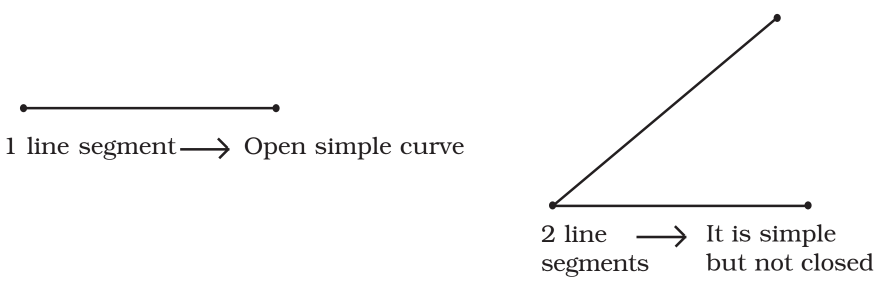


Fig. 2.1(ii)

Thus, the only way to make a simple closed figure is to involve at least 3 line segments, i.e.

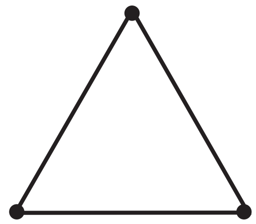


Fig. 2.1(iii)

It is a simple closed curve entirely made up of line segments.

Thus, triangle is the smallest simple closed curve entirely made up of line segments or triangle is the smallest polygon.

Then, she may conclude that the particular simple closed shape, made by three line segments only, is 'the triangle'. Some of the main properties of a triangle will be considered and these will be taught to students with the help of activities only and not by the logical proof.

Main Concepts and Sub-concepts

- Polygons and their classification
- Classification of Triangles
- Altitudes, Medians, Angle Bisectors and Perpendicular Bisectors of sides
- Exterior Angle property
- Angle sum property
- Equilateral and Isosceles Triangle
- Relation between lengths of sides of a Triangle
- Pythagoras property

Objectives

After teaching these concepts, the students can

- understand the meaning of a polygon;
- identify various polygons; e.g. triangle, quadrilateral, pentagon, hexagon, etc.
- identify and draw rough sketches of various types of triangles;
- state the exterior angle property of a triangle and apply it;
- state that the sum of the angles of a triangle is 180° and apply this property;
- state the relationship between lengths of sides of a triangle and apply it;
- state and apply the Pythagoras property.

Teaching Points

1. Polygons

A simple closed curve entirely made up of only line segments is called a **polygon**. The following are some shapes in the form of a polygon:

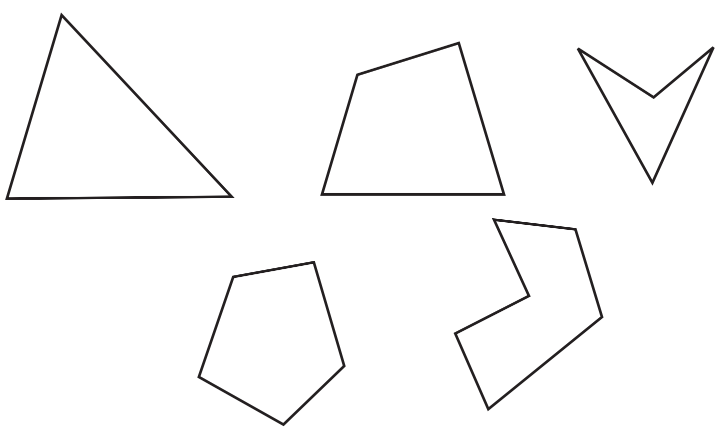


Fig. 2.1(iv)

The polygons are classified according to the number of line segments by which it is made (called their sides). For example,

Number of sides	Name of the polygon	Number of sides	Name of the polygon
3	Triangle	7	Septagon or Heptagon
4	Quadrilateral	8	Octagon
5	Pentagon	9	Nonagon
6	Hexagon	10	Decagon
		n	n-gon

- Three should be related to Tri of Triangle
- They know quarter in a clock means division into 4 and so 4 is related to quad of quadrilateral
- Five is related to Penta of Pentagon
- Six is related to Hexa of Hexagon
- Seven is starting with same letter as Septa of Septagon
- Eight is related to Oct and even Octopus has eight tentacles
- Nine starts with same letter as Nona of Nonagon
- Ten is related to Deca and otherwise they also study decimal system (System of Tens)

The teacher may draw several polygons and ask the students to identify them.

2. Diagonals

Teacher can ask 5 students to come and stand in the class. The teacher may then arrange them as 5 vertices of a Pentagon. Now she can explain the students about consecutive and non-consecutive vertices.

Then she can ask children representing consecutive vertices to join hands with each other. She may now reinforce that whenever we join consecutive vertices we get a side of the pentagon and then by the joining of non-consecutive vertices she can give the idea of a diagonal. This activity is also useful to give children the idea about adjacent sides.

Two adjacent sides share a vertex.

Thus, a diagonal of a polygon is a line segment joining its two non-consecutive vertices. For example, a pentagon ABCDE has AC, AD, BE, BD and CE, i.e. five diagonals (see Fig. 2.2). The following table gives the number of diagonals for different polygons. The teacher may help students to complete the table by observing a pattern.

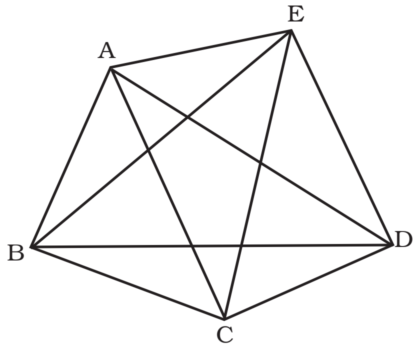


Fig. 2.2

Number of sides	3	4	5	6
Number of diagonals	0	2 (0 + 2)	5 (2 + 3)	9 (5 + 4)

Number of sides	7	8	9	10
Number of diagonals	14 (9 + 5)	–	–	–

3. Convex and Concave Polygons

A diagonal of a polygon may or may not lie completely in its interior (see Fig. 2.3)

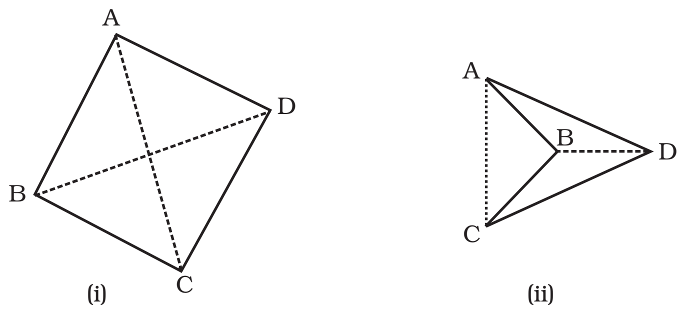
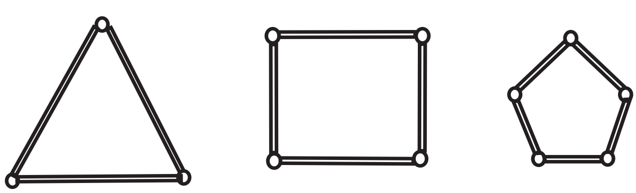


Fig. 2.3

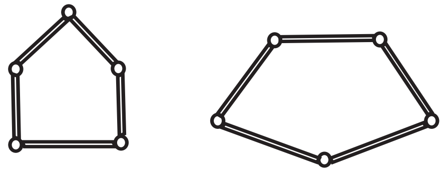
If all the diagonals of a polygon lie completely in its interior, the polygon is **convex** [(Fig. 2.3(i)], otherwise it is **concave** [Fig. 2.3(ii)]. Another way of testing, whether a given polygon is convex or not, is to check whether each of its interior angles is less than 180° or not. If each of the interior angles of a polygon is less than 180°, then it is convex, otherwise it is concave. The geometry at this stage will involve only convex polygons. The teacher may tell the students that now onwards a polygon will mean only a convex polygon.

4. Regular and Irregular Polygons

If all the sides and all the angles of a polygon are equal, the polygon is called a **regular polygon**. Students may be asked to make regular polygons using matchsticks. Further they may be asked to make polygons which are not regular.



Regular Polygons

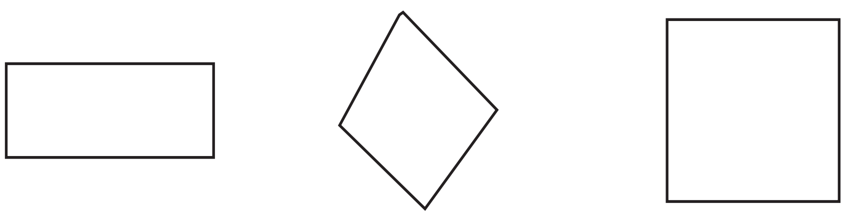


Irregular Polygons

Fig. 2.4(i)

(Each matchstick is of same length, hence sides of these irregular polygons are equal but interior angles are not equal.)

The teacher should clearly tell the students that for a regular polygon both the conditions are essential. In particular, the teacher may tell the students that all the four sides of a rhombus are equal and all the four angles of a rectangle are equal but none of them is a regular polygon. The only regular polygon is a square [see Fig. 2.4(ii)].



Not regular Not regular Regular

Fig. 2.4(ii)

The teacher may tell the students that an equilateral triangle is a three-sided regular polygon (triangle). However, an isosceles triangle is not a regular polygon. Here, teacher may evolve these results from students with proper discussion.

5. Triangle

A polygon having three sides is called a triangle. Students may be asked to draw a set of three collinear points and a set of three non-collinear points.

Then they may be asked to join each set of points. They can observe that they can make a triangle by joining three non-collinear points. With the help of teacher they can explore that a triangle is made up of three points (vertices), three line segments joining these points (sides) and three angles (at a point where two sides meet). Thus, a triangle has three vertices, three sides and three angles. Note the meaning of the word TRIANGLE = TRI (Three) + ANGLE. The three sides, three angles and three vertices are called its parts. If ABC is a triangle, its parts are $\overline{AB}, \overline{BC}, \overline{CA}$, $\angle A$, $\angle B$, $\angle C$ and vertices A, B and C. Students may be asked to draw a triangle in their notebooks and name the vertices with the alphabets of their own choice. Further they write the name of the triangle and its parts. Region enclosed by the triangle can be shaded by some colour to understand the interior of a triangle. A triangle has no diagonals. Also it is always a convex polygon. In the triangle ABC the side opposite to A is BC which may also be denoted by $\overline{BC}$. The angle opposite to the side CA is $\angle B$ [see Fig. 2.5(i)]. The 'triangle' symbolically is denoted as Δ. So, you may either write triangle ABC or ΔABC.

After showing a simple triangle as in Fig. 2.5(ii), students should be made familiar about all possible names that it could have. Also teacher may highlight about the case of naming its angles with the help of vertex letter. But she should also classify the situation where naming of angles with only vertex letters may be incorrect.

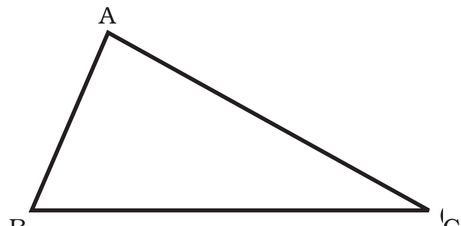


Fig. 2.5(i)

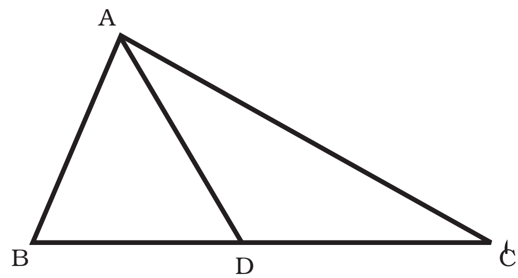


Fig. 2.5(ii)

6. Altitudes, Medians, Angle Bisectors and Perpendicular Bisectors

Let ABC be a triangle. From A, draw a line segment AD, perpendicular to the opposite side BC (or CB produced). This line segment AD is called the altitude corresponding to the side BC of the triangle ABC (see Fig. 2.6).

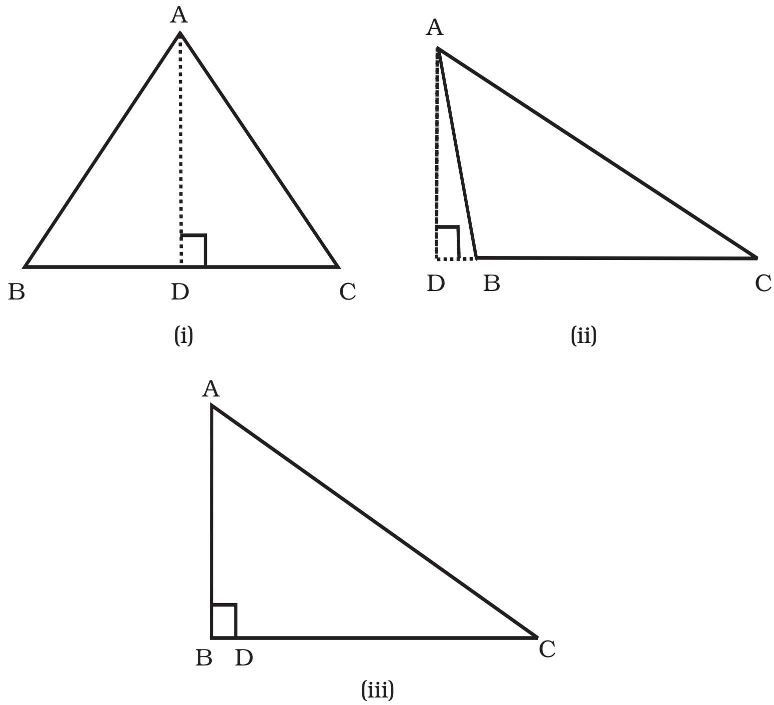


Fig. 2.6

[Note that Fig. 2.6(i) is an acute triangle, Fig. 2.6(ii) is an obtuse triangle and Fig. 2.6(iii) is a right triangle.]

In the same way, students may be asked to explore two more altitudes corresponding to sides AC and AB. By actual drawing, it can be seen that the three altitudes of a triangle always meet at a point. Altitudes and their point of intersection may or may not lie in the interior of ABC. (Fig. 2.6a).

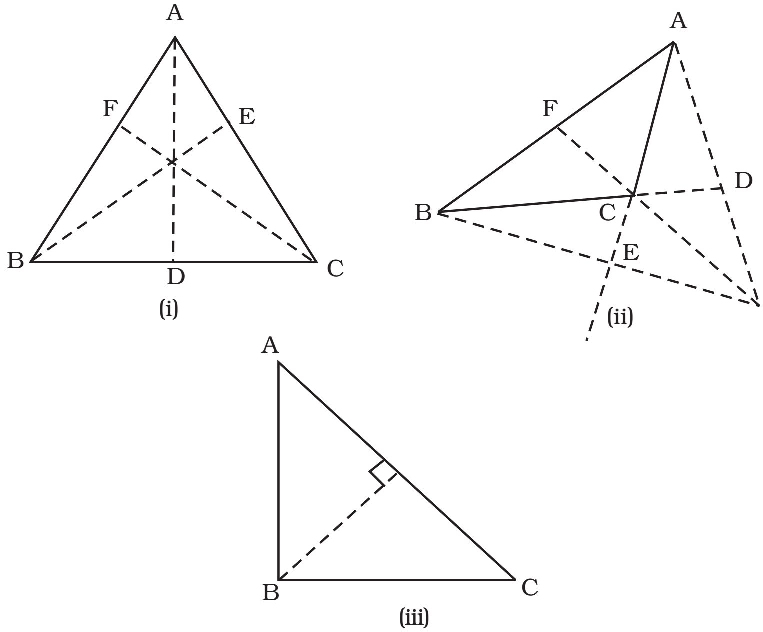


Fig. 2.6a

Teacher is supposed to emphasize upon altitude in Fig. 2.6(iii) in particular, so that children may realize that in a right Δ its 2 sides become altitudes.

Now in triangle ABC if the point A is joined to the mid-point say D of the side BC, then the line segment AD is called a **median** of the triangle (see Fig. 2.7). Here again students may be asked to explore two more medians of the triangle. It may be observed by actual drawing that these medians always meet at a point. The medians and the point of intersection of medians always lie in the interior of the triangle ABC. (See Fig. 2.7a).

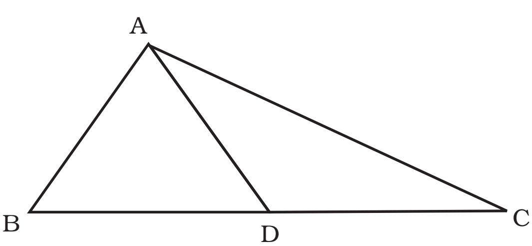


Fig. 2.7

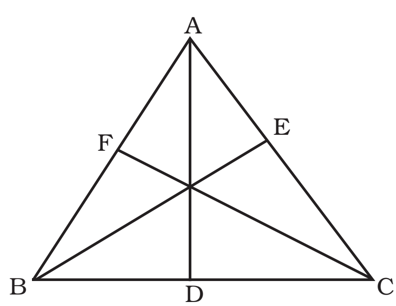


Fig. 2.7a

Next draw the bisectors of the angles A, B and C of a triangle ABC by paper folding method or by construction. Folding of the paper can be done by folding through a vertex, say B, so that BA falls on BC and the crease will be the angle bisector of ∠B.(see Fig. 2.8). It may be observed by actual drawing that bisectors of the three angles lie in the interior of ΔABC and also meet at a single point which is inside the triangle always (Fig. 8a).

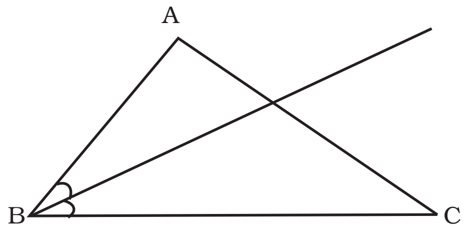


Fig. 2.8

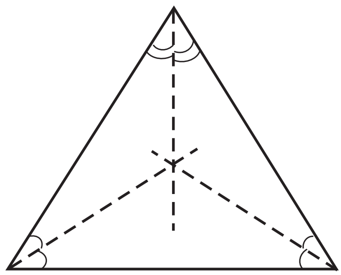


Fig. 2.8a

Finally, if perpendicular bisectors of the line segments BC, CA and AB of a triangle ABC are drawn by paper folding method or by construction, the line representing these also meets at a point (See Fig. 2.8b). For this, the teacher may ask the students to draw a number of triangles and the perpendicular bisectors of their sides using ruler and compasses. Perpendicular bisectors and their point of intersection may or may not lie inside the triangles.

