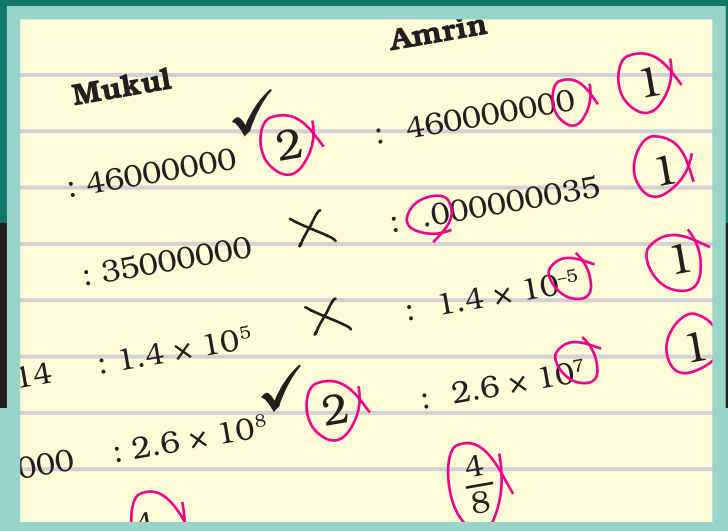
NCERT

Source Book on Assessment for Classes VI - VIII



Mathematics

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Source Book on Assessment

for Classes VI - VIII

MATHEMATICS

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FOREWORD

National Council of Educational Research and Training (NCERT) has published innovative textbooks based on National Curriculum Framework (NCF)–2005. The new approach advocated in NCF–2005 attempts to infuse subject knowledge with the awareness of children's own learning strategies. While reforms in teacher training are being initiated as a parallel process, the challenge of changing the traditional system of examination and evaluation has emerged as a major focus. The present system is accustomed to classifying and labeling children on the basis of a test or examination. The Source Book signifies NCERT's resolve to provide to teachers and administrators a new vision and approach for assessing children's progress.

We earnestly hope that the Source Book will receive attention in all segments of our complex system of education in India, both in its centrally-run components, such as Kendriya Vidyalayas, Navodaya Vidyalayas and the State-controlled schools of different types. It highlights the importance of cooperative classroom culture in the learning process at Upper Primary Stage. The vision of a universalised system of education implies patience and kindness for every child, irrespective of her/his style and pace of learning. Only a system which is kind and gentle to every child can ensure the joyful learning.

The development of this Source Book was a collaborative exercise. We wish to thank all the experts and institutions which participated in this exercise at different stages. NCERT's Department of Education in Science and Mathematics (DESM), especially its *Head*, Professor Hukum Singh and other colleagues have accomplished a major breakthrough in the long and painful history of evaluation reforms. This project received vital inputs from several other departments of NCERT and its Regional Institutes.

The vision of NCF–2005 emanates from the great struggle symbolised by Gandhi and Tagore to soften the heart of educationists towards the little child. This Source Book is dedicated to that struggle.

New Delhi
19 April 2011

Director
National Council of Educational
Research and Training

Nirmalya Chakraborty, College of Art, New Delhi

PREFACE

National Curriculum Framework (NCF)–2005 brought about a major shift in the education system by focusing on child-centered education. The child-centered education system requires that a child gets complete attention in the teaching-learning processes by the teacher, parents and school administration. This is possible if the child receives encouragement to do better, participates actively in the classroom and learns by exploring concepts. At present the child's learning is centered around the examination system. The obsession of scoring marks and the fear of failure in the examination has diverted her/his attention from the process of learning. The real joy of exploring and learning has thus declined.

If the child is made free from this anxiety of examinations and receives attention in the different processes of learning mathematics, then she/he will grow up to be confident in mathematics and will be ready to face challenges in life. This does not mean that no assessment should be carried out. In fact more emphasis should be laid on learning and the assessment should be done continuously and comprehensively with as little stress on the child as possible. A good evaluation and examination system can become an integral part of the learning process and benefit both the learners and the educational system by giving credible feedback.