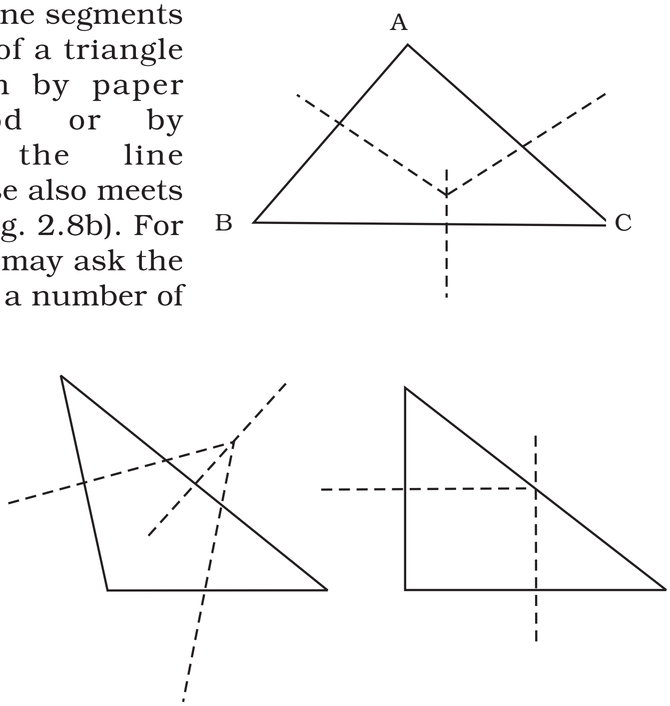


Fig. 2.8b

7. Classification of Triangles

The teacher may ask the children to draw several triangles and measure their angles. Then, she can ask the following questions:

(i) Is there a triangle having two right angles?

(ii) Is there a triangle having two obtuse angles?

(iii) Is there a triangle having one right angle and one obtuse angle?

(iv) Is there a triangle having two acute angles?

The answer for each of (i), (ii) and (iii) is 'no' while the answer for (iv) is 'yes'. Therefore, triangles can be divided into three categories according to their angles:

a. All the three angles are acute. Such triangles are called **acute angled triangles**.

b. Any one angle is a right angle. Such triangles are called **right angled triangles** or simply **right triangle**.

c. Any one angle is obtuse. Such triangles are called **obtuse angled triangles**.

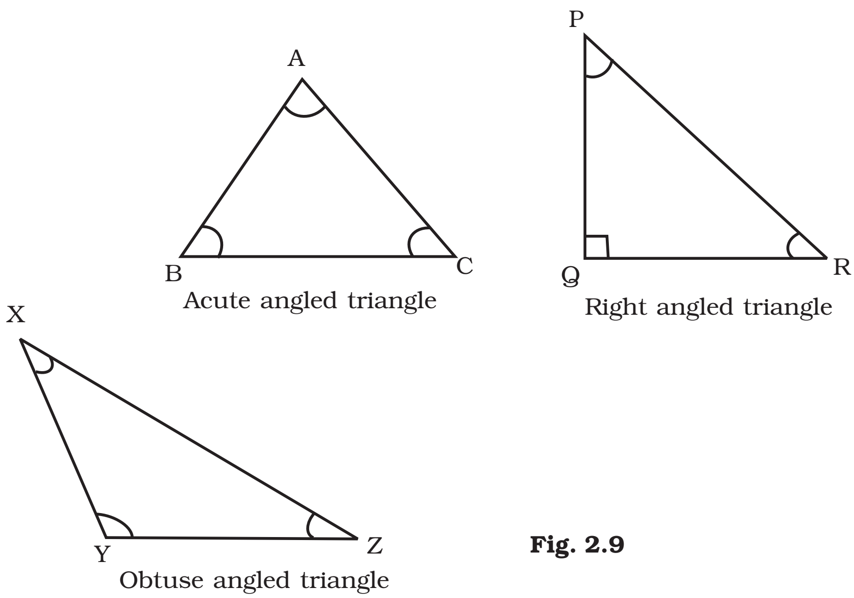


Fig. 2.9

Teacher may summarise it as:

	Acute∆	Right angled∆	Obtuse angled∆
Any 2 angles	Always acute	Always acute	Always acute
3rd angle	Acute	Right	Obtuse

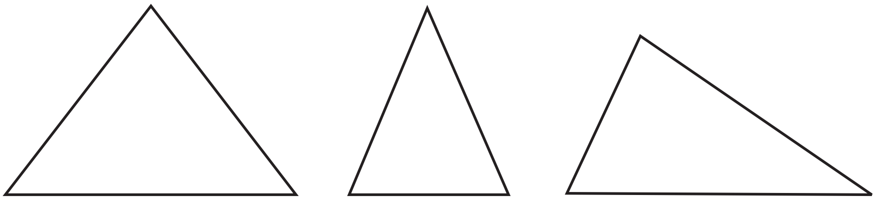
Now teacher may ask students to measure the sides of these triangles.

The triangles can also be classified according to the lengths of their sides (See Fig. 2.10).

(a) If all sides are of unequal lengths, the triangle is called a **scalene triangle.**

(b) If two sides are of equal length, the triangle is called an **isosceles triangle.**

(c) If all the three sides have equal length, it is called an **equilateral triangle.**



Equilateral Triangle   Isosceles Triangle   Scalene Triangle

Fig. 2.10

Anglewise Classification	
1. Acute angled	All angles are acute
2. Obtuse angled	One angle is obtuse
3. Right angled	One angle is right
Sidewise Classification	
4. Equilateral	All sides are equal
5. Isosceles	Two sides are equal
6. Scalene	No two sides are equal

Teacher may draw some triangles on the blackboard and ask the students to classify them according to their angles and also according to their sides. The teacher may help the students to understand the above classification of triangles using an Activity 'Geoboard' given in Mathematics Kit. It may be noted that a triangle can be acute angled isosceles or right angled isosceles or obtuse angled isosceles. Teacher may ask students to explore this type of classification also.

8. Exterior Angle Property of a triangle

In a triangle, angles between the sides are also called the interior angles of the triangle. Let ABC be any triangle. If any of its sides, say BC, is produced to a point D, then an angle ACD called an exterior angle is formed at the vertex C. Observe that one arm of ∠ACD is a side CA of the triangle and the other arm CD is not a side of the triangle (See Fig. 2.11), The teacher may ask the students how many such angles can be made? If AC is produced to a point E (See Fig. 2.11), then ∠BCE is also an exterior angle at C but the angle DCE is not an exterior angle at C. The teacher should explain that for an exterior angle one arm of the angle should be a side of the triangle and other should be an extension of the other side of the triangle.

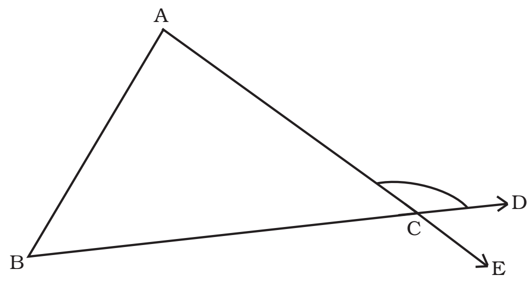


Fig. 2.11

Now ∠ACB is adjacent angle to ∠ACD and the two angles form a linear pair of angles. The remaining angles, i.e. ∠A and ∠B of the triangle are called interior opposite angles to exterior ∠ACD.

The teacher is expected to perform the activity given on page 116 of Class VII, Mathematics, NCERT to verify that the measure of an exterior angle is equal to sum of the measures of the two interior opposite angles. Then she may give the formal proof given on page 117.

This relation is known as **Exterior angle property of a triangle.**

9. Angle Sum property of a Triangle

Students are already aware that a triangle has three angles and three sides. There must be some relation between the angles of a triangle. Let the students try to find relationship, if any, between the angles of a triangle. Teacher may now explain 'Angle Sum Property of a Triangle' through the following activity.

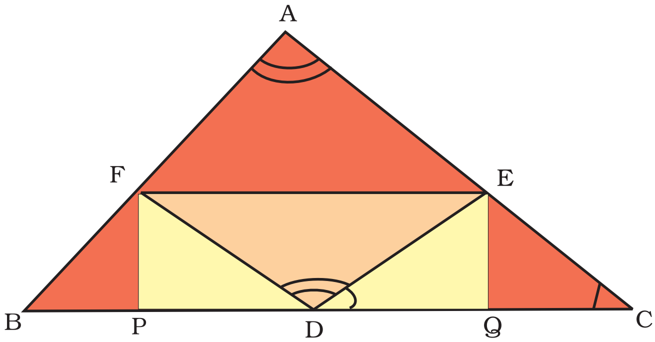


Fig. 2.12

Cut out a triangular piece of paper ABC. Fold it in such a way that all the vertices should meet at a point D on its base BC (See Fig. 2.12). The creases of the foldings are EF, FP and EQ.

The vertices A, B, and C all coincide at the point D. Therefore, $\angle FDE + \angle FDP + \angle EDQ = \angle A + \angle B + \angle C$ = straight angle = 180°.

The teacher may explain this property of the triangle by actual measurement of the angles and summing them and also by other activities given on pages 119-120 of Class VII, Mathematics, NCERT. The students now clearly understand the fact that **sum of interior angles of a triangle is 180°.**

Let the students draw triangles of different types and measure the angles of each triangle and verify the above property.

10. Equilateral and Isosceles Triangles

From a piece of paper cut out an equilateral triangle, say ABC. Fold it along the line AD through the vertex A such that C falls on B. Show the students that two parts of the triangle divided by the line segment AD completely overlap each other (Fig. 2.13). Repeat similar activity for the points B and C. Here again the two portions overlap each other. This shows that $\angle B = \angle C$, $\angle C = \angle A$. Therefore, $\angle A = \angle B = \angle C$. Since, $\angle A + \angle B + \angle C = 180°$, therefore measure of each angle is 60°. Thus,

Angles of an equilateral triangle are all equal and each measures 60°.

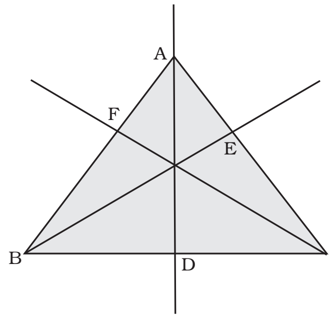


Fig. 2.13

Here, teacher may encourage children to observe that equilateral Δ is the smallest regular polygon.

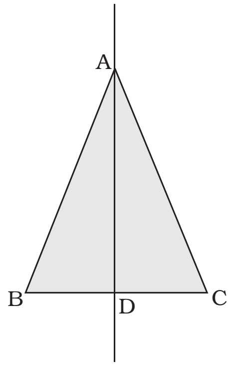


Fig. 2.14

Now cut out an isosceles triangle from a piece of paper. Name it as Δ ABC where AB = AC. In general, unequal side BC is said to be the base of the triangle and angle A opposite to base BC is said to be the vertical angle. Now teacher may help the students to fold it along the line AD through A so that C falls on B. Let the students observe that the two parts formed by the line segment AD overlap each other, i.e. $\angle B = \angle C$ (See Fig. 2.14). Therefore,

The base angles of an isosceles triangle are equal.

Students may recall these points as follows:

1. Types of Δs (classification by side and classification by $\angle$s).
2. Isosceles and Equilateral Δ property.
3. Angle sum property.

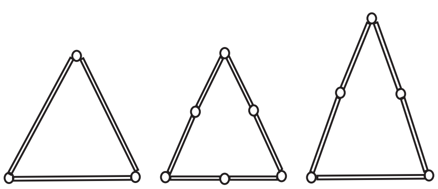
A table showing relation between types of Δ s is as follows:

On the basis of sides	On the basis of $\angle$s		
	Acute Δ	Right Δ	Obtuse Δ
1. Scalene	one ex. (40°, 60°, 80°)	one ex. (90°, 30°, 60°)	one ex. (110°, 50°, 20°)
2. Isosceles	one ex. (80°, 80°, 20°)	one ex. (90°, 45°, 45°)	one ex. (110°, 35°, 35°)
3. Equilateral	(60°, 60°, 60°)	Not possible	Not possible

11. Property of Lengths of Sides of a Triangle

Teacher may ask the students to draw several triangles. Measure the lengths of sides of each triangle and write them along the respective sides. Find the sum of the lengths of any two sides and compare it with the length of the third side.

Students may be asked to make triangles using match sticks.



Taking one matchstick as one unit find the sum of the lengths of any two sides and then compare it with the length of third side.

And let the students observe that

Sum of any two sides of a triangle is always greater than the third side.

Next the teacher may take a piece of wire say of length 15 cm and try to make a triangle by bending it at two places in different ways. It may be shown to the students that in some cases a triangle is formed while in other ;some cases a triangle is not formed. Let them observe the lengths of the three parts of the wire when the triangle is formed. In such cases, let them see that the sum of any two parts is greater than the third. Let the students also observe the lengths of parts of the wire when the triangle is not formed. In such cases, let them see that the sum of lengths of any two parts is either less than or equal to the third pieces (See Fig. 2.15).

The teacher may help the students to explore the fact that the difference of any two sides of a triangle is always less than the third side. Teacher may use Activity 6 'Properties of a Triangle' in the Mathematics Kit given with the package for a better presentation of these facts.

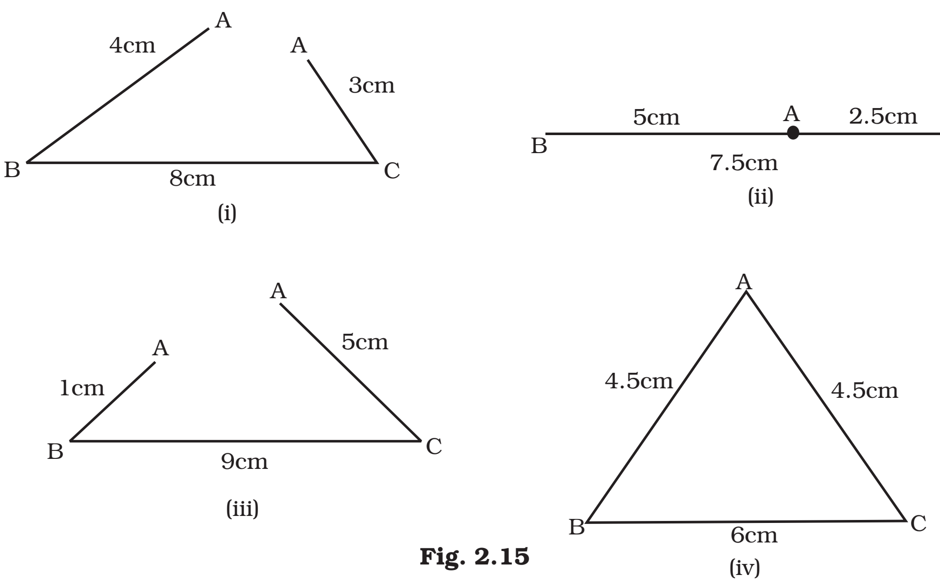


Fig. 2.15

12. Pythagoras Property

Consider a triangle ABC right angled at C. The side AB opposite to the right angle C will be the largest side of the triangle [Fig. 2.16(i)].

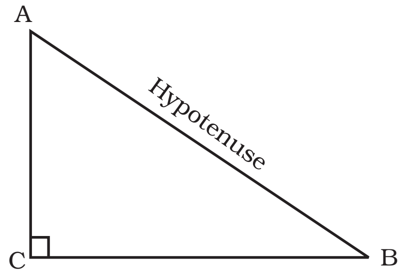


Fig. 2.16(i)

It is called the hypotenuse and other sides AC and BC are called the legs of the triangle ABC. The students may be told that there is an important relationship between the length of hypotenuse and lengths of legs which was found by a Greek Philosopher Pythagoras around 500 BC. This relation is:

Square of hypotenuse is equal to the sum of squares of legs.

If AB = c, BC = a and AC = b, then this relation is $c^2 = a^2 + b^2$. Some activities for its verification are given in Mathematics Textbook for Class VII (pages 127–128). The teacher may explain them to the children by actually performing at least one of them. Let us see one activity on graph paper.

Teacher can encourage children to draw a right Δ on a graph sheet so that construction of squares on each side is easy and areas can be calculated easily.

In a Right ΔPQR

$(PQ)^2 + (QR)^2 = (PR)^2$

or

The area of square on the hypotenuse = Sum of the areas of squares on the other two sides [Fig 2.16(ii)].

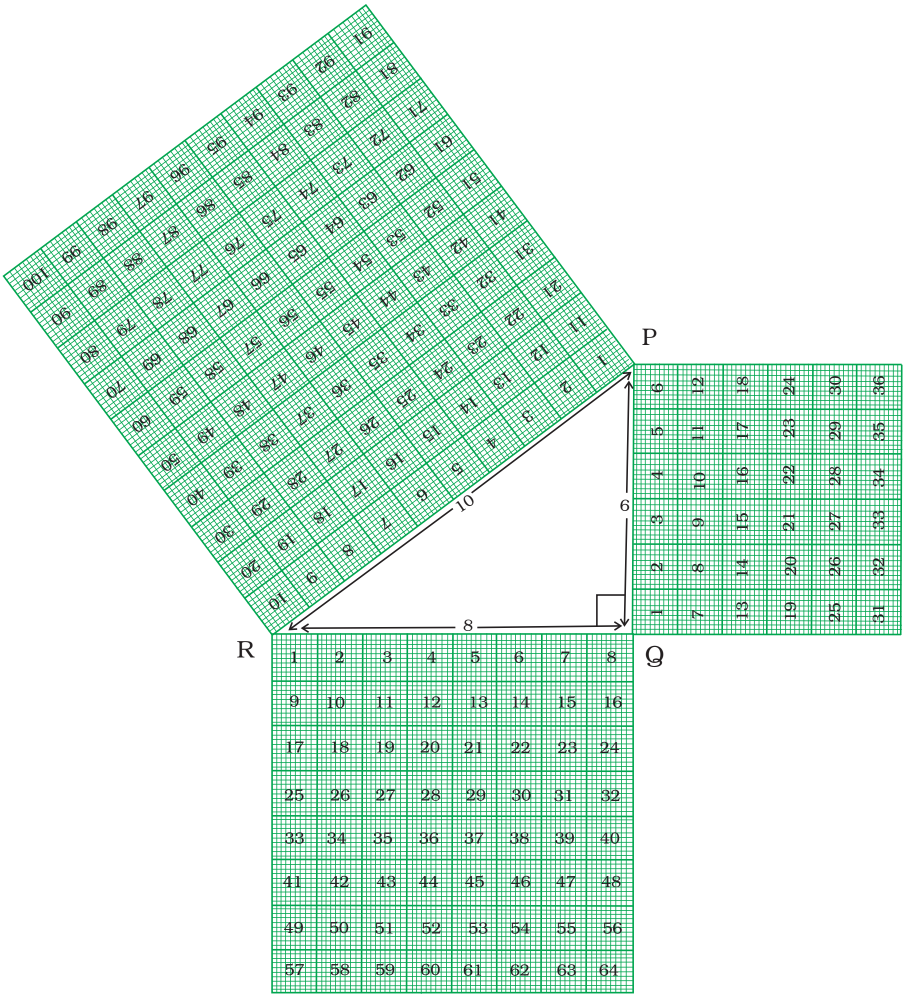


Fig. 2.16(ii)

Teacher may also encourage the students to perform some more activities for verifying Pythagoras property specially Bhaskar's method.

In Fig. 2.17 four exactly same right angled triangles are placed together to form a square of side C in such a away that one more square inside the big square with sides $a–b$ is made.

Area of bigger square = c^2

Area of smaller square = $(a–b)^2 = a^2 + b^2 - 2ab$

Area of bigger square – Area of smaller square = $4 \times \frac{1}{2} (a \times b)$

Hence $4 \left(\frac{1}{2} a \times b\right) = c^2 - (a^2 + b^2 - 2ab)$

i.e. $2ab = c^2 - a^2 - b^2 + 2ab$

or $a^2 + b^2 = c^2$

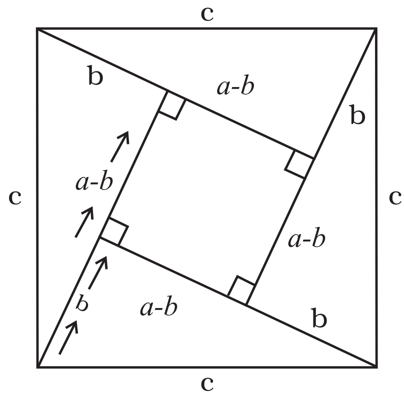


Fig. 2.17

Now, if AC = 3 cm, BC = 4 cm, then $AB^2 = 3^2 + 4^2 = 25 = 5^2$ or AB = 5 cm.

The natural numbers representing the sides of a right angled triangle give rise to a triplet which is called a **Pythagorian Triplet**. Some of such triplets are given below:

(3, 4, 5), (5, 12, 13), (7, 24, 25), (8, 15, 17), (9, 40, 41), (11, 60, 61), (12, 35, 37), (13, 84, 85), etc.

The multiples of the triplets also give such triplets. For example, (6, 8, 10), (9, 12, 15), (10, 24, 26), etc. are all **Pythagorian Triplets**.

The teacher may emphasise that if the sides of a triangle are forming a Pythagorian triplet, then it must be a right angled triangle and if they do not satisty Pythagoras property, it is not a right angled triangle. This can be stated as:

If sides *a, b, c* of triangle ABC are such that $a^2 + b^2 = c^2$, then ABC is a right angled triangle.

Teacher should allow the children to leave the answers in square root till they know how to find square root of 3 digit numbers like 289, 676, etc.

Teacher may reinforce this concept by performing activities as suggested on page 128 of Class VII, Mathematics Textbook, NCERT.

Common Errors

(i) One may think that if all the sides of a polygon are equal, it is regular as is in the case of a triangle.

(ii) Some students are not able to locate the altitudes of a right angled triangle and sometimes the students consider the altitudes as median or perpendicular bisector.

(iii) Obtuse angled triangle may be mistaken as having all angles obtuse as is in the case of acute angled triangle.

(iv) If side AB of a triangle ABC is produced to E and CB is produced to F, then $\angle$EBF is taken as exterior angle of the triangle.

(v) Isosceles triangle is taken as a regular triangle.

(vi) In right angled triangle, hypotenuse is taken to be the sum of its smaller sides.

Teacher should emphasise on the following during the transaction of this unit.

(i) Labelling of figures.

(ii) Writing the name of the property along the step, wherever it is used.

You may evaluate the students through the following exercise.

Exercise

1. How many diagonals does each of the following have?

   (a) A quadrilateral (b) A regular hexagon (c) A triangle

2. Can you think of a triangle in which two altitudes are two of its sides?

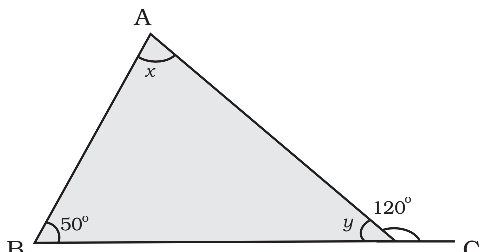
3. Can an altitude and a median be same for a triangle? If so, what type of triangle will it be?

4. Can all the altitudes and all the medians be same for a triangle. If so, what type of triangle will it be?

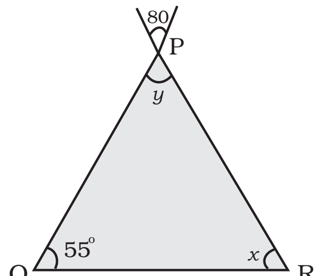
5. What can you say about each pair of the interior opposite angles when the exterior angle is:

   (i) a right angle? (ii) an obtuse angle? (iii) an acute angle?

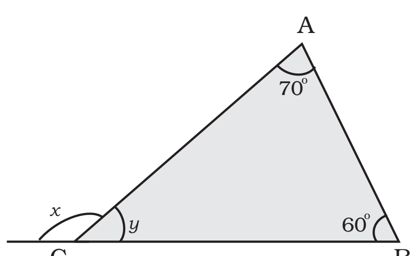
6. Find x and y in the following figures:



(i)



(ii)



(iii)

7. AM is a median of a triangle ABC. Is AB + BC + CA > 2 AM? Justify.

8. ABCD is a quadrilateral. Is AB + BC + CD + DA > AC + BD? Justify.

____

9. Which of the following can be the sides of a right triangle?

(i) 2.5 cm, 6.5 cm, 6 cm

(ii) 2 cm, 4 cm, 5 cm

(iii) 1.5 cm, 2 cm, 2.5 cm

10. Find the perimeter of a rectangle whose length is 40 cm and a diagonal is 41 cm.

UNIT 3

Congruence of Triangles

Structure

- Introduction
- Main Concepts and Sub-concepts
- Objectives
- Teaching points
  1. Congruence of plane figures
  2. Congruence of triangles
     - (i) SSS criterion
     - (ii) SAS criterion
     - (iii) ASA criterion
     - (iv) RHS criterion
- Common Errors
- Exercise

Introduction

The congruency of two figures is one of the most important concepts in geometry. It helps us in studying a large number of figures with the help of a limited number of figures which are congruent to these figures. The word congruent means having the same shape and having the same size, i.e. geometrically the two congruent figures are exactly the same. Congruent figures may be solid figures or plane figures. As a first introduction of the word 'congruent', teacher may take a couple of pages of same size from the notebook of different colours and show the students that when they are overlapped only one paper is visible. In this unit, discussion will be mainly on congruence of triangles.

Main Concepts and Sub-concepts

- Congruency of line segments, angles and circles
- Congruency of Triangles

  (i) SSS (ii) SAS (iii) ASA (iv) RHS

Objectives

After teaching these concepts/sub-concepts, the students can

- understand the meaning of congruent figures;
- understand the importance of congruency;
- state the different criteria for congruency of two triangles; and
- apply the criteria for congruency of triangles in various situations.

Teaching Points

1. Congruence of Plane Figures

If two plane figures are such that the trace copy of one figure exactly covers the other, then the two figures will be of the same shape and of the same size or in other words, such figures are called **congruent figures.** The method of placing one trace copy of one figure over the other is known as method of superposition. The teacher may explain the idea of congruence of two figures by taking two photocopies of the same size of the map of India, photographs of the same size of the same negative, etc. and superposing them one over the other.

If F_1 and F_2 are two congruent plane figures then it is symbolically expressed as $F_1 \cong F_2$, pronounced as F_1 is congruent to F_2. The teacher may explain the congruency of two line segments as follows:

Consider three line segments AB = 4cm, A'B' = 5cm and A"B" = 4cm (See Fig 3.1). If the trace copy of AB is superposed over A'B', A coincides with A', but B will not coincide with B' i.e. AB does not cover A'B' completely. However, when it is placed over A"B", A coincides with A" and B coincides with B" as AB = A"B" = 4cm.

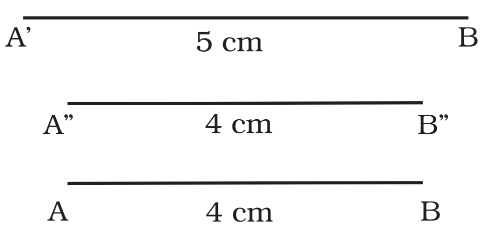


Fig. 3.1

Therefore, line segment AB $\cong$ line segment A"B". And line segment AB is not congruent to A' B'.

Two line segments are congruent if they are of the same length.

Proceeding in a similar way, the teacher may ask to perform the following:

Students can be asked to draw an angle in their notebook. They can then be told to take straw and try to make a congruent copy of angle drawn by them. They can then measure it using a protractor and infer that 'congurent angles have same measure'.

Two angles are congruent if they are of the same measure.

Teacher may ask the students to bring bangles of different sizes and then compare the congruent ones. She can also ask them to draw few circles with common centre on a sheet of paper. This activity will help them visualise that for drawing a circle that overlaps any previously drawn circle, radii of both the circles have to be the same.

Two circles are congruent if they have the same radii.

Application of congruence

To express the beauty of congruence teacher can show certain real life examples, such as:

- Pile of notebooks of similar size making a solid cuboid;
- Pen holder made with the help of bangles; and
- Cylinder made with the help of circles of same size overlapped on each other.

2. Congruence of Triangles

Similarly two triangles ABC and PQR are congruent if trace copy of Δ ABC covers Δ PQR exactly, i.e., AB falls on PQ, BC falls on QR and AC falls on PR (see Fig. 3.2).

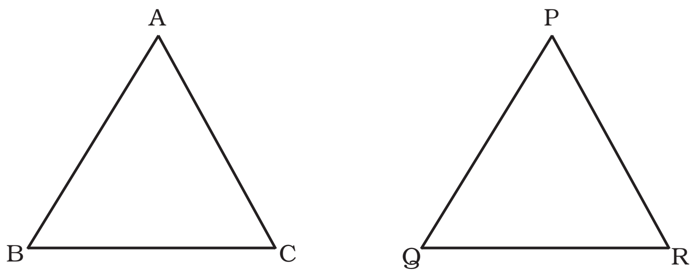


Fig. 3.2

It is symbolically written as $\Delta\ ABC \cong \Delta\ PQR$.

The teacher may explain the students that in writing the congruence, the order of correspondence must be correct. Here A corresponds to P, written as $A \leftrightarrow P$; B corresponds to Q; written as $B \leftrightarrow Q$; and C correspond to R; written as $C \leftrightarrow R$. It is incorrect to write $\Delta\ BAC \cong \Delta\ PQR$ or $\Delta\ CAB \cong \Delta\ PQR$, because here correspondence is disturbed. However, in case of congruency of equilateral triangles the order of correspondence is immaterial. This fact may be illustrated by taking some congruent equilateral triangles.

It is to be stressed by the teacher that if two triangles are congruent they are equal in all respects, i.e. all the six parts of one triangle are equal to the corresponding parts of the second triangle. In other words if Δ ABC $\cong$ Δ PQR, then

1. Corresponding sides: AB and PQ, BC and QR, AC and PR are equal
2. Corresponding angles: $\angle$A and $\angle$P, $\angle$B and $\angle$Q, and $\angle$C and $\angle$R are equal.

To check the congruence of two triangles it is not necessary to show that all the six parts of one triangle are respectively equal to the corresponding six parts of the second triangle but equality of few parts is enough to show that the two triangles are congruent. Teacher may explain the following criteria for congruence of two triangles with the activities as stated in Class VII, Mathematics, NCERT.

a. **SSS criterion** Three sides of one triangle should be equal to the three corresponding sides of the other triangle.

b. **SAS criterion** Two sides and the angle included between them of one triangle should be equal to the two corresponding sides and the angle included between them of the other triangle.

c. **ASA criterion** Two angles and the included side of one triangle should be equal to two corresponding angles and the included side of the other triangle.

(Note: It may be noted that if two angles of one triangle are equal to the two corresponding angles of the other triangle, then the third angle of both triangles is also equal. Thus, ASA criterion can be taken as AAS criterion in which any two angles and one side of one triangle are equal to the corresponding two angles and corresponding side of the other triangle.

d. RHS criterion The hypotenuse and one side of one right angled triangle should be equal to the hypotenuse and one side of the other right angled triangle.

After explaining these criteria of congruency of triangles, the teacher may ask the students to solve various questions given in Chapter 7 of Class VII, Mathematics, NCERT. Before giving questions on SAS criterion, students must be provided enough practice for identifying 'included angle' between two sides. Similarly, before giving questions on ASA criterion, students may be provided practice of identifying included side between two angles. Initially, questions may be given on individual criterion and then some questions may be given where the students need to decide which criterion is to be used. While solving these questions, students may be asked to state different corresponding parts of two congruent triangles, every time.

Teacher should also tell that AAA is not a congruence condition by showing two different examples of equilateral triangles of different sizes (Fig. 3.2a).

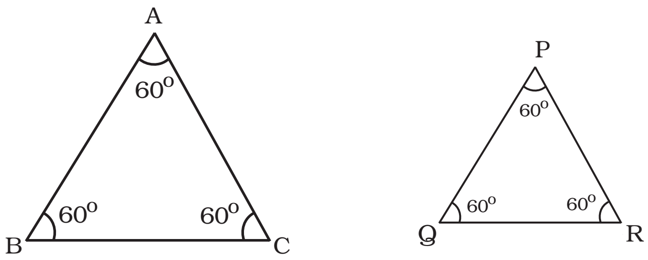


Fig. 3.2a

Common Errors

(i) Students sometimes do not care to write the congruency in a correct order, they write it in arbitrary manner.

(ii) In SAS and ASA congruency, sometimes students do not bother to consider included angle and included side and in their place they take any angle and any side respectively.

(iii) Students get confused over the fact that if areas are same the shapes are congruent, so they should be provided suitable examples to explain it.

Teacher may evaluate students through the following exercise.

Exercise

1. If Δ DEF $\cong$ Δ BCA, write the part(s) of Δ BCA that correspond to

   (i) $\angle$E (ii) EF (iii) $\angle$F (iv) DF

2. ABC is an isosceles triangle in which AB = AC and AD is its median (Fig. 3).

   (i) Name three parts of Δ ABD which are equal respectively to three parts of Δ ACD.

   (ii) Are the two triangles congruent?

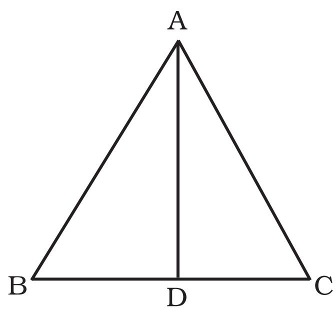


Fig. 3.3

3. In Figure 4, AC = BD, AD = BC and $\angle$DAB = $\angle$CBA = 90^0. Which of the following statements is correct? Give reasons.

(i) Δ ABC ≅ Δ ABD

(ii) Δ ABC ≅ Δ BAD

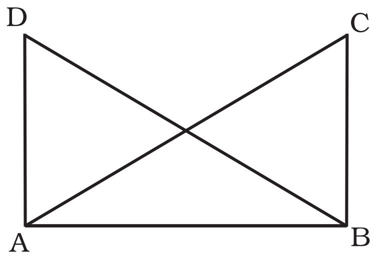


Fig. 3.4

UNIT 4

Understanding Quadrilaterals

Structure

- Introduction
- Main Concepts and Sub-concepts
- Objectives
- Teaching Points
  1. Angle sum property of a polygon
  2. Exterior angle sum property of a polygon
  3. Types of quadrilaterals
     - (i) Trapezium
     - (ii) Kite
     - (iii) Parallelogram
  4. Properties of a parallelogram
  5. Property of Diagonals of a parallelogram
  6. Special parallelograms
     - (a) Rhombus
     - (b) Rectangle
     - (c) Square
- Common Errors
- Exercise

Introduction

A quadrilateral is a polygon having four sides only. In daily life and in surroundings, the students may find quadrilaterals everywhere. Edges of window, door, floor of a house, blackboard, table top, book, photo frame are all in the shape of a quadrilateral. In this unit, some geometrical properties of quadrilaterals, specially parallelograms, shall be discussed.

Main Concepts and Sub-concepts

- Angle sum property of a quadrilateral and polygons
- Exterior angle sum property of polygons
- Characteristics of trapezium, kite and parallelogram
- Opposite sides and opposite angles of a parallelogram
- Diagonals of a parallelogram
- Rhombus, Rectangle and Square as special parallelograms

Objectives

After teaching these concepts and sub-concepts, the students can:

- understand that the sum of the angles of a quadrilateral is 360° and extend it to find the angle sum property of other polygons also.
- understand that the sum of exterior angles of any polygon taken in order is 360°.
- understand that the trapezium, kite and parallelogram are special quadrilaterals in which some extra property is satisfied, e.g.
    - (i) In a trapezium, a pair of opposite sides is parallel.
    - (ii) In a kite, adjacent sides are equal pair wise.
    - (iii) In a parallelogram, both pairs of opposite sides are parallel.

- understand that in parallelograms, opposite sides are equal, opposite angles are equal and diagonals bisect each other.
- understand that rhombus, rectangle and square are special types of parallelograms.
- apply the properties of parallelograms in solving some problems.

Teaching Points

1. Angle Sum Property of a Polygon

Teacher may ask the students to draw a quadrilateral ABCD in their notebooks and divide it into two triangles by joining AC (or BD) (See Fig. 4.1) to get two triangles ABC and DAC.

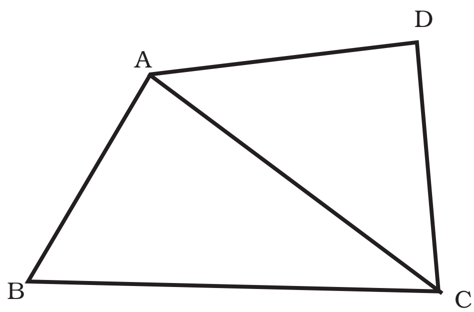


Fig. 4.1

The students can be asked to find the sum of all the angles of these two triangles. The sum so obtained will be very near to 360°. Now help them to find the sum of all the angles of the quadrilateral ABCD as;

$\angle A + \angle B + \angle C + \angle D$
$= (\angle BAC + \angle CAD) + \angle B + (\angle ACB + \angle ACD) + \angle D$
$= (\angle BAC + \angle B + \angle ACB) + (\angle CAD + \angle D + \angle ACD)$
$= 180° + 180° = 360°$ = 2 straight angles = 4 right angles.
[using angle sum property of a triangle]

This shows that sum of all the angles of a quadrilateral is 360°.

Now, ask the students to find the sum of all the angles of a pentagon in the same way as done for a quadrilateral.

In Fig. 4.2, pentagon ABCDE is divided into (5 – 2), i.e. 3 non overlapping triangles. Since sum of the angles of a traingle is 180°, so

Sum of all the angles of a pentagon = 3 × 180° = 540° = 6 right angles = (2 × 5 – 4) right angles.