This Source Book tries to address the concerns related to the assessment of the children in mathematics at Upper Primary Stage. The book also tries to clear the misconceptions built around the words 'continuous assessment' and 'comprehensive assessment'.

We urge teachers as well as the Administrators to think over the different aspects of assessment and the processes of assessment outlined in this book with an open and positive frame of mind. We hope that it will bring about a change in their mind set thereby benefiting the future generations of this nation and, in turn, the nation itself.

The Source Book has been divided into five chapters.

Chapter 1 introduces the teachers to, how a child learns, what is assessment and how the child should be assessed. It also compares the existing practices of assessment with the proposed continuous and comprehensive evaluation.

Chapter 2 deals with the instruments for evaluation. It discusses, at length, both the indicators of learning and the reason for indicator-based assessment.

Chapter 3 discusses the different tasks in various areas of mathematics, which if followed can make assessment more comprehensive and qualitatively better. However, these tasks are only suggestive. The teacher has the freedom to modify them according to the needs of the learner and learning environment.

Chapter 4 provides the details and the mechanism for recording and reporting the assessment of an individual learner over the entire session (year). The need for maintaining the individual learner's records is also discussed.

Chapter 5 contains case studies directly from the classrooms. These might prove useful to the teachers for carrying out the assessment through the proposed ways given in the Source Book.

This book has been made possible because of the active participation of many people. I wish to thank *Director*, NCERT, Professor G. Ravindra, specially for his keen interest in the development of the book and for all the administrative support. I acknowledge with thanks the dedicated efforts and valuable contribution of Ashutosh K. Wazalwar, *Coordinator* of this Source Book. I am also grateful to the review committee for their comments and suggestions. I express my gratitude to the members of the Editing Team who worked tirelessly in bringing this book to the present form. We warmly welcome comments and suggestions for improvement from our readers.

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CONTENTS

CHAPTER 1

INTRODUCTION

Education aims at reflecting the current needs and aspirations of a society, its lasting values, and the immediate concern of a community as well as local human ideals. Thus, it must aim to build a commitment to the lasting values and human ideals, which are based on reasoning and understanding at the school level.

To realise all these values and ideals there is a provision of curriculum in a formal educational set up. The curriculum provides gradual direction to all sort of educational activities done inside and outside the classrooms by the teachers in the form of teaching-learning processes. Thus, there is a need to have learning to learn and the willingness to unlearn and relearn as means of responding to new situations in a flexible and creative manner. The curriculum should emphasise the process of constructed knowledge. Now, the question arises – how to know whether the child has constructed the knowledge through the learning or not? Here comes the role of teachers, learners, teaching-learning process and the process of assessment.

> *Education is. .. to educate your children to understand the whole of life and not merely segment of life like the physical, emotional, mental, psychological or spiritual; to have not the compartmental, divided outlook but a whole total integrated outlook on life…" to bring out through education a human being who is creative, who is capable, who possesses that intelligence which is not burdened and which is not shaped in any particular direction but is total, who is not belonging to any particular society, caste or religion so that through that education and with that intelligence he arrives at a maturity and, therefore, is capable of making his life, not merely as a technician but as a human being".*
>
> *J.Krishnamurti*

National Council of Educational Research and Training (NCERT) has already brought out a *Source Book on Assessment* for Classes I-V in Mathematics based on National Curriculum Framework (NCF)–2005 to provide assistance and guidance to the teacher for the process of assessment. In continuation to this, the Source Book in hand is another added document for all of us who are concerned with education, in general, and assessment in mathematics, in particular, for Upper Primary Classes.