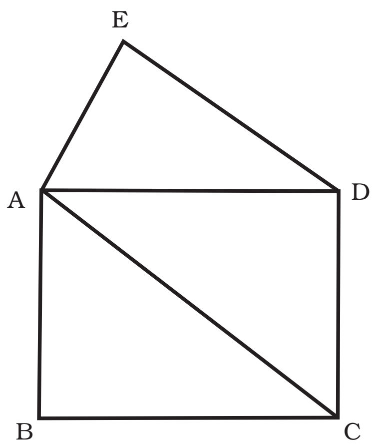


Fig. 4.2

Let the children observe the pattern:

Figure	Number of sides	Number of non overlapping triangles	Sum of angles
Quadrilateral	4	4 – 2 = 2	360° = 2(4 – 2) right angles
Pentagon	5	5 – 2 = 3	540°= 2 (5 – 2) right angles
Polygon	n	n – 2	2(n – 2) right angles

Example: In case of hexagon sum of angles will be equal to 2 (6 –2) right angles (here 6 is the number of sides of hexagon) = 8 right angles = 720°.

2. Exterior Angle Sum Property of a Polygon

If the sides of a polygon are produced in an order (clockwise or anticlockwise), exterior angles are formed. The teacher may consider a pentagon ABCDE and show the five exterior angles ∠1, ∠2, ∠3, ∠4 and ∠5 formed in an order as shown in Fig. 4.3.

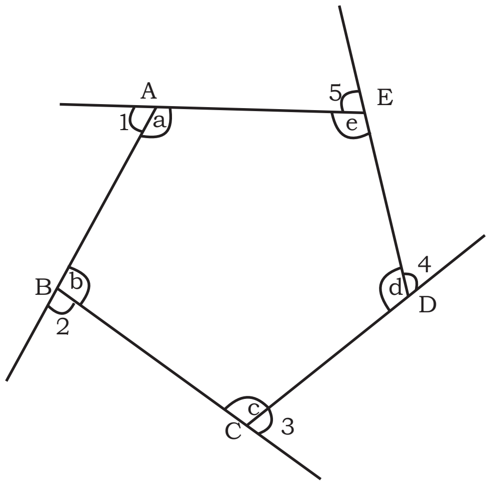


Fig. 4.3

Now, $\angle 1 + \angle a = \angle 2 + \angle b = \angle 3 + \angle c = \angle 4 + \angle d = \angle 5 + \angle e = 1$ straight angle = 180°

Therefore, $\angle a + \angle b + \angle c + \angle d + \angle e + \angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5$ = 5 straight angles,

But, $\angle a + \angle b + \angle c + \angle d + \angle e$ = 3 straight angles

This gives, $\angle 1 + \angle 2 + \angle 3 + \angle 4 + \angle 5$ = (5 – 3) straight angles = 2 straight angles = 360°

The same is true if instead of pentagon, any of the polygons is considered. So sum of exterior angles of a polygon (taken in order) is 360°, i.e. a complete angle.

As an activity, the teacher may ask the students to construct trace copies of angles equal to ∠1, ∠2, ∠3, ∠4 and ∠5 of Fig. 4.3 and place them at a point in an order and observe that they form a complete angle.

3. Types of Quadrilaterals

The teacher may tell the students that the quadrilaterals they see normally in their surroundings are generally of special shape such as kite, the shape of sweet *burfi*, sheet of paper etc. The teacher may then ask students to draw several quadrilaterals on blackboard and through interaction, she can divide these into following categories.

(a) Trapezium – If a pair of opposite sides of a quadrilateral is parallel, it is called a trapezium. In Fig. 4.4, ABCD is a trapezium in which AD is parallel to BC. Recalling the properties of parallel lines, teacher may help students to conclude that $\angle A + \angle B = \angle C + \angle D = 180°$.

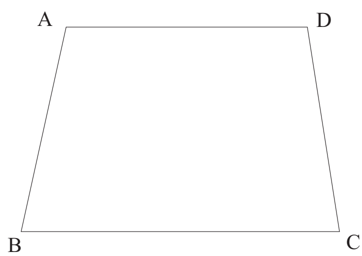


Fig. 4.4

(b) Kite – Kite is a quadrilateral in which two distinct consecutive pairs of sides are of equal length, i.e. ABCD will be a kite if either AB = AD and BC = CD [see fig. 5(i)] or AB = BC and CD = AD [See Fig. 4.5 (ii)].

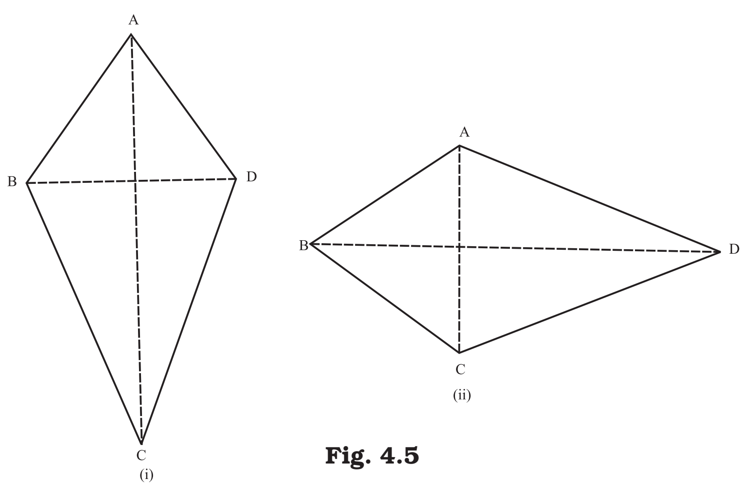


Fig. 4.5

The teacher may ask students to draw a kite on a paper, cut it out and by folding the cut-outs along the diagonals, they can observe that diagonals intersect at a right angle (see Fig. 4.5).

(c) Parallelogram – A quadrilateral in which both pairs of opposite sides are parallel is called a parallelogram. So ABCD will be a parallelogram if AB is parallel to CD and AD is parallel to BC (See Fig. 4.6).

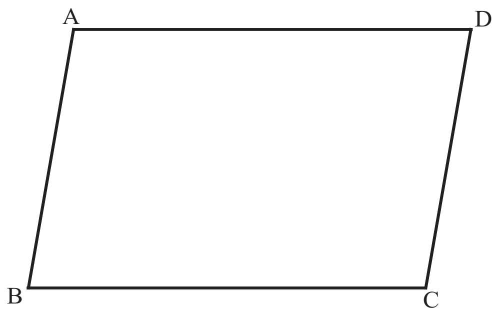


Fig. 4.6

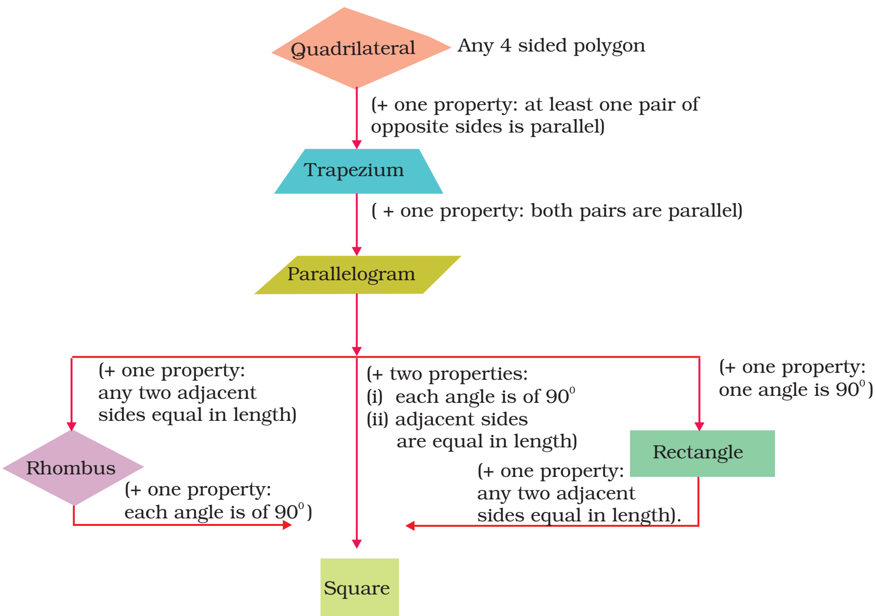
The teacher is expected to draw several figures of quadrilaterals on the blackboard and ask the students to identify trapeziums, kites, parallelograms and also those which are neither of these. It may also be explained to the students that a parallelogram is a special type of trapezium in which the other pair of opposite sides is also parallel.

Teacher can draw a chart/tree to summarise the types of quadrilaterals as follows:

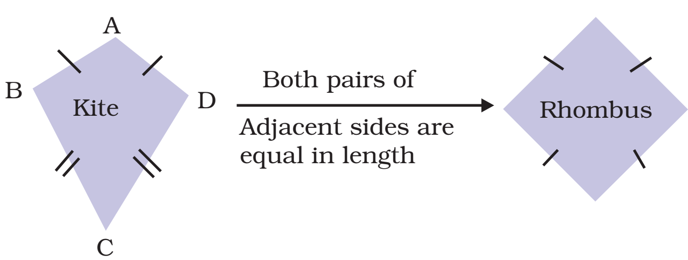
This tree is helpful in concluding the following:

- Every trapezium is quadrilateral, but not conversely.
- Every parallelogram is quadrilateral, not conversely.
- Every parallelogram is trapezium, not conversely.
- Every rhombus is quadrilateral, not conversely.
- Every rhombus is trapezium, not conversely.
- Every rhombus is parallelogram, not conversely.
- Every rectangle is quadrilateral, not conversely.

- Every rectangle is trapezium, not conversely.
- Every rectangle is parallelogram, not conversely.
- Every square is quadrilateral, not conversely.
- Every square is trapezium, not conversely.
- Every square is parallelogram, not conversely.
- Every square is rhombus, not conversely.
- Every square. is rectangle, not conversely.



For Kite and Rhombus



Thus,

Every rhombus is a kite, but not conversely.

4. Properties of a Parallelogram

The teacher may ask the students to draw a parallelogram (parallel lines can be drawn with the help of set squares or using the opposite edges of a ruler) and measure the lengths of its opposite sides. They will find that lengths of opposite sides are equal. The teacher may also give the logical argument as follows:

Let ABCD be a parallelogram. Join A to C (See Fig. 4.7).

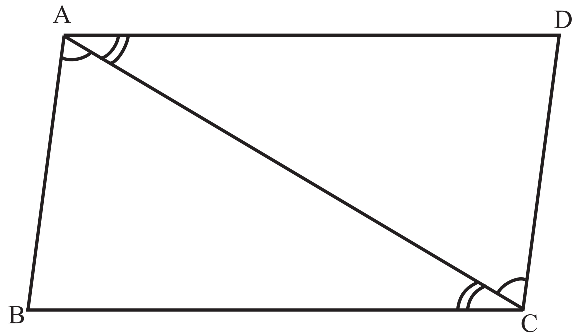


Fig. 4.7

Then in triangles ABC and CDA

$\angle BAC = \angle ACD$ (AB | |CD and AC is transversal)

$\angle ACB = \angle CAD$ (AD| |BC and AC is transversal)

AC = AC (common sides)

Therefore, $\Delta ABC \cong \Delta CDA$ (ASA criterion)

This shows that AD = CB and AB = CD (corresponding parts of congruent triangles)

Also, $\angle B = \angle D$ and $\angle BAC + \angle CAD = \angle ACD + \angle ACB$ which is same as $\angle A = \angle C$. These give that:

The opposite sides of a parallelogram are equal.

The opposite angles of a parallelogram are equal.

Also, $\angle BAD + \angle B = \angle BCD + \angle D = 180°$, because AD| |BC and transversals AB and CD cut them. Also, $\angle BAD + \angle D = \angle B + \angle BCD = 180°$.

Therefore, the adjacent angles of a parallelogram are supplementary.

From the result ΔABC ≅ ΔCDA obtained above, the teacher may apprise the students with the following important property.

Diagonal of a parallelogram divides it into two congruent triangles.

By paper folding it can be verified that opposite sides and opposite angles of a parallelogram are equal. Also by using a tracing paper, it can be verified that diagonal of the parallelogram divides it into two congruent triangles.

5. Property of Diagonals of a Parallelogram

Students may be asked to draw several parallelograms in their notebooks and also draw their diagonals. The two diagonals of a parallelogram intersect and divide each other in two parts. Let students measure the parts of each diagonal. What will they observe? The two parts of a diagonal are equal. This fact (property) should be logically justified as follows:

Let ABCD be a parallelogram in which diagonals AC and BD intersect at O (Fig. 4.8).

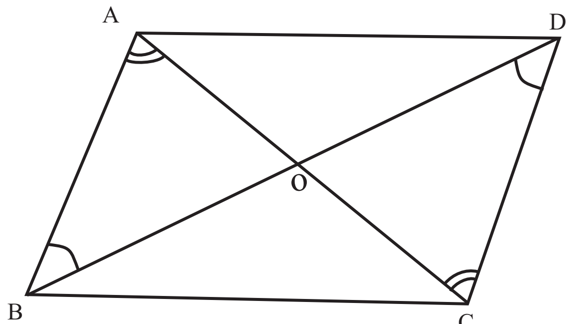


Fig. 4.8

Now, in triangles AOB and COD,

∠ABO = ∠ODC (AB||CD and BD is transversal)

∠BAO = ∠OCD (AB||CD and AC is transversal)

AB = CD (opposite sides of parallelogram)

Therefore, ΔAOB ≅ ΔCOD (ASA criterion)

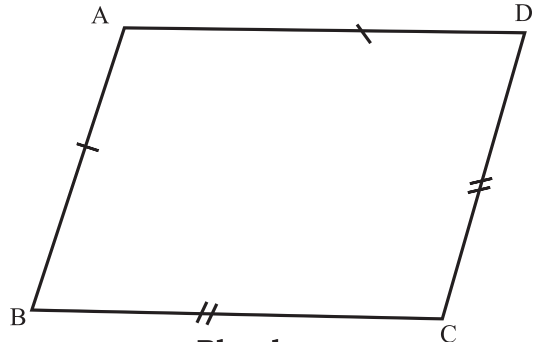
This gives AO = OC and BO = OD (corresponding parts of congruent triangles)

Thus,

The diagonals of a parallelogram bisect each other.

6. Special Parallelograms

(a) Rhombus: The teacher may draw a parallelogram on the blackboard in which two adjacent sides are equal. The students will find that all the sides become equal. Such a parallelogram is called a **rhombus**. Rhombus can also be treated as a kite because here AB = AD and BC = CD (Fig. 4.9).



Rhombus

Fig. 4.9

A rhombus is a parallelogram as well as a kite. Therefore, all the properties of a parallelogram and a kite will also hold for it. On the basis of their properties, you may say that in a rhombus,

(i) all sides are equal;

(ii) the diagonals bisect each other at right angles;

(iii) opposite angles are equal; and

(iv) each diagonal bisects opposite angles.

Note that diagonals of a rhombus may not be equal.

(b) Rectangle: The teacher may ask the students to draw a parallelogram in which one angle is a right angle. The students will find that all other angles will also be right angles (See Fig. 4.10).

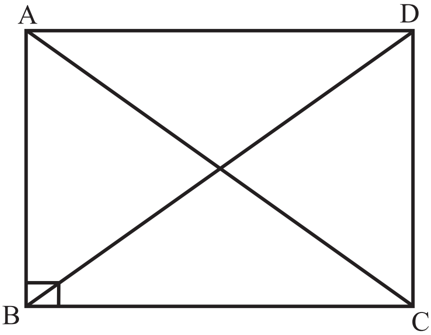


Fig. 4.10

Such a figure is called a **rectangle**. So a rectangle is a parallelogram in which all the angles are right angles.

Now ask the students to measure its diagonals AC and BD. They will find that they are equal. It can also be verified by using Pythagoras property as follows:

$AC^2 = AB^2 + BC^2 = AB^2 + AD^2$ (as BC = AD) $= BD^2$

Therefore, AC = BD

Thus,

The diagonals of a rectangle are equal.

Since rectangle is also a parallelogram, therefore in a rectangle,

(i) opposite sides are equal;

(ii) all angles are right angles; and

(iii) diagonals are equal and bisect each other.

(c) Square: A square is a rectangle in which all sides are equal. Thus in a square all sides are equal and all angles are equal. So, it is the only quadrilateral which is *regular*. A square is a rectangle and it is also a rhombus (See Fig. 4.11).

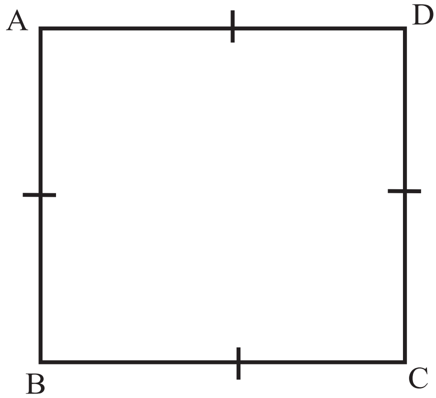


Fig. 4.11

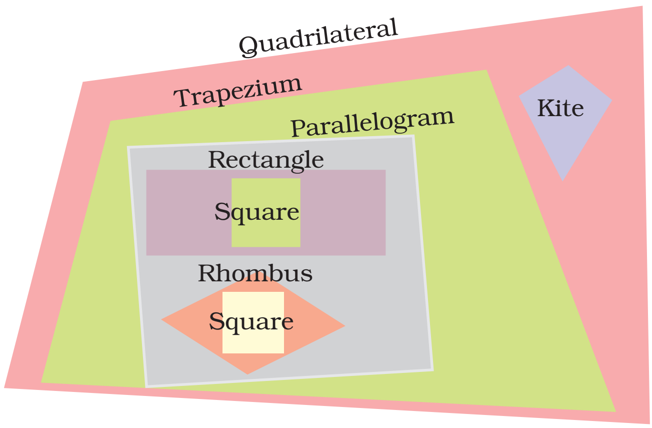
Therefore, a square has the following properties:

(i) All sides are equal.

(ii) All angles are equal.

(iii) Diagonals are equal and bisect each other at right angles.

Teacher may encourage the children to do Activity 7, 'Quadrilaterals and their properties' in the Mathematics Kit given with this package.



Teacher should draw the above diagram to explain the relation between various types of quadrilaterals. Every figure which is inside a figure also represents the outside figure.

For example, every square is a rhombus, since a square is drawn inside a rhombus. So a square carries all the properties of rhombus plus some additional properties. But every outside figure is not the inside figure. So, every kite is a quadrilateral, every rectangle is a parallelogram and so on.

Common Errors

1. Taking diagonals equal in a parallelogram.
2. Taking diagonals perpendicular in a parallelogram.
3. Taking non-parallel sides equal in a trapezium.
4. Taking diagonals perpendicular in a rectangle.

Teacher may evaluate students through the following exercise.

Exercise

1. If an interior angle of a regular polygon measures 165°, find the number of sides
2. If diagonals of a quadrilateral intersect at right angles and
   (i) If they bisect each other, then it can be a ....
   (ii) If they do not bisect each other, then it can be a ....
3. Which type of special parallelogram will it become if in a parallelogram ABCD
   (i) AB = BC (ii) $\angle ABC = 90°$
   (iii) $\angle ABC = 90°$ and AB = BC
4. If a diagonal of a rhombus is equal to its side, find all the angles of the rhombus.
5. Two adjacent sides of a parallelogram are in the ratio 1 : 2. If its perimeter is 27 cm, find its sides.