1.1 Know Your Student – A Learning Perspective

You will agree that each child is different from the other. Each child has her/his likes, dislikes, interests, attitudes, mental abilities, skills and way of thinking and is unique in her/his own way. Thus, for teachers, it is important to understand how children develop competency by their teaching so that maximum learning may take place conceptually (thinkable) and operationally (do–able).

While assessing children, it is important to appreciate differences amongst them and respect the fact that they will understand and respond in different ways while learning on the basis of the past experiences. You know that children start learning from an early age, and continue learning throughout their lives. You would also have noticed that when a child enters in Class VI, she/he acquires lot of different experiences and knowledge from her/his primary schooling as well as from the environment. New learning is thus to be developed-based on what the child already knows and understands.

Inspite of all these differences, there are certain commonalities in them as far as learning is concerned. These include:

- Every child can learn if allowed to do so, at her/his own pace and follow her/his own way of learning.
- Children learn more through activities done cooperatively and learn better from each other, if they actually 'do' things by themselves.
- Learning is a continuous process. Thus, children's learning does not take place in the school only. Classroom learning is linked to what happens outside the classroom and at home.
- Children 'construct' their own knowledge in a unique style and do not learn only what and when the teacher teaches. This means that every child makes sense of what information she/he is exposed to, based on her/his previous experiences and learning. Only then the child arrives at her/his own understanding and conclusions.
- Children learn in a spiral way, not in a linear way only. Thus, revisiting concepts again and again helps them to understand in a better way. The act of learning involves a process of establishing connections among facts observed, experienced or realised by children. The new learning, therefore, is related to things acquired long back in school, home or elsewhere.
- Children learn through the mistakes and errors they make.
- Learning takes place in a holistic manner.
- Children in the Upper Primary Classes learn better while interacting with others, more so if they are encouraged. Their learning is promoted much more through activity, meaningful imitation, practice, proceeding from concrete to abstract, simple to complex, provided learning process is also stimulating.

In the light of the above discussion about difference and commonalities, there is a need to understand your children for better learning and hence for effective transaction of concepts. For this purpose, it is imperative to understand the meaning of the terms measurement, assessment and evaluation associated with the process of learning.

1.2 Measurement, Assessment and Evaluation

The word "measurement" is derived from the Greek word "metron," which means to measure or an act of measuring.

Measurement is also the process of assigning numbers to individuals or their characteristics according to some specified rules.

For example, we say that area of the floor of a room is $36m^2$, weight of a student is 42kg, marks obtained by Hari are 18/100 and so on. In these cases, we have simply collected some information using some established rule or standard. All these are examples of measurement. If weight of a girl of 14 years is 25kg then we can say that it is too less, or the marks of a student in mathematics test is 18/20 then we can say that she/he has performed well. Giving this type of meaning to the measurement obtained is known as assessment. Thus, 25 kg or 18/20 in the above cases is measurement while 'too less' and 'very good' are examples of assessment.

Suppose a student gets marks in mathematics as follows:

1st term	10/30
2nd term	15/30
3rd term	20/30

From this, one can infer or judge that she/he has improved continuously. This way, we have not only assessed the student but evaluated also. This evaluation includes measurement and value judgement. Thus, evaluation may be understood as:

Evaluation = Measurement + Value Judgement

We have seen above that measurement is the process of obtaining a numerical description of the degree to which an individual possesses a particular characteristic. Assigning meaning to the measurement is assessment. The final opinion or overall judgement which you form on the basis of several assessments is evaluation. The term measurement is limited to quantitative description of pupils, i.e., the results of measurement are always expressed in numbers. Evaluation on the other hand, includes both quantitative description (measurement) and qualitative description (non-measurement) of pupils. In addition, evaluation always includes value judgements concerning the desirability of the results.

To sum up, assigning marks is an example of measurement, marks plus meaning to marks is an example of assessment and finally marks plus meaning to marks along with value judgement is an example of evaluation.