6. In Fig. 4.12, ABCD is a rhombus in which $\angle ACD = 30°$. Find all the angles of the rhombus.

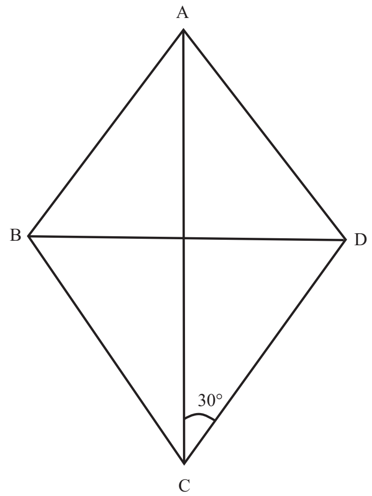


Fig. 4.12

7. In Fig. 4.13, ABC is an isosceles triangle with AB = AC and $\angle A = 50°$. If M, N are the mid-points of AB and AC respectively, find all the angles of the trapezium MBCN.

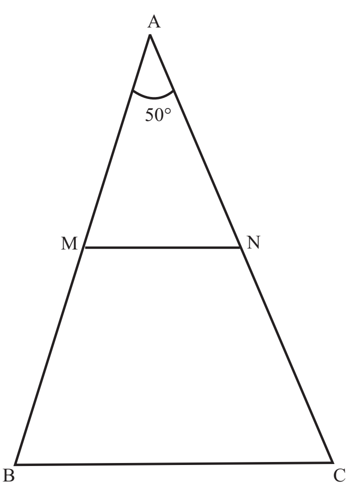


Fig. 4.13

8. The diagonals of a rhombus measure 16 cm and 30 cm. Find its perimeter.

UNIT 5

Symmetry

Structure

- Introduction
- Main Concepts and Sub-concepts
- Objectives
- Teaching Points
  1. Symmetric Figures
  2. Lines of Symmetry
  3. Reflection and Symmetry
  4. Lines of Symmetry for Regular Polygons
  5. Rotational Symmetry
- Common Errors
- Exercise

Introduction

Symmetry plays a very important role in many fields of work. Some examples are: making toys, idols, household goods, manufacturing many items, construction of buildings, etc. Symmetry in any item produces beauty. The students, while studying mathematics, come across a number of geometrical figures out of which some are symmetrical about some line, some are symmetrical about some point while others are not symmetrical. The symmetrical figures are easier to handle while studying their geometrical properties.

Strategies to Teach Transformations

The following types of transformations should be developed at upper primary stage:

a. **Translations** – a transformation that slides all points of a figure through the same distance in the same direction.

b. **Rotations** – a transformation that rotates all points of a figure about a point through an angle of x degrees.

c. **Reflections** – a transformation in which a line of reflection acts like a mirror reflecting all points of figure to their images.

d. **Dilations** – a transformation that expands or contracts a figure by k times, where k is the scale factor.

Teachers can use one of the following processes to teach transformations depending on the students' readiness:

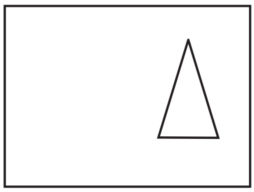
(i) Tracing paper

(ii) Using graphs

Tracing Paper

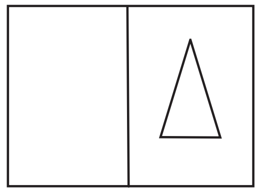
Students at the visualisation level focus on properties of the whole figure and can begin their exploration of transformations using tracing paper. For example, students can draw a figure and then use the paper to do the transformation as illustrated.

Step 1



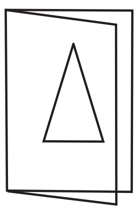
Draw a figure on tracing paper

Step 2



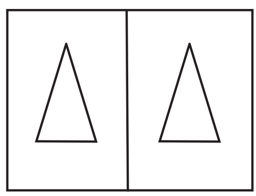
Fold the paper to form a line of reflection on the paper

Step 3



Trace the image on the folded paper

Step 4

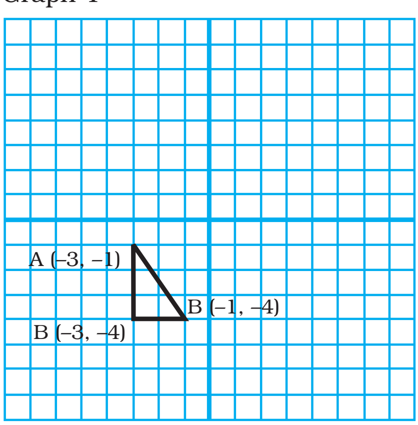


Open the paper to show both the images

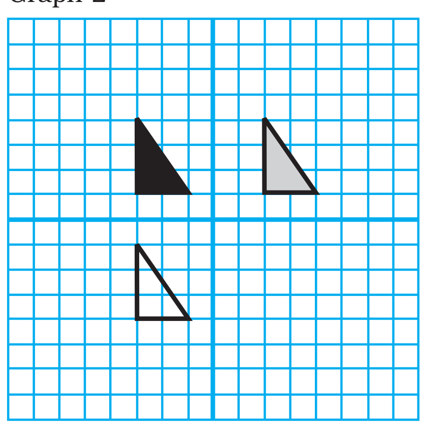
Using Graphs

To differentiate, teachers can use graphs to investigate the properties of transformations with students who demonstrate the ability to work with more than one concept at a time. An understanding of the plotting on graph papers is required for this type of exploration. For example,

Graph 1



Graph 2



Students can be given a figure as in Graph 1. Students are then asked to translate the figure by 5 units up and then 5 units to the right. Students must redraw the figure and name the new coordinates of the vertices of figures. In the above example the black triangle's vertices will be A (–3, 4), B (–3, 1) and C (–1, 1) and the vertices of the gray triangle will be A (2, 4), B (2, 1) and C (4, 1).

Main Concepts and Sub-concepts

- Symmetric figures
- Lines of symmetry
- Lines of symmetry of regular polygons
- Regular triangles, regular quadrilaterals
- Rotational symmetry
- Order of rotational symmetry

Objectives

After teaching these concepts and sub-concepts, the students can

- understand the concept of symmetry;
- find the difference between a mirror image of a figure and a figure congruent to that figure;
- understand line symmetry and lines of symmetry of a figure;
- complete a symmetric figure when its line of symmetry and its half part on one side is given; and
- understand rotational symmetry and its order.

Teaching Points

1. Symmetric Figures

The teacher may tell the students that in nature, creatures, leaves, flowers, moon, sun, etc. appear to be beautiful. What property is common in these? In human body, the legs, hands, eyes, ears which are in pairs are situated in a symmetric manner in the sense that they appear to be the mirror images of each other. Similarly the man-made most of the things, e.g. bulbs, tube lights, ceiling fans, wheels of vehicles, vehicles themselves, doors and windows of a house, chairs, tables, books, etc., are all symmetric figures. Some of these can be divided into two parts such that one part is the mirror image of the other (See Fig. 5.1). Teacher may make efforts to draw half part of a symmetrical figure on an edge of a cardboard sheet and she should take a long mirror along with it in the class. Let students see themselves that when they place the half figure in front of the mirror, they can see a complete figure (half part of cardboard and remaining half as the mirror image in the mirror). By actually placing the mirror in front of the children, teacher can not only create interest in lesson, but can really make them understand the word 'mirror image'. Teacher should make the students clear that line of symmetry is an imaginary line and it might not appear in the figure explicitly.

The students can be asked to collect some pictures of flowers, monuments, leaves, animals and draw their lines of symmetry.

Fig. 5.1

The teacher may make an ink-blot devil on a paper by folding it after putting an ink-blot on it and then telling the students that it is also a symmetric figure (See Fig. 5.2).

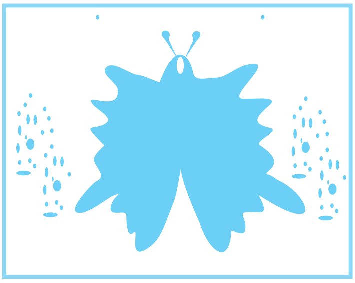


Fig. 5.2

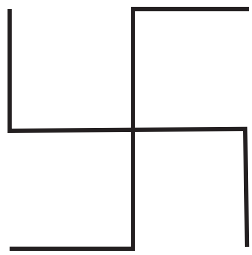


Fig. 5.3

The teacher may ask the students to locate the two parts of a table, a blackboard, a sheet of paper such that one is the mirror image of the other with respect to a fixed line.

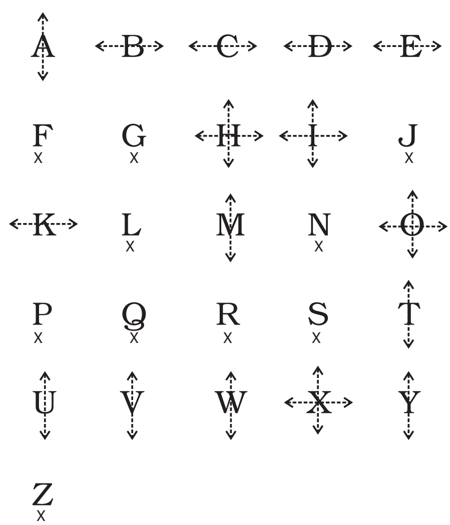
Teacher may tell the students that all the above figures are symmetrical about some line. They may also be told that there are some figures which are symmetrical about a point (See Fig. 5.3).

2. Lines of Symmetry

If there is a symmetric figure about a line, then it can be considered as a figure made up of two parts such that one is the mirror image of the other with respect to the line and the line is called the line of symmetry of the figure. In other words, the line along which if a mirror is placed then one part is the image of the other is the line of symmetry. For example, in case of ink-blot devil, the crease of fold is the line of symmetry. The teacher may ask the students to locate lines of symmetry in different symmetric figures.

Teacher can show symmetry line in all alphabets (wherever it exists).

Symmetry in Alphabets



Note: 1) Here O is drawn as oval.

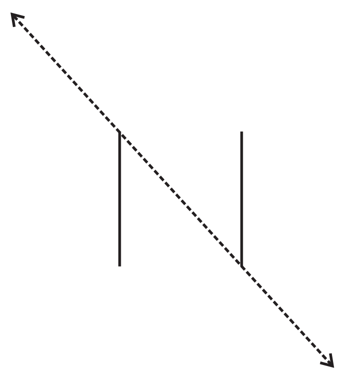
2) Here X is different from cross.

3) Students can be asked to name alphabets with more than one line of symmetry.

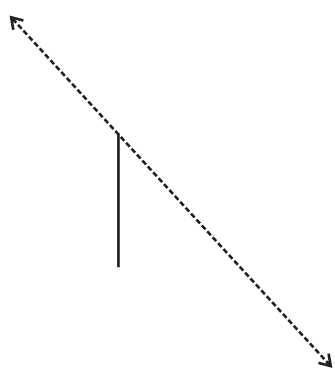
Letters F, G, J, L, etc. have no line of symmetry, i.e. these are not symmetrical about a line.

Teacher can make clarity about line of symmetry as follows:

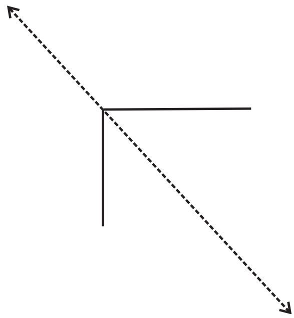
Suppose a student has a doubt saying ‘N’ has a line of symmetry along the diagonal. She should tell the student to draw this line of symmetry as:



She can then ask student to rub any one side of the line of symmetry as:



The students should now be asked to reflect on the left out on the other side of the line of symmetry, if the original shape returns then it is a line of symmetry, otherwise not.



Original shape does not return. So, it is not a line of symmetry.

3. Reflection and Symmetry

It is to be understood by students that mirror image of any figure is congruent to the given figure but if two figures are congruent, then they may not be the mirror images of each other. If any figure is placed in front of a mirror, then the figure with its image forms a symmetric figure and the mirror is the line of symmetry (see Fig. 5.4).

A figure can have several lines of symmetry. For example, a flower having six petals has six lines of symmetry (see Fig. 5.5). A rectangle has two lines of symmetry (see Fig. 5.6).

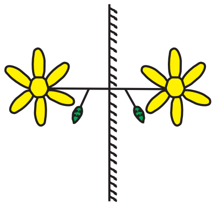


Fig. 5.4

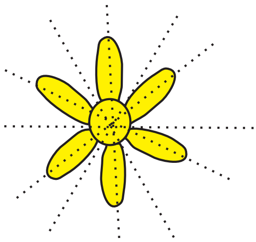


Fig. 5.5

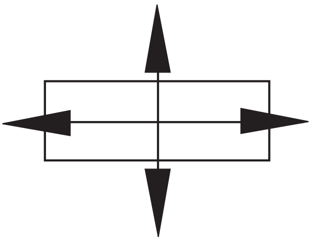


Fig. 5.6

4. Lines of Symmetry for Regular Polygons

The students know that an equilateral triangle is the regular triangle and a square is regular quadrilateral. Ask the students to draw an equilateral triangle and a square and try to find different lines of symmetry. Help them to find all of them by the process of paper folding (See Fig. 5.7).

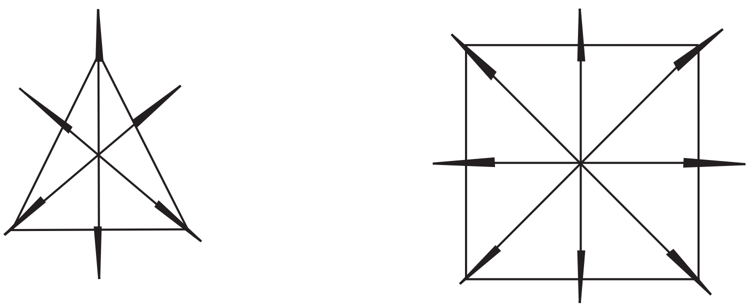
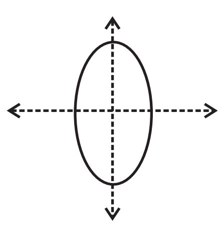
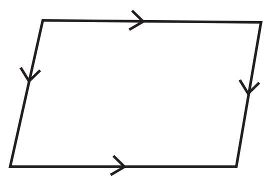


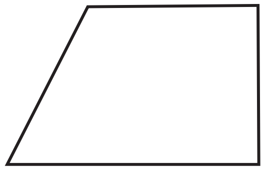
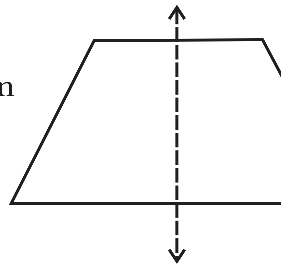
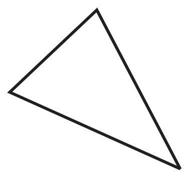
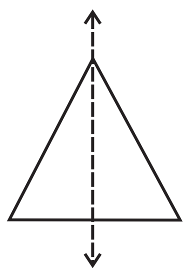
Fig. 5.7

In an equilateral triangle there are three and in a square there are four lines of symmetry. Similarly, they may be encouraged to explore that in a regular pentagon there are five, in a regular hexagon there will be six lines of symmetry. In general in a regular n-gon there will be n lines of symmetry.

Teacher may ask the students to draw all important geometrical shapes known to them other than regular figures.

		Line of Symmetry
1. Oval		2
2. Parallelogram		0

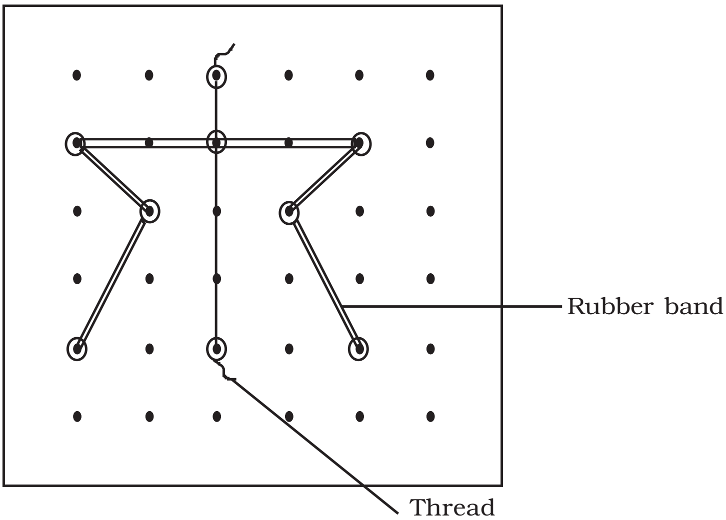
(A student can be explained this by same method as used for N earlier by rubbing and completing.)

3. Trapezium		0
4. Isosceles trapezium		1
5. Scalene Δ		0
6. Isosceles Δ		1

Activity

Take a Geoboard. Tie a streched thread in between two nails of the board as shown in the figure.

Ask the students to put streched rubber bands on both sides of the thread such that they make a symmetric shape along the thread.

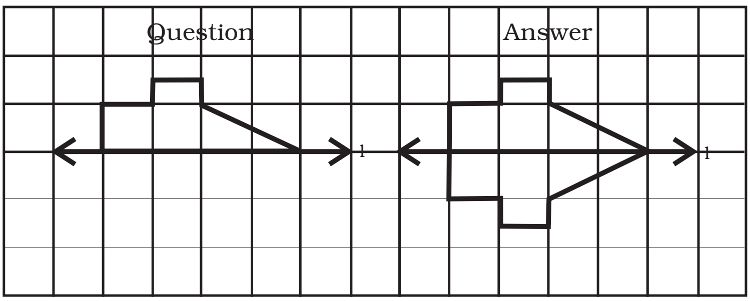


Geoboard

Similarly students can use dot paper or squared paper to make symmetrical figures about a line or a dot.

Completion of figure along the line of symmetry

Teacher may ask the students to follow the method of completion of figures along the line of symmetry on a graph paper as follows:



Note that doing this on a graph makes their understanding about size of figure easier.

5. Rotational Symmetry

The students must have seen many things which rotate about a fixed point. For example, ceiling fan, wheels of vehicles, windmills, etc. rotate about their centre called the **centre of rotation**. In full turn, the object comes to its original position. The angle by which it rotates is called the angle of rotation. In Fig. 5.8 when the point P comes at P′, ∠POP′ is the **angle of rotation**. Teacher may ask the students questions such as, What is the angle of rotation during (i) full turn, (ii) half turn? These will be 360 degrees and 180 degrees, respectively. Sometimes, when the object rotates, its position changes and after a rotation of some angle, new position coincides with its original position. This type of symmetry is called **rotational symmetry** for **that angle of rotation.**

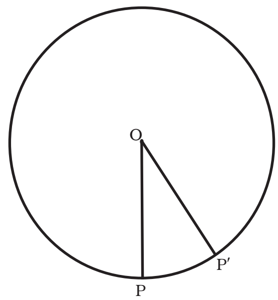


Fig. 5.8

The number of times an object does not change its shape (i.e. its new position coincides with its original position) in one complete turn is called the **order of rotational symmetry**. For example, if you rotate an equilateral triangle about its centre three times (at an angle of 120°, 240° and 360°) in one turn, it does not change its shape (See Fig. 9).