> Marks ~ measurement
>
> Marks + Meaning to marks ~assessment
>
> Marks + Meaning to marks + value judgement ~ evaluation

However, for all practical purposes, we would be using the term assessment equivalent to evaluation as the term assessment has multiple meanings such as:

- It is measurement and testing;
- It is evaluation;
- It helps to diagnose individual difficulty; and
- It is a procedure to gather information on student's performance.

1.3 Assessment at Upper Primary Stage – Why, What, When and How ?

1.3.1 Why should children be assessed?

Since all of us are concerned about children's learning and providing quality education to all the children, the reasons as to why assessment is undertaken at upper primary stage will be varied. You may already be aware of some of them. Some of the important reasons to assess are to:

- find out what learning and progress took place in a child over a period of time in the discipline of mathematics and in her/his mathematical thought process.
- identify individual and specific needs and requirements of the child.
- plan and improve teaching-learning situations in a more suitable and organised way.
- help the child understand and realise what she/he can or cannot do and what she/he likes or does not like to do.
- find out to what extent aims/objectives of mathematics curricula have been achieved or are yet to be achieved.
- obtain evidence of childrens' progress so as to communicate the same to different stake holders in education.
- support and improve every child's learning and development as per her/his own style of learning.

- encourage a feeling of confidence and accomplishment amongst the children.

1.3.2 What should be assessed?

The most common question that most of the teachers ask is – "What should be assessed with regard to the mathematical learning at this stage?"

We need to ask ourselves what is it that we are looking for. As far as mathematical learning is concerned, if we look at the broader perspective of school mathematics, it is generally accepted that children must find mathematics as something to "talk about, to communicate through, to discuss amongst themselves, to work together on".

The students must be assessed with regard to the following capabilities:

Are they able to :

- consolidate and reason out mathematical facts, figures etc.?
- use abstractions to perceive relationships, to see structures, to argue logically the truth and falsity of statements?
- understand the basic structure of different branches of mathematics such as arithmetic, algebra, geometry, data handling, mensuration etc.?
- familiarise themselves with different ways of dealing with and handling abstractions and their generalisations?
- apply mathematical concepts learnt at this stage to solve problems related to daily life situations?

1.3.3 When should assessment be done?

Closely related to what needs to be assessed is the critical question expressed by most of us – when or how often should a child's learning and progress be assessed. Ideally, assessment of the outcomes of mathematics learning must go along with the teaching-learning process itself in a continuous manner. But for all practical purposes, the assessment must be done periodically to be followed by unit tests, periodical tests in the form of formative as well as summative form. There is a need to maintain a profile for each child. This is required in order to reflect upon, derive feedback, plan and implement measures to enrich and enhance child's learning.

1.3.4 How should assessment be done?

Teaching strategies should be so designed that they involve both learning as well as assessment. For this purpose, various skills may be used in the form of specific questioning techniques, well designed handouts or some other suitable activities.

The children can be assessed by the teacher by asking different questions related to the topic being taught at the time of teaching-learning and the questions raised by the students. Some other ways of assessments are:

- **Individual Assessment:** It focuses on one child while she/he is doing an activity/task and thus on individual work and accomplishments.
- **Group Assessment:** It focuses on the learning and progress of a group of children working on a task together with the objective of completing it. This method of organisation is found to be more useful in order to assess social skills, co-operative learning processes and other value-related dimensions of a child's behaviour.
- **Self Assessment:** It refers to the child's own assessment of her/his learning and progress in knowledge, skills, processes, interests, attitudes etc.
- **Peer assessment** refers to a child assessing other children. This can be conducted in pairs or in groups.

At present what is observed across all the schools is that during assessment the most commonly used methods are those developed by teachers themselves. Among these are pen-paper tests/tasks, written and oral tests, questions on pictures, simulated activities and conversations with students. Small class tests are used by most teachers as a quick and easy way of assessing the learning progress of children. These are generally conducted at the end of a unit/month/cycle/term. No doubt these are useful but need to be used carefully. The kind of questions and items used, as far as possible, should not have pre-determined answers but be worded in ways so that children have space to generate and express individual thoughts and ideas in a variety of ways. Test items that promote thinking, analysis and not merely recall of textbook material need to be included. Put simply, test items should provide the space for a variety of responses from children.