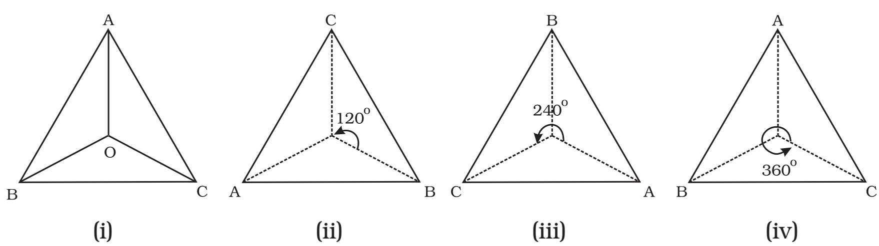


Fig. 5.9

Note: It can be seen that every figure has rotational symmetry of order 1 (i.e. angle of 360°).

Therefore, it has the order of symmetry three. Clearly, it has a rotational symmetry with angles of rotation as 120°, 240° and 360°. In Fig. 5.10, the order of rotational symmetry of the wheel is four (90°, 180°, 270° and 360°). The teacher may ask the students to think and draw figures whose orders of rotational symmetry are one, two, three and four, respectively.

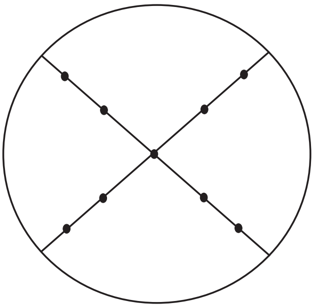
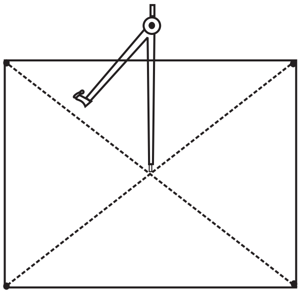


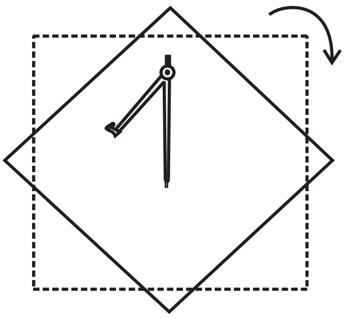
Fig. 5.10

The teacher may ask the students to check the order of rotational symmetry of a square by the following activity.

Cut a square cardboard. Put it on the table top. Draw its boundary with the help of chalk. Mark its centre by joining its diagonals. Put the pin of the compasses on its centre and press.



Now start rotating the square, clockwise or anti clockwise.



Now the students can observe that the square coincides with its initial position four times in one complete rotation.

Hence, its order of rotational symmetry is four.

Also ask the students to repeat the same activity for other plane figures such as triangle, rectangle, etc.

The teacher may highlight that there are many figures which have only line of symmetry (say, isosceles triangle), there are certain figures having only rotational symmetry (say, parallelogram) and there are certain figures which have both types of symmetries (say rectangle, square, circle).

It may also be highlighted that a circle has infinitely many lines of symmetry and it also has rotational symmetry of infinite order.

Common Errors

1. It appears that 卍 has two lines of symmetry, but, in fact there is no line of symmetry, though it has rotational symmetry.

Teacher may evaluate the students through the following exercise:

Exercise

1. How many lines of symmetry are there in the following?

   a. A kite b. A parallelogram

   c. A rhombus d. A rectangle

   e. A square f. A circle

2. Find the lines of symmetry in Fig. 5.11.

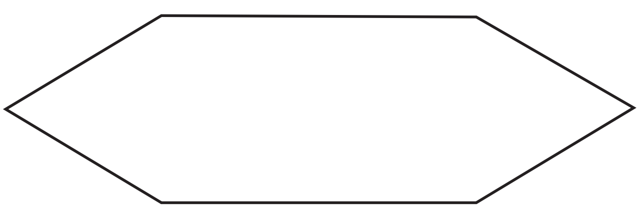


Fig. 5.11

3. What is the order of rotational symmetry of a scalene triangle?

4. What is the order of rotational symmetry of a rectangle?

5. Find the order of rotational symmetry of the wheel given in Fig. 5.12.

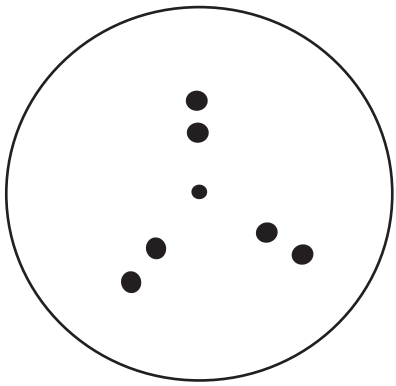


Fig. 5.12

UNIT 6

Visualising Solid Shapes

Structure

- Introduction
- Main Concepts and Sub-concepts
- Objectives
- Teaching Points
    1. 2D and 3D shapes
    2. Polyhedrons
    3. Euler's formula
    4. Viewing 3D objects from different positions
    5. Cross sections and shadows of 3D objects
    6. Drawing 3D shapes in 2D surfaces
    7. Nets of some 3D shapes
    8. Maps
- Common Errors
- Exercise

Introduction

So far, students have learnt only plane figures, i.e. figures which lie wholly in a plane. They are called two dimensional figures or simply 2D figures. However, in practical life, most of the objects we come across do not lie wholly in a plane. For example, objects like a ball, a box, a tumbler, etc. do not lie wholly in a plane. They are called **three dimensional objects** or briefly 3D objects. In mathematical sense, a ball is termed as a sphere, a box is termed as a cuboid and so on. In general, the geometrical representation of each of these 3D objects is called a 3D figure or a 3D shape. Due to their practical importance, it is essential to visualise these 3D shapes (solid shapes) in reference to some known 2D figures. In this unit, students may be exposed to various ways in which 3D shapes can be visualised. Through this visualisation, students may also be exposed to ideas like faces, edges and vertices of a polyhedron, Euler's formula, nets of solids and so on.

Main Concepts and Sub-concepts

- 2D and 3D shapes
- Common 3D shapes such as cuboid, cube, cylinder, cone, sphere, prism and pyramid
- Polyhedron as a special 3D shape
- Faces, edges and vertices of a polyhedron
- Euler's formula
- Nets of various 3D shapes
- Drawing of 3D shapes on a flat (2D) surface
- Different views of a 3D shape (top, side and front)
- Cross sections and shadows of some 3D shapes
- Drawing and interpretation of maps in the surroundings

Objectives

After teaching this unit, students can

- classify the given objects or figures as 2D or 3D objects or figures;
- recognise some simple solid shapes such as cuboid, cube, cylinder, cone, sphere, prism, pyramid, etc.;
- give examples of some 3D shapes from environment;
- identify polyhedrons from a given collection of 3D shapes;
- distinguish convex and non-convex polyhedrons;
- find the number of edges, faces and vertices of a polyhedron and verify the Euler's formula;
- identify front, top and side views of a 3D object from a given collection;
- identify cross-sections and shadows of different 3D objects;
- draw a sketch of 3D shape on a 2D surface or plane;
- interpret some simple maps of the surroundings; and
- draw maps in simple situations without scale.

Strategies to Teach Geometric Solids

In teaching geometric solids to upper primary students, teachers and students will benefit by using the following phases of learning.

a. Inquiry/information

b. Directed Orientation

c. Explication

d. Free Orientation

e. Integration

The phases and related activities follow.

a. Inquiry/information

The purpose at this stage is to assess students' prior knowledge to guide the instruction of the unit of study. To begin this phase, the teacher can ask the following types of questions:

- What is a cube?

- What is a cone?
- What is a prism?
- What is a sphere?
- How are they alike and how are they different?

b. Directed Orientation

During the directed orientation stage, the teacher can lead guided explorations of the solids. Activities may include the following:

- Gather different household items that resemble a cone, cube, sphere, cylinder, pyramid, and prism. Let the students sort them out by various teacher-described attributes.
- Have students trace each side of the solid to determine its net (a two-dimensional figure when folded forms the surface of a three-dimensional object). Students could then construct the object from the net.
- Using a flashlight, cast a shadow of each face of the solid onto the wall to help determine its net.
- Have the students use diagrams and pictures of different solids to analyse similarities and differences between them.
- Create rough sketch plot by looking at each solid and determine the number of edges, surfaces, or vertices for each.

c. Explication

In the third phase, students write and discuss the observations from the second phase. Students express their ideas about the solids. Examples of such activities include:

- Write a paragraph describing each of the solids; include diagrams with important parts labelled.
- Use nets for observing relationships between the different types of solids.
- Discuss the similarities and dissimilarities between the solids.

d. Free Orientation

In the next phase, students learn about the solids by performing more complex tasks. These tasks will be of multi-steps and will require a higher level of thinking than in directed orientation.

Examples include:

- Fill hollow solids with rice, sand, or water and explore properties of volume.
- Cut apart cardboard solids such as small carton boxes to explore the surface area of solids.

e. Integration

In the final phase, students do not learn any new material. Instead they review and summarise the work done in this unit through activities such as the following:

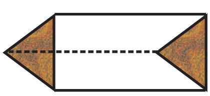
- Create a poster describing the properties of the solids.
- Demonstrate understanding of the solids through a brochure developed to teach their peers.

Teaching Points

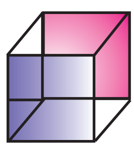
1. 2D AND 3D SHAPES

Students are familiar with a number of 2D shapes such as a rectangle, a square, and 3D shapes such as ball as a sphere, box as a cuboid from primary classes. Teachers may ask the students to recall these shapes through some examples as given in Class VI and VII Mathematics textbooks. It may be explained that a 2D shape (square, triangle) lies wholly in a plane, while a 3D shape (like a sphere) does not lie wholly in a plane. The teacher may encourage the students to make some of the 3D shapes, such as sphere (for a ball), cuboid (for a box), cylinder (for a drum), and so on. For explaining the terms like cuboid, cube, cylinder, cone, sphere, prism and pyramid, their wooden or plastic models may be displayed in the classroom. This topic has been discussed in Classes VI, VII and VIII. The teacher may also discuss these shapes in a phased manner, starting from simple to difficult cases. For example, in the beginning the teacher may confine to single shapes and gradually come to combination of different shapes (see pages 108 of Class VI, Fig. 15.1 of page 277 of Class VII, and page 153 (Do This) of Class VIII Mathematics, NCERT). Teacher may also help the students to understand the terms like surface, face, edge, base, lateral surface, lateral face or side face (wherever possible) for 3D shapes. It may be pointed out that some 3D shapes have flat surfaces (cuboid, cube), some have only curved surfaces (sphere) and some have curved as well as flat surfaces (cone, cylinder). Students may be encouraged to give examples of each type of such 3D objects from their environment. Finally, they may be told that

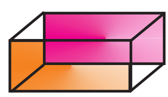
(i) Base and top of a prism are congruent polygons and its side faces are parallelograms. It may also be pointed out that usually we consider prisms whose side faces are rectangles. Such prisms are called right prisms. It may also be pointed out that a cuboid or a cube is also a type of right prism, cuboid as rectangular prism, cube as square prism. Teacher may tell students that prism is named in accordance with base (or top), e.g.



Triangular prism



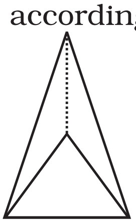
Square prism



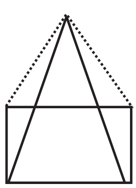
Rectangular prism

Fig. 6.1

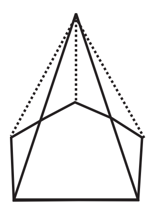
(ii) Base of a pyramid can be any polygon and its side faces are triangles. Likewise a pyramid is also named according to its base, e.g.



Triangular pyramid



Square pyramid



Pentagonal pyramid

Fig. 6.2

Teacher should display edges, faces, vertices in their models.

(iii) A right circular cylinder can be considered as a special form of a right prism, when the base and top become two congruent circles. Similarly, right cone can be considered as a special form of pyramid when the base becomes a circle.

(iv) It may also be explained to the students that just as in a 2D shape like a rectangle (or a triangle), it is said that a rectangle consists of only its sides, a 3D shape consists of height or thickness along with its sides of the base. A cuboid has six rectangular surfaces or faces with height or thickness.

2. Polyhedrons

From the above discussions, students may be encouraged to point out some 3D shapes which have only flat surfaces or which have curved as well as flat surfaces both. From this information, it may be brought home to the students that 3D shapes which are made up of only polygonal regions are called **Polyhedrons.** Clearly, a cylinder is not a polyhedron, while a pyramid is a Polyhedron. The teacher may give some collection of 3D shapes (as given on page 164 of Class VIII Mathematics, NCERT) and ask them to identify polyhedrons and non-polyhedrons from this collection.

Discussion of convex and non-convex polyhedrons may also be done by taking the figures given on the same page 164 of Class VIII Mathematics, NCERT. Here, the teacher may point out that in a non-convex polyhedron, polygons involved in the polygonal regions may not be all convex polygons. Some may be convex and some may be non-convex.

Regular Polyhedron: Such shapes may be explained as given on pages 164 and 165 of Class VIII Mathematics, NCERT. It may be pointed out that a cube is a regular polyhedron, while a cuboid is not a regular polyhedron. A regular tetrahedron is a 3D figure whose all the four faces are congruent regular triangles, i.e. equilateral triangles (in fact, triangular regions).

It is interesting to note that there are only five possible regular polyhedrons. They are:

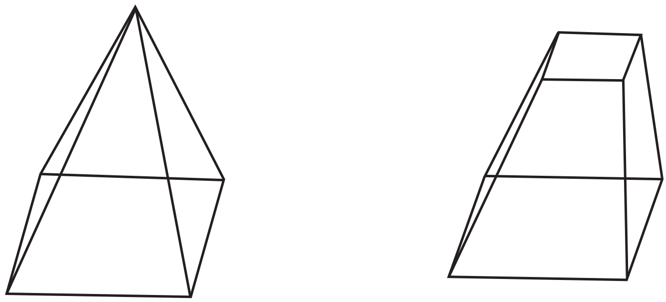
1. A regular tetrahedron (All faces are congruent equilateral triangles)
2. A regular hexahedron or a cube (All the faces are congruent squares)
3. A regular octahedron (All the eight faces are congruent equilateral triangles)
4. A regular dodecahedron (All the twelve faces are congruent regular pentagons)
5. A regular icosahedron (All the twenty faces are congruent equilateral triangles)

It may also be stated by the teacher that these five solids are known as **Platonic solids**. However, this must be limited for the purpose of information and students should not be expected to answer questions based on these shapes.

3. Euler's Formula

Students have been exposed to the idea of faces, edges and vertices of different types of polyhedrons such as cuboid, cube, pyramid, prism, etc. in Class VI. Discussion has been continued in Classes VII and VIII also. Finally, students have been exposed to the Euler's formula, $F + V = E + 2$, where F, V and E are respectively the number of faces, vertices and edges of any polyhedron in Class VIII (Pages 165 and 166 of Class VIII, Mathematics, NCERT).

The teacher may adopt the same strategy, i.e. the Euler's formula must be introduced in a phased manner, starting with Class VI and giving the name Euler's formula in Class VIII only. The teacher should show some polyhedrons to the students and ask the students to verify the Euler's formula (Fig. 6.3). The teacher may help the students to count the faces in Figure 6.3(i), $F = 5$, $V = 5$ and $E = 8$ and in Fig. 6.3(ii), $F = 6$, $V = 8$ and $E = 12$



(i) (ii)

Fig. 6.3

In (i) F + V = 5 + 5 = 10 and E + 2 = 8 + 2 = 10

In (ii) F + V = 6 + 8 = 14 and E + 2 = 12 + 2 = 14

Thus, Euler's formula is verified in both the cases.

4. Viewing 3D Objects from Different positions

The teacher may tell the students that one of the ways of visualising solid shapes is to look at them from different positions (angles). Students may be exposed to different solid objects and look at them from different angles. Then they may be asked to observe their 'Front view', 'Side view' and 'Top view'. They need not draw their actual views, but they should only be asked to match the given views as done in Chapter 15 (Class VII) and Chapter 10 (Class VIII) of Mathematics, NCERT. Students may also be encouraged to build different solid shapes using unit cubes given in Mathematics and then draw their front views, side views and top views.

5. Cross-sections and Shadows of 3D Objects

The teacher may encourage the students to suggest some more ways of visualising the 3D objects. This will lead them to the idea of slicing or cutting the solid and observing its cross section as discussed in section 15.5.1 of Class VII, Mathematics, NCERT. Let them cut the solid in different ways (say horizontally, vertically or inclined position) and observe their cross sections accordingly. Let them also enjoy the cutting of different vegetables (potatoes, lady fingers, etc.) of different shapes and observe their cross-sections, observe the shadows of some 3D objects through different sources say sunlight, projector and so on. The teacher may explain that observing shadows is another way of visualising solid shapes and it has been discussed in Section 15.2.2 of Class VII, Mathematics, NCERT. Now the students may be asked to attempt questions on Exercise 15.4 of Class VII, Mathematics, NCERT. At this stage, the teacher may point out that observing cross sections and shadows do not give a real picture about the solids, because the answers in many cases will not be unique. They are good for fun activities only.

6. Drawing 3D Shapes in 2D Surfaces

The teacher may explain in the beginning that it is not possible to draw an exact sketch of a 3D object on a 2D surface such as on a blackboard or on a notebook. However, these sketches are drawn by observing certain conventions. It is natural that whole of the solid cannot be shown on the blackboard or notebook. In general, its hidden faces or portions are shown by dotted lines or curves. These may be explained as discussed in Section 15.4 of Class VII, Mathematics, NCERT and pages 154 and 155 of Class VIII, Mathematics, NCERT.

In Class VII, drawings of single solids have been considered, while in Class VIII, drawings of combination of two solids have been considered. The teacher may explain the following two types of sketchings as discussed in Chapter 15 of Class VII Mathematics, NCERT.

1. Oblique sketches
2. Isometric sketches

The teacher may point out that for isometric sketches, isometric dot papers have been used, in which equilateral triangles are formed by joining the dots, while in oblique sketches, square dot papers are used. However, it may be made clear to the students that this method will not work in the case of drawing or sketching a cylinder, cone, etc. Further, it may also be stated that for an oblique sketch, there is no need of a square dot paper. For example, Fig. 15.11 on page 282 of Class VII Mathematics, NCERT may be considered as an oblique sketch of a cuboid or a cube.