Though many of the methods presented in the table given below are being used in schools, there are a large number of teachers who are not seen adopting all the different tools/techniques to the extent desired, thereby limiting their own understanding of the child's learning and progress. Let us now consider why different methods need to be used?

Sl.No.	Types of assessment tools and techniques	What are the strengths and advantages?	What should be taken care of ?
1.	**Observation** Information can be gathered about children in 'natural' settings. Some are based on teachers' observations about child in the course of teaching. Others are based on planned and purposeful observation	• Various aspects of personality development can be assessed through observations. • Can be used to assess individuals as well as groups. • Assessments can be made during varying time periods. • Evidence of child's performance/	• Avoid arriving at inferences/ interpretations or jumping to conclusions • Important to take down more than what is actually seen. • Dependent on the skill of the observer which

	of children on activities/tasks.	knowledge is based on an 'on-the–spot' record. • Over time, detailed observations of behaviour as well as interests, challenges, – patterns/trends emerge which allow teachers to create a comprehensive picture/view of the child.	determines 'what' is observed. • Requires sensitivity and unobtrusiveness in the way the observation is done. • Observations are to be made over a period of time, across different activities and settings.
2.	**Assignments** Written assignments are designed to allow the child to plan, compose and report about a unit/area/theme of learning. These can be completed as class work and/ or home work. Can be open ended or structured. Some could be based on contexts outside textbooks as well. Presentations can also be used on an individual or group basis.	• Provides children an opportunity to search for information, construct their own ideas and articulate the same ideas through spoken, written and/or visual expressions. • Helps assess a wide range of objectives and content of learning. • Provides children an opportunity to relate and synthesise within and outside school learning.	• Not too much home work or class work should be given which is currently the normal practice. • Assignments should be such that they can be managed by children on their own. • Should not become the only method of assessment.
3.	**Projects** These are undertaken over a period of time and generally involve collection and analysis of data. Projects are useful in theme-based learning.	• Provides opportunities to explore, work with one's hands, observe, collect data, analyse, organise and interpret data and draw generalisations. • Provides an opportunity to work in groups and in real life situations. • Helps develop a positive attitude towards group work,	• The nature and difficulty level of the projects should be such that children can do it by themselves. • Materials to be used for the project should be available in the school, neighbourhood or home setting. These should not put a financial burden on the parents.

		sharing and learning from each other.	• Each school could go in for a Resource Centre, which would have locally-available materials.
4.	**Portfolio** Collection of a child's work over a period of time. It could be day-to-day work or selection of the child's best piece of work.	• Provides a cumulative record. In the process, a picture of how a skill or knowledge area develops emerges. • Enables the child to demonstrate to others, her/his learning and progress. • The child becomes an active participant in learning and assessment.	• Select works to be put into the portfolio should have a specific reason. • Not all papers/ items of work are to be included. This will become unmanageable.
5.	**Checklists** A systematic way of recording specific behaviour/action helps focus attention on particular aspects. Observation checklist records the presence or absence of a particular knowledge item, skill or process. Checklists are a list of criteria that the teacher determines are important to observe in a child at a particular time.	• Quick and easy to implement. • Provides specific information about specific objectives. • Can point towards a trend of how and when skills have been acquired by the child as well as a group of children.	• Limited information, only indicating presence of a skill. • Does not indicate the child's response to different situations or provide specific examples of responses. • Does not provide information about context. • Can at times become unwieldy because of the number of specific items. • If developed by others, may not be suitable for the objectives that you as teachers have in mind or, for the group, you wish to use it.

6.	**Rating Scales** Rating scales have the same usage as observation checklists. They record the degree to which the presence or absence of a particular knowledge item/ skill of a process or quality performance are found. Used to record and judge the quality of a child's work against specified criteria. Holistic rating scales require a single, overall assessment of a piece of work.	• Various aspects of development that can be described along a continuum can be assessed. • Can be used to assess individuals as well as groups. • Assessments can be made during varying time periods and in different environmental settings. • Evidence of the child's performance/ knowledge is based on 'on the spot' record. • Over time, detailed observations of behaviour as well as interests, challenges, patterns /trends emerge which allow teachers to have a comprehensive picture/view of the child.	• Avoid inferences/ interpretations or giving judgements. Concentrate on taking down what is seen. • The skill of the observer may determine what is observed. • Be sensitive and unobtrusive in the way the observation is done. This does not necessarily mean being at a distance (physically). • Make the observations over a period of time, and across different activities and settings.
7.	**Anecdotal Records** Refers to written descriptions of a child's progress that a teacher keeps on a day-to-day basis. Provides observational narrative records of significant incidents in a child's life.	• Provides a wealth of information across different developmental areas. • Facilitates taking of notes on the child's social and emotional development, choices, interests and relationships etc. • Identifies strengths and weaknesses in the child. • Helps assess children's progress over time.	• A single anecdote does not give conclusive information. • Only 'problematic' situations may be noticed. It would be better to describe incidents rather than making statements of judgement. • Selecting amongst the many interesting classroom events and not including all. • Avoiding general comments.

This is done so that:

- learning in mathematical areas and aspects of development can be assessed.
- children get an opportunity to respond better to one method as compared to another.
- each method contributes in its own way to the teacher's understanding of children's learning.

No single assessment tool or method is capable of providing information about a child's progress and learning in different areas of development. While teaching, you would have realised that a lot can be understood by observing pupils, listening to them, discussing informally with their peers and parents, talking to other teachers, reviewing written work (class work and home work) and other articles made by children and observing the self-assessments done by them. In addition to the techniques detailed out in the table, photographs and audio-video recordings, if possible, can also be used. They provide a documentation of children's experiences while doing tasks or even finished products.

Assessment in Mathematics — Present Status

Assessment or evaluation is an essential component of all teaching-learning situations. It not only measures the extent of learning by students, but also reflects the performance of the teacher. In this sense, it is both a reflected (student's performance) and a reflective (teacher's performance) process. The reflective assessment occurs when the teacher looks back over a lesson or a series of lessons and analyses what went well and what needs improving. During reflection, we ask ourselves questions such as: What went well? How can I improve upon the lesson? What would I change if I did the lesson again? In short, it helps teachers to assess their performance and, where necessary, enable them to evolve new methods of teaching.

Now, let us look at the scenario in our mathematics classes and evaluate if it caters to the two features of assessment mentioned earlier. What is our usual method of assessment? It is the paper-pencil or written test. We call these tests by different names like, unit test, mid-term examination, preparatory examination, annual examination, etc. Paradoxically, these have only different names but in reality, they all have essentially similar characteristics limited to testing mostly what has been memorised by the students.

Such an assessment will give us only quantitative scores or achievement levels that may not be reflective of the actual learning that has taken place in mathematics classroom. They are, in fact, quite arbitrary because they do not holistically assess the cognitive abilities. With the

prevalence of such dubious and questionable assessment methods, the extent of reliability of such quantitative scores becomes questionable. However, without giving a serious thought to these limitations, we award numerical scores or grades to the students and rank them based on these scores. Poor performance and low scores are often interpreted as 'student failures'. Repeated branding of students as 'failed' may demotivate them and in some cases may even lead to their opting out of the school system altogether. These tests never indicate 'teacher inadequacies' such as faulty teaching strategy or defective evaluation methods. Such an assessment does not help a teacher to reflect upon her/his performance and provide clues and pointers that will help her/his modify her/his teaching methods. In conclusion, this method of assessment is of little help either to the student or to the teacher. It may even prove detrimental to the academic health of the students. It is possible that memorisation has completely bypassed the essential learning that was desired. Therefore, it may be wrong to presume that a higher achievement in a 'test' reflects better learning.