A cylinder and a cone can be drawn as follows (Fig. 6.4):

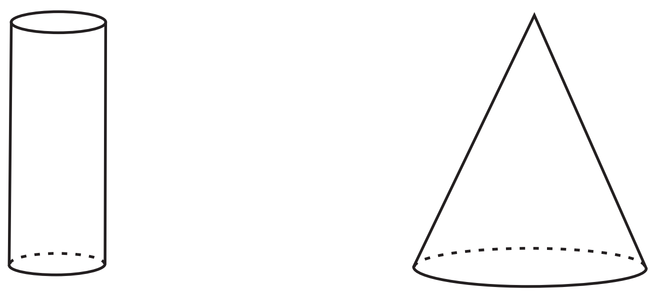


Fig. 6.4

7. Nets of some 3D Shapes

Ideas of nets of 3D shapes may be explained through activities as suggested on pages 279 and 280 (Chapter 15) of Class VII, Mathematics, NCERT. Then the students may be asked to attempt Questions 1 and 2 of Exercise 15.1 of Class VII, Mathematics, NCERT. For this, students may be advised to take the enlarged versions of figures given in the book and then decide which of these are nets and which of these are not nets for a cube. The students may be asked to do a similar exercise for nets of a cuboid.

In this regard, some items given in the Mathematics Kit, Activity 12 'Nets of solid shapes' may also be used. It is suggested that for nets of cylinder and cone only matching type of questions may be asked from the students (see Question 3 of Exercise 15.1).

8. MAPS

An elementary idea of reading and drawing of maps has been discussed in Section 10.3 (Chapter 10) of Class VIII, Mathematics, NCERT. Students have learnt these ideas in Primary Classes also. The teacher may explain these ideas as given on pages 160 to 163 of Class VIII, Mathematics, NCERT. For better understanding about the maps, students may be given some maps drawn to a clear cut scale and then they may be asked to locate different places in the map along with the distances between some pairs of places.

As an activity, students may also be advised to note down certain landmarks while going from their home to school or from home to some other place and draw maps depicting these landmarks. However, for this purpose, they need not use any scale. They may use different colours for depicting different places.

Common Errors

(i) Some students may not be able to distinguish between a general solid and a polyhedron.

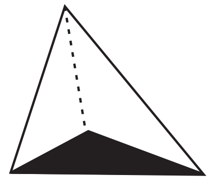
(ii) Students may not be able to count the number of faces, edges and vertices of a polyhedron correctly. Sometimes students count only horizontal lines in oblique sketch for edges and many times only vertical lines.

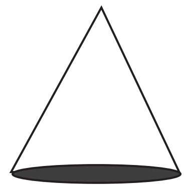
(iii) Some students write the Euler's formula as E + V – F = 2, which is not correct.

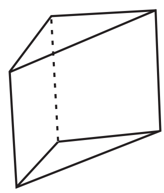
You may evaluate the students through the following exercise:

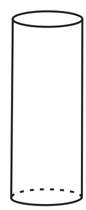
Exercise

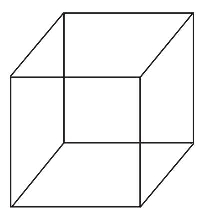
1. Match the following

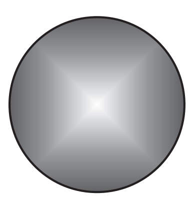
(i) Cuboid — (a) 

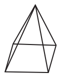
(ii) Sphere — (b) 

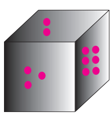
(iii) Triangular prism — (c) 

(iv) Pyramid — (d) 

(v) Cylinder — (e) 

(vi) Cone — (f) 

2. Find the number of faces, edges and vertices of square pyramid  and verify the Euler's formula for it.
3. Is cuboid a regular hexahedron? Why or why not?
4. Is  a net of a cone? Why or why not?
5. Draw an isometric as well as an oblique sketch of a cuboid of dimensions 5 cm × 4 cm × 3 cm.
6. Draw a front view, top view and side view of the solid.



7. Draw a map of the path from your school to your home giving important landmarks without scales.

UNIT 7

Practical Geometry

Structure

- Introduction
- Main Concept and Sub-concepts
- Objectives
- Teaching Points
    1. Basic constructions
    2. Construction of triangles
    3. Construction of quadrilaterals
    4. Construction of some special quadrilaterals
- Common Errors
- Exercise

Introduction

By now, the students have become familiar with various geometrical shapes such as point, line segment, angle, triangle, quadrilateral, circle, etc. They have also studied these geometric shapes by drawing their rough sketches. In this unit, we will discuss various methods of constructing these shapes with given measurements, using geometrical instruments, such as protractor, compasses, divider, ruler and set squares.

Main Concepts and Sub-concepts

- Basic constructions such as construction of a line segment, circle, perpendicular to a line through a point, perpendicular bisector of a line segment, angle of a given measure, a copy of a given angle, bisector of a given angle, Angles of 60° and 90°, the line parallel to a given line through a given point.
- Construction of triangles when
  - (i) three sides are given;
  - (ii) two sides and included angle are given;
  - (iii) two angles and included side are given; and
  - (iv) hypotenuse and a leg of a right angled triangle are given.
- Construction of quadrilaterals when
  - (i) four sides and a diagonal are given;
  - (ii) both diagonals and three sides are given;
  - (iii) two adjacent sides and three angles are given;
  - (iv) three sides and two included angles are given.
- Construction of a parallelogram, rectangle, rhombus and square.

Objectives

After teaching these concepts and sub concepts, the students can

- construct perpendicular to a line through a point. The point may be on the line or outside the line;
- construct perpendicular bisector of a line segment;
- construct a copy of the given angle;
- construct bisector of a given angle;
- construct angles of 60°, 90°, 30°, 120°, 45°, 135° using protractor and compasses;
- construct a line parallel to a given line through a point;
- construct triangles under given measurements;
- construct quadrilaterals under given measurements; and
- decide least number of measurements required to construct a geometrical figure such as a triangle and a quadrilateral.

Teaching Points

1. Basic Constructions

Instructions for construction

Teachers should emphasise on the correct use of geometrical instruments. Also, while doing constructions, instruct children to keep their notebook flat on the table and ask them not to have anything under their notebooks. It is extremely important for the teacher to use geometric kit on blackboard while teaching constructions. This will give a clear understanding to their students.

Students initially may not be comfortable about using the compasses. They should be encouraged to draw beautiful circle patterns of any size. This would help them in using compasses with accuracy. They can even colour those patterns, so that they enjoy beauty of geometry.

For example:

Following constructions except part (h) are from class VI; part (h) is from class VII.

a. A line segment of a given length and a circle of given radius: The students know that a line segment is a part of a line (straight), having two end points and its length is fixed. Help the students to draw a line segment of 5 cm. Let them draw a line segment AX of any length (more than 5 cm) with the help of a ruler and a pencil. Use graduated ruler and open the legs of compasses such that the distance between the pointed needle and the tip of the pencil is 5 cm. Putting its pointed leg on point A, mark a point B using pencil end. Then, AB is the required line segment (See Fig. 7.1).

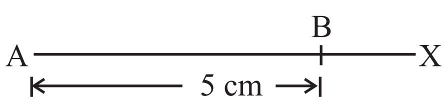


Fig. 7.1

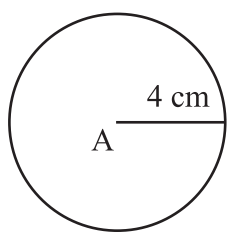


Fig. 7.2

To draw a circle of radius, say 4 cm with centre A, help the students take the compasses fitted with a pencil and measure 4 cm on the graduated ruler. Putting the pointer of the compasses at A, help them to draw the circle gently by the pencil fitted in it by making a full turn about the point A (See Fig. 7.2).

b. Perpendicular to a line through a point

i. ***When the point is on the line*** **:** The teacher may tell the students that drawing of perpendicular to a line through a point can be done by using a set square and a ruler and also by using ruler and compasses.

(x) *Using ruler and set squares* : Let a point A be given on a line *l*. To draw perpendicular to *l* through A, just put the ruler along the line and a set square resting on the edge of the ruler. Slide the set square till the lower end of the vertical straight edge falls on A. Draw a line segment BA along this edge using a pencil. This line segment BA is the required perpendicular (See Fig. 7.3).

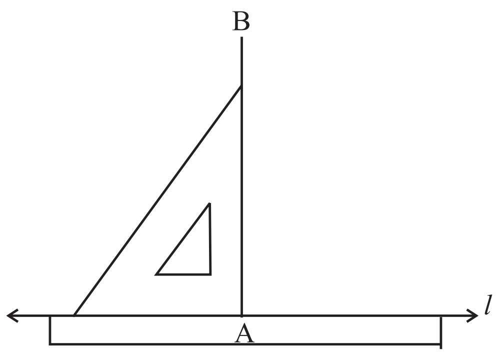
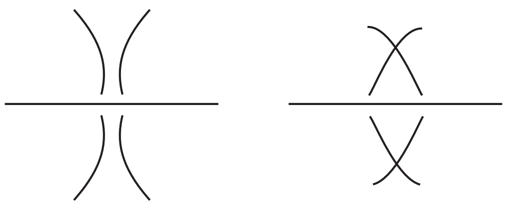


Fig. 7.3

(y) *Using ruler and compasses* : Let A be the given point on the line *l*. Taking A as centre, help the students to draw an arc (of any radius) cutting *l* at, say B and C. Then, taking centre as B draw an arc of any radius which is more than the length of half of length BC and then taking C as centre and with the same radius, draw another arc intersecting the former arc at, say D. Join AD. Then, AD is the required perpendicular [See Fig. 7.4(ii)].

Here at least once student should be provided few examples justifying the need of taking radius more than half of BC and also meaning of intersection of arcs as follows:

Intersecting arcs   Non-intersecting arcs



(If radius is less than half of BC, arcs do not intersect.)   (If radius is more than half of BC, arcs intersect.)

Fig. 7.4(i)

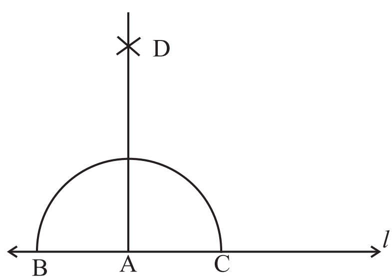


Fig. 7.4(ii)

The teacher should tell the students that the method (y) is actually a geometrical construction while method (x) is just drawing and not a geometrical construction. She may also tell that in a geometrical construction only a ruler and compasses should be used and not a set square or protractor, as far as possible. She should stress on constructions using ruler and compasses only.

(ii) When the point is not on the line : Let l be a line and A be a point lying outside l. Taking A as centre and any radius, help the students to draw an arc so that it cuts the line l at two points, say B and C. Teacher should make it clear that the radius of the arc should be such that the arc cuts the line l in two distinct point B and C. Then, taking B as centre and a radius, greater than half of BC draw an arc on that side of the line which is opposite to A and taking C as centre and with the same radius, draw another arc intersecting the previous arc at the point, say D. Join AD. Then AD is the required perpendicular (See Fig. 7.5).

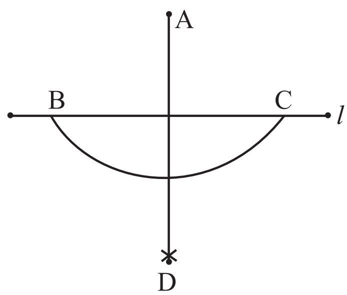


Fig. 7.5

(c) Perpendicular bisector of a line segment: Let a line segment AB be given and you have to construct its perpendicular bisector. Taking A as centre and any radius (more than half of AB), draw two arcs on both sides of AB. Then taking B as centre and the same radius, draw two arcs on both sides of AB so that they cut the earlier drawn arcs at, say points C and D. Join CD. Then, CD is the required perpendicular bisector of AB (See Fig. 7.6).

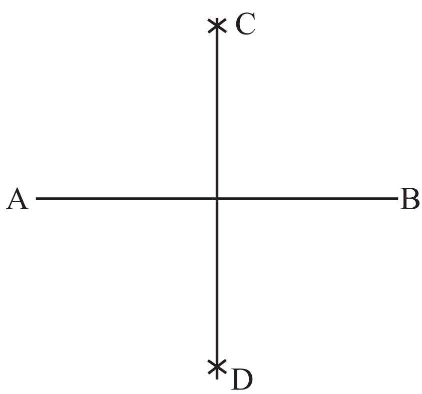


Fig. 7.6

(d) Angle of a given measure: Protractor is provided in the geometry box for drawing an angle of any measure. The teacher should explain the use of protractor in drawing an angle of any measure as given in section 14.5 of Class VI, Mathematics Textbook, NCERT. Students often get confused about which scale to use while reading a protractor. For this definition of angle could be made clear by using terms as initial ray and final ray. After drawing initial ray they should be made familiar with 0° angle present for ray in a specified direction and so final ray could be visualised after moving pencil over it.

For example:

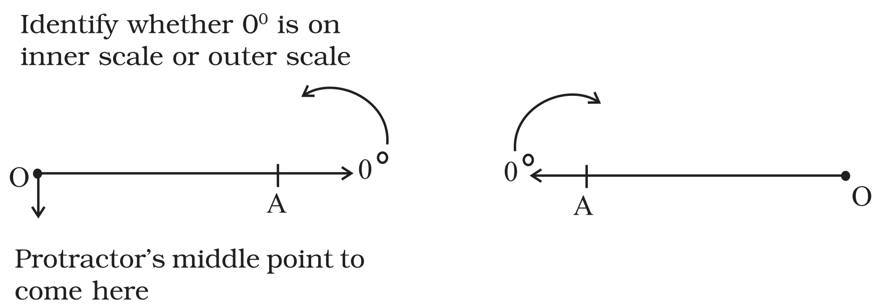


Fig. 7.7(i)

(e) A copy of a given angle : Let ∠AOB be given and you have to construct an angle which is congruent (equal) to it at the point O´ on line segment O´A´. Taking O as centre help the students to draw an arc XY with any convenient radius cutting arms OA and OB at X and Y respectively. Taking O´ as centre and same radius, draw an arc cutting line segments O´ A´ at X´. Now taking X´ as centre and XY as radius, help

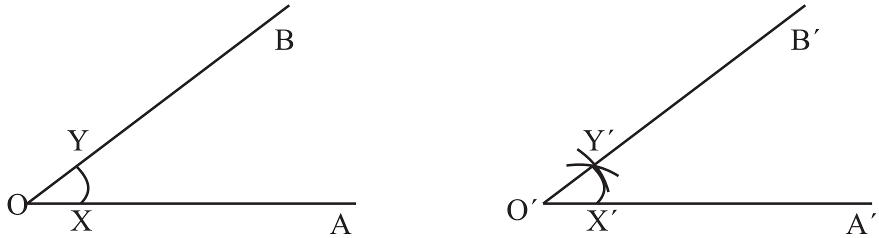


Fig. 7.7(ii)

the students to draw an arc intersecting the previously drawn arc at, say, Y´. Join O´ Y´ and produce it, say up to B´. Then ∠A´O´B´ is a copy of the angle AOB [See Fig. 7.7(ii)].

(f) Bisector of a given angle: Let an angle AOB be given and you have to bisect it. Taking O as centre draw an arc of any radius interesting OA at X and OB at Y. Then, taking X and Y as centres and with radius more than half of XY, draw arcs to intersect at a point, say C. Join OC. Then, OC is the required bisector of ∠AOB (See Fig. 7.8). Teacher should encourage students to draw an arc of bigger radius initially (when drawing from O), so that bisection of angle is easier.

Teacher may provide sufficient practice to the students for bisecting different types of angles (acute, right and obtuse).

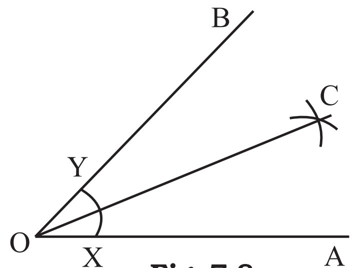


Fig. 7.8

(g) Angles of 60° and 90°: To construct an angle of 60° ask the students to draw a ray OA. Taking O as centre and any radius, draw an arc XY intersecting OA at X. Then, taking X as centre and same radius, draw an arc cutting the first arc at the point, say C. Joint OC. Then ∠COA = 60° (See Fig. 7.9).

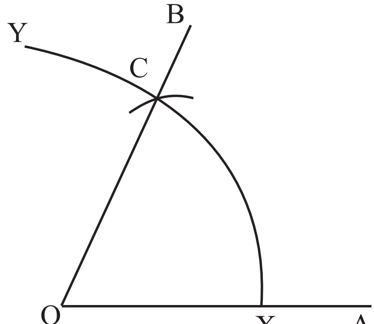


Fig. 7.9

The teacher may tell the students that an angle of 120° can be made by making an angle of 60°, say ∠DOC at line segment OC with O as centre so that ∠DOA will be 120°. Here if necessary, teacher may give justification that by joining X and C an equilateral triangle OXC is formed. So each angle of this triangle is 60°.

Next let OA be a ray and you have to construct an angle of 90° at O. Taking O as centre and any radius, draw an arc XY intersecting OA at X. With the same radius and centre X, cut the arc XY at C and then taking C as centre and the same radius, draw another arc to intersect arc CY at D. Now taking C and D as centres and radius more than $\frac{1}{2}$ CD, draw arcs to intersect each other at the point E. Join OE. Then, ∠EOA = 90° (Note that ∠DOA = 120°). (See Fig. 7.10).

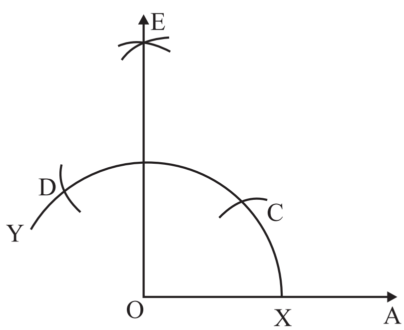


Fig. 7.10

The teacher may tell the students that for constructing an angle of 30°, they must first construct an angle of 60° and then bisect this angle. Similarly bisecting an angle of 90°, you can construct an angle of 45°. Also, since 135° = 180° – 45° or 90° + 45°, you can construct an angle of 135° also by using compasses and ruler.

(h) The line parallel to a given line through a given point: Let a line AB and a point P outside it be given and you have to draw a line through P parallel to AB. Join the point P to any point Q on AB. Make ∠QPR equal to ∠PQB, using the method given in (e). Then the line CD is the required parallel line (See Fig. 7.11).

Here, teacher may recall the properties of parallel lines and properties of angles formed by a transversal and set of parallel lines and may give reasoning of this construction in terms of making alternate angles equal, in simple words.