Following is an example of the responses of a test given by Mukul and Amrin, students of Class VIII :

		Mukul		**Amrin**	
1.	**Usual form of**				
(i)	4.6×10^7	: 46000000 ✓	2	: 460000000	1
(ii)	3.5×10^{-7}	: 35000000 ✗		: .000000035	1
2.	**Standard form of**				
(i)	0.0000014	: 1.4×10^5 ✗		: 1.4×10^{-5}	1
(ii)	260000000	: 2.6×10^8 ✓	2	: 2.6×10^7	1
		$\frac{4}{8}$		$\frac{4}{8}$	

In this example, Mukul is not able to expand negative exponent or write the number in standard form whereas Amrin is ignoring decimal in all the questions though she/he can expand negative exponents and can write them in the form of exponents also.

Here, we can see that both got the same marks but can we say that their level of understanding is the same? Or have they made the same kind of error? Do the marks assigned to them by the teacher explain the mistakes done by them? If the answer to the above questions is 'No' then is this the right kind of assessment?

Another feature of such an assessment is that it is entirely a 'product assessment' and not 'process assessment'. Product assessment refers to a method of assessment that is done at the end when the teacher feels that "presumably something has been taught or learnt". So only the product of the teaching-learning situation is assessed. This promotes only rote memorisation and completely overlooks the learning processes of children and the various skills, which they may have acquired during the learning process. Unfortunately, in our school system, the 'process assessment' is almost entirely overlooked in spite of the fact that the product assessment offers only a very limited scope for us to comprehensively judge the students capabilities. Such an assessment ultimately boils down to testing the ability of students to memorise facts and concepts of mathematics and reproducing them when asked for. We do not even give them the freedom of expressing in their own individual styles. Can we ever be happy with such a type of assessment?

The tests we conduct are too formal, structured and cause a great deal of stress to the students, teachers and parents, as they ignore the pace and the learning styles of the students which depend on their socio-cultural and intellectual levels.

There is another facet of the existing system. Teachers award marks for the test that is conducted. On the basis of the marks, teacher ranks them and often students also rank themselves. Those with higher marks consider themselves 'superior' to others even when the difference is as little as one mark! Is our system of evaluation so perfect that it can discriminate between students even on the basis of one mark? This half-a-mark or one mark is sufficient to create a superior-inferior complex which may be detrimental to the student's progress and self-esteem.

Given so many limitations of the present evaluation system in our mathematics classes, can we devise a method of evaluation that helps us overcome the deficiencies of the present system? Can we wean ourselves away from a system of product evaluation to a more dynamic, informal, broad-based and comprehensive evaluation that assesses each and every inherent skill of a student? Yes, this is possible if we can integrate assessment with the teaching-learning process. Such a method of assessment is at the core of the ethos of continuous and comprehensive assessment.

1.4 Continuous and Comprehensive Evaluation

It is generally agreed upon by different stake holders that learning is not merely a one time activity. It is of course, on-going and continuous. Since, teaching-learning process is continuous, our assessment or evaluation should also be a continuous process and not a one time affair. Moreover, the emphasis

should be on a type of evaluation which takes place while teaching-learning is still on i.e., it should be an integral part of it. This type of evaluation is known as Continuous and Comprehensive Evaluation (CCE). The CCE scheme refers to a school-based evaluation of students that covers all the aspects of a student's development. The term 'continuous' refers to regularity in assessment. Since the growth of a child is a continuous process, evaluation has to be completely integrated with teaching and learning. The process of the students' learning should be evaluated regularly and frequently i.e., during the process of teaching-learning itself. Continuous also means regular testing, analysis of learning gaps, applying corrective measures, retesting and feedback to teachers and students for their self-evaluation etc. Thus continuous evaluation provides an opportunity to understand regularly what the child knows, what she/he does not know, what difficulty she/he is facing in the learning situation, and her/his specific progress in learning the objective of education. In short, continuous evaluation provides feedback in improving the levels of achievement and proficiency among students through corrective procedures and improvement of instruction.