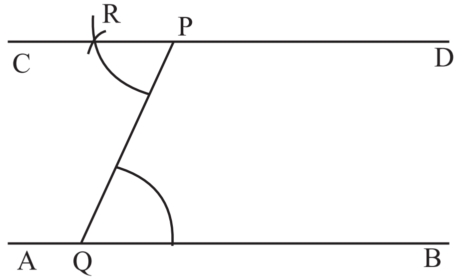


Fig. 7.11

2. Construction of Triangles

(a) When three sides are given: The teacher should tell the students that before doing any construction, they must first draw a rough sketch without measurement and think how to get it constructed. Suppose, you have to construct a triangle whose sides are 4 cm, 5 cm and 6 cm. Looking at the rough sketch, first draw a line segment BC equal to 6 cm. Then, taking B as centre and radius equal to 4 cm, draw an arc and taking C as centre and radius equal to 5 cm, draw another arc cutting the previous arc at point A. Join AB and AC. Then, ABC is the required triangle (See Fig. 7.12)

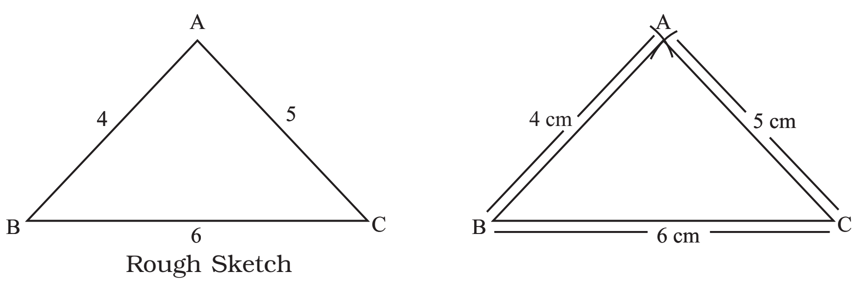
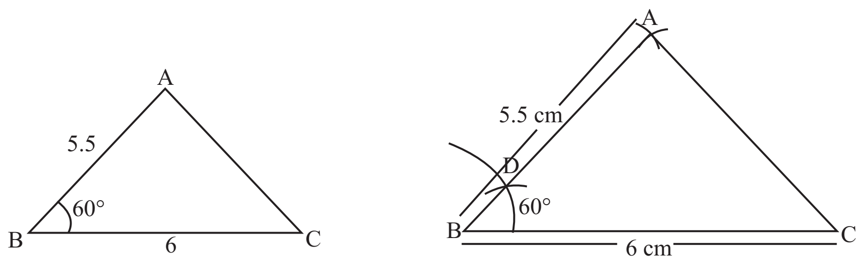


Fig. 7.12

Teacher may tell the students that the point A could be obtained on the other side of line segment BC.

(b) When two sides and the included angle are given: Suppose you have to construct a triangle in which two sides are of lengths 6 cm and 5.5 cm and included angle is of 60°. Analysing the rough sketch, draw a line segment BC equal to 6 cm. Then, at the point B construct an angle equal to 60°. Let it be ∠DBC. Taking B as centre, draw an arc of radius 5.5 cm cutting the arm BD at A. Join AC. Then, ABC is the required triangle (See Fig. 7.13).

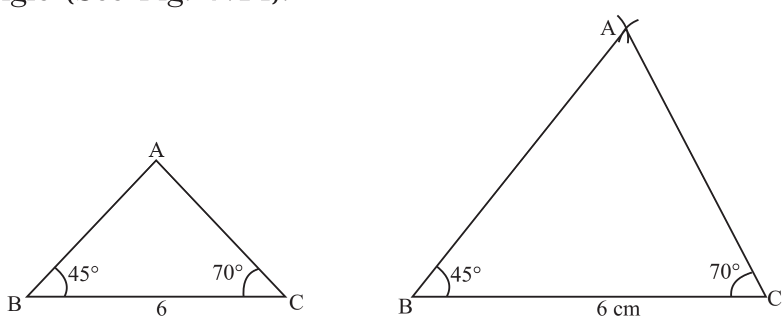


Rough Sketch

Fig. 7.13

The teacher may tell the students that ∠DBC could also be drawn on the other side of BC. It is to be noted that if the measure of the angle is such that it cannot be constructed with the help of a ruler and compasses (say of 40°), then use of protractor in such case is allowed.

(c) When two angles and the included side are given: Suppose you have to construct a triangle ABC in which BC = 6 cm, ∠B = 45° and ∠C = 70°. Then, look first at the rough sketch. First, draw a line segment BC equal to 6 cm. Then make angles of 45° at B and 70° at C. Let the arms of the two angles meet at A. Then, ABC is the required triangle (See Fig. 7.14).

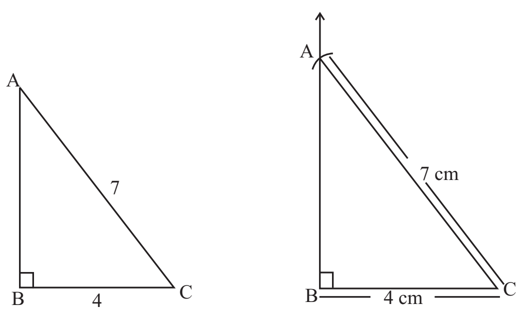


Rough Sketch

Fig. 7.14

It must be explained to the students that a triangle can be constructed, if two angles and any one side of it are given, using angle sum property of a triangle. This will help the students to solve the problem (2) of Exercise 10.4 on page. 202 of Class VII, Mathematics, NCERT.

(d) When hypotenuse and a leg of a right angled triangle are given: Suppose you have to construct a right angled triangle ABC in which hypotenuse AC = 7 cm and leg BC = 4 cm. Draw a line segment BC equal to 4 cm. Then, construct an angle of 90° at B (i.e. draw perpendicular to BC at B). Then taking C as centre and 7 cm radius, draw an arc to intersect the perpendicular, say at A. Join AC. Then, ABC is the required right angled triangle (See Fig. 7.15).



Rough Sketch

Fig. 7.15

3. Construction of Quadrilaterals

It can be observed from the above construction that in constructing a triangle at least three measurements must be given. In constructing quadrilaterals, it might be imagined that its construction will be possible with four given measurements. In fact, it is not true. It can be seen that for the construction of a unique quadrilateral, at least five measurements must be given. The teacher may take these cases one by one.

(a) When four sides and a diagonal are given: Suppose you have to construct a quadrilateral ABCD in which AB = 4 cm, BC = 6 cm, AC = 7 cm, AD = 3 cm and CD = 5 cm. Look at the rough sketch. It is just the construction of two triangles ABC and ACD on the same base AC and lying on opposite sides of AC with given measurements (See Fig. 7.16).

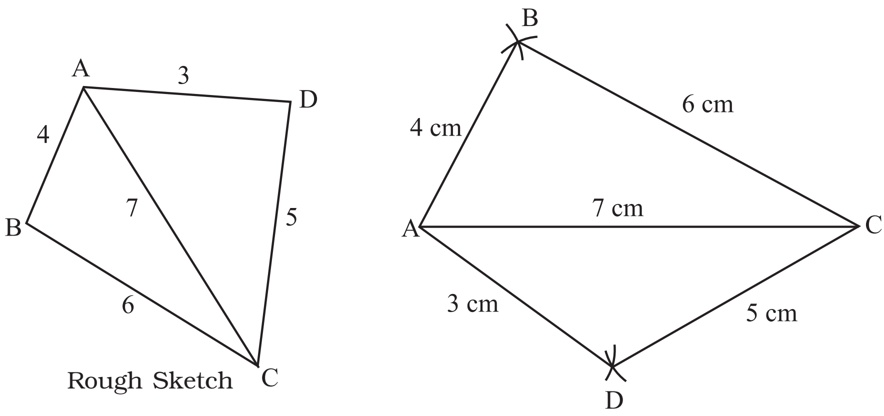


Fig. 7.16

(b) When both the diagonals and three sides are given: Suppose you have to construct a quadrilateral ABCD in which AB = 3 cm, BC = 6 cm, AC = 8 cm, BD = 7 cm and AD = 6 cm. Observing the rough sketch you can construct the triangle ABC with sides 3 cm, 6 cm and 8 cm.

Next taking A as centre and 6 cm radius and then B as centre and 7 cm radius, draw two arcs to intersect each other at the point D. Join DA and DC. Then, ABCD will be the required quadrilateral (See Fig. 7.17).

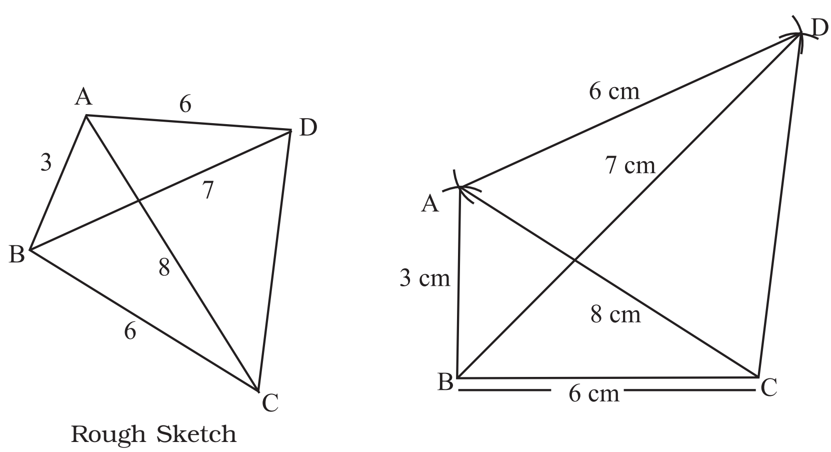


Fig. 7.17

(c) When two adjacent sides and three angles are given: Suppose you have to construct a quadrilateral ABCD in which AB = 3 cm, BC = 6 cm, ∠A = 110°, ∠D = 95° and ∠C = 70°. See the rough sketch. ∠B = 360° – (110° + 95° + 70°) = 85°. Draw a line segment BC equal to 6 cm. Draw an angle XBC of 85° at B and taking B as centre cut an arc of radius 3 cm on it to get the point A . At A, make an angle of 110° and at C, make an angle of 70°. The point of intersection of arms of these angles is taken as D. Then ABCD is the required quadrilateral (See Fig. 7.18).

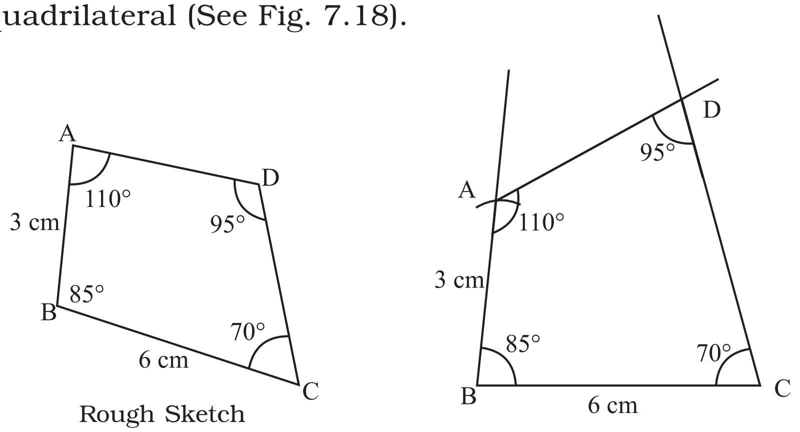


Fig. 7.18

(d) When three sides and two included angles are given: Suppose you have to construct a quadrilateral ABCD in which BC = 6 cm, CD = 5 cm, AD = 4 cm and ∠C = 60°, ∠D = 90°.

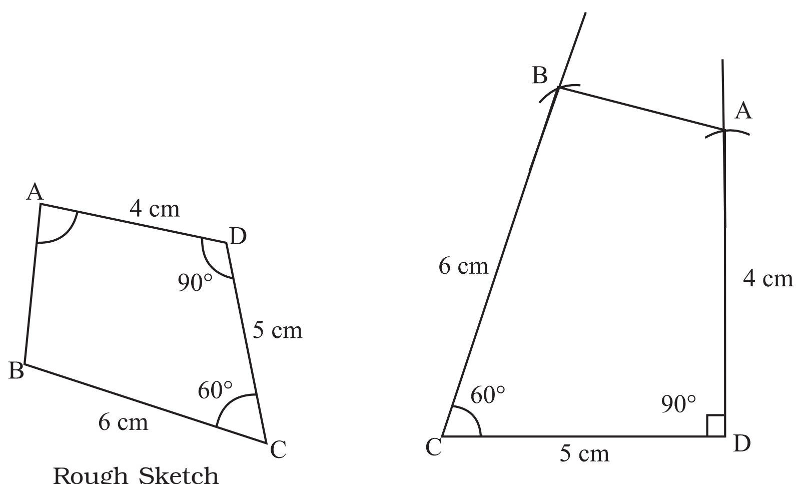


Fig. 7.19

Then, draw the rough sketch. From the rough sketch, it may be observed that you must first draw line segment CD equal to 5 cm. Then construct angles of 60° and 90° at C and D respectively. Now cut BC = 6 cm (on the other arm of angle 60°) and AD = 4 cm (on other arm of angle 90°) and join AB. Then, ABCD will be the required quadrilateral (See Fig. 7.19).

4. Construction of Some Special Quadrilaterals

Now you may discuss how to construct some special quadrilaterals, e.g. parallelogram, rhombus, rectangle and square.

(a) Parallelogram : When adjacent sides and included angle are given. Suppose you have to construct a parallelogram ABCD in which AB = 3 cm, BC = 5 cm and ∠ABC = 45°. Then looking at the rough sketch, draw a line segment BC = 5 cm and make angle equal to 45° at B. Now with B as centre and radius 3 cm make an arc which cuts the other arm of angle 45^0 at A. Taking A as centre and radius 5 cm and taking C as centre and radius 3 cm, draw two arcs intersecting at D. Join DA and DC. ABCD will be the required parallelogram (See Fig. 7.20). (It can also be constructed by drawing lines parallel to BC through A and parallel to BA through C intersecting at D).

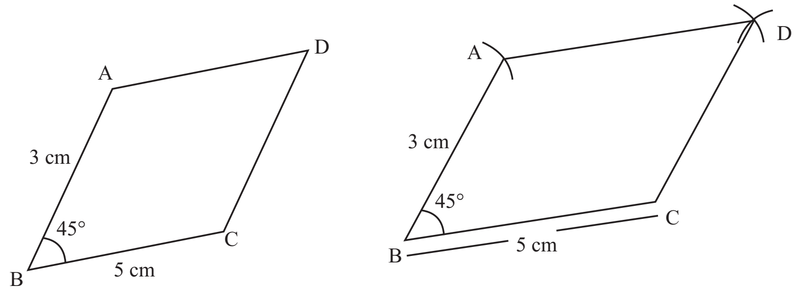


Fig. 7.20

(b) Rhombus : In a rhombus, all sides are equal. So it can be constructed if a side and one angle are given or a side and a diagonal are given or both the diagonals are given. Let us consider one case when both diagonals are given. Suppose you are required to construct a rhombus with diagonals 6 cm and 4 cm. You know that in a rhombus, diagonals bisect each other at right angles. So draw a line segment BD equal to 6 cm and draw the perpendicular bisector PQ of it intersecting BD at the point M. Taking M as centre, draw arcs of radii 2 cm $\left(\frac{4\text{ cm}}{2}=2\text{ cm}\right)$ on both the sides of BD intersecting PQ at A and C. Join AB, AD, CB and CD. Then ABCD is the required rhombus (See Fig. 7.21).

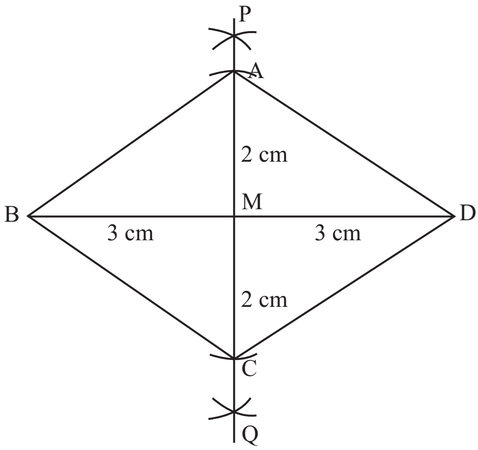
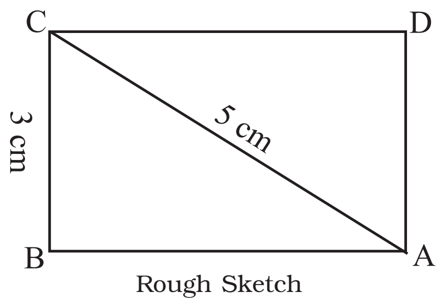


Fig. 7.21

(c) Rectangle: In constructing a rectangle, only two measurements are required–either two sides or one side and one diagonal. Suppose you have to construct a rectangle ABCD in which AB = 3 cm and AC = 5 cm. Draw a line segment AB equal to 3 cm. Construct an angle equal to 90° at B. Taking A as centre and 5 cm, as radius draw an arc cutting the perpendicular through B at C. Complete the rectangle ABCD as shown in the figure (See Fig. 7.22). Alternatively, taking C and A as centre, draw arcs of radius equal to AB and BC intersecting at D.



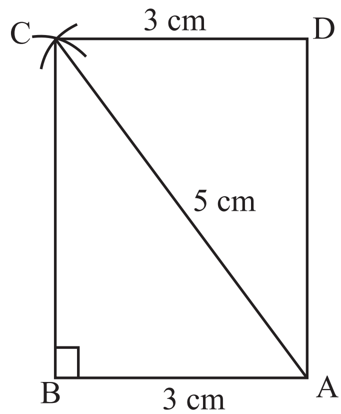


Fig. 7.22

(d) Square: Square is a rectangle whose all sides are equal. It can easily be constructed when its side is given.

The teacher may note that all these constructions are based on the properties of geometric shapes learnt so far. If some students demand, the teacher may explain the justification of these constructions.

Teacher may point out that in the construction of parallelograms, rhombus, rectangles and squares it appears that the number of measurements required is less than five. In fact, due to properties of these shapes, this number is less than five.

Common Errors

(i) In drawing a circle or an arc, the pointer is not firmly put. It is disturbed many a times.

(ii) After use, the sharpness of the pencil is not retained and so the figure is not accurate.

(iii) In drawing an angle using a protractor, the correct scale from the two scales on the protractor is not read.

(iv) Sometimes, students write an arc of length 3 cm in place of an arc of radius 3 cm.

(v) If something is lying on the table, students' notebook becomes slanting so exactness of construction is disturbed.

You can evaluate the students through the following exercise.

Exercise

1. Construct angles equal to 135° and 30°.
2. Construct a triangle ABC is which AB = 2.5 cm, BC = 6 cm and AC = 6.5 cm.
3. Construct a triangle PQR in which PQ = 5 cm, ∠Q = 105° and ∠R = 40°.
4. Construct a triangle ABC in which ∠A = 30°, BC = 4 cm and AB = 5 cm.
5. Construct a quadrilateral ABCD in which AB = 4.5 cm, BC = 5.5 cm, CD = 4 cm, AD = 6 cm and AC = 7 cm.
6. Construct a quadrilateral PQRS in which QR = 7.5 cm, PR = 6 cm, PS = 6 cm, RS = 5cm and QS = 10 cm.
7. Construct a right triangle ABC right angled at C in which AB = 7 cm and BC = 5 cm.
8. Construct a quadrilateral ABCD in which AB = 4 cm, BC = 5 cm, CD = 4.5 cm, ∠B = 60° and ∠C = 90°.