The term 'comprehensive' refers to both the scholastic and co-scholastic areas of pupil growth. The function of school is not only to build up the cognitive abilities (abilities related to mind) only but also to develop non-cognitive abilities (abilities related to hand and heart) too. The scholastic area/domain includes the desirable behaviour related to the student's knowledge and understanding in the subjects and her/his ability to apply it in an unfamiliar situation. The non-scholastic areas include the desirable behaviour related to student's attitude, interests, personal/social qualities and physical health. The scholastic areas are meant for intellectual growth whereas co-scholastic areas are required to develop physical growth, deployment of socio-personal qualities, interests, attitudes, values etc. For evaluating all the aspects of pupil's growth multiple techniques of evaluation need to be used. CCE involves a set of multiple techniques and different persons like teacher, pupil, peer, parent and community etc.

The need for introducing CCE in schools in an effective and systematic manner has been felt for a long time. CCE needs to be institutionalised for all stages of school education. In the present set up, the importance attached to the assessment by Boards and school-based assessment is driven to back seat. The scenario is now changing. Many school education Boards are now emphasising the importance of CCE and have taken measures to implement it in schools with the cooperation of the State Education Departments. CCE should be viewed not as an alternative but complementary to Board evaluation.

1.4.1 Purpose of Classroom Evaluation

Usual tests, in general, and other evaluation procedures, in particular, can also be classified in terms of their functional role in classroom instruction. The functional role of evaluation procedure follows a set of well identified sequence. On the basis of this sequence, the evaluation procedure may be classified as follows:

1. Placement Evaluation

The evaluation which is used to determine pupil performance at the beginning of instruction is known as placement evaluation, It is concerned with the pupil's entry performances i.e., whether a student is suitable for placement in the class for the course or not. For this purpose, a variety of techniques like readiness test, aptitude test, self-report inventories, observational techniques or entrance test may be used.

2. Formative Evaluation

The evaluation which is used to monitor learning progress during instruction is known as formative evaluation. Its purpose is to provide continuous feedback to both the pupil and the teacher about the learning success and failure. Feedback to pupils provides reinforcement of successful learning and identifies the specific learning errors that are in need of correction. Also, feedback to the teacher provides information for modifying instruction and for prescribing group and individual remedial work.

Tests used for formative evaluation are mostly teacher made, but customised tests made by other agencies can also supplement this purpose. Observational techniques are also useful for monitoring pupil's progress and identifying learning errors. Since, formative evaluation is mainly directed towards improving learning and instruction, the results may typically not be used for assigning course grades. For that purpose, summative evaluation may be used (discussed later).

3. Diagnostic Evaluation

This is a typically specialised procedure which is concerned with the persistent or recurring learning difficulties. If a student continues to experience failure in learning mathematics despite the use of prescribed alternative methods of instruction, a more detailed diagnosis is required. The main aim of diagnostic evaluation is to determine the causes of persistent learning problems and to formulate a plan for remedial action. For example, a Hindi-medium student taking admission in an English-medium school, understands the concepts but cannot express them simply because of language deficiency and not due to conceptual deficiency.

Let us understand the procedure to carry out a diagnostic evaluation through an example [also see Chapter 3].

Procedure to Carry out Diagnostic Evaluation

Conceptual Area: Decimal fractions

Task: To divide 13.5 by 0.25

Indicator: Child can divide a decimal number by a decimal number

How to Conduct: Student is asked to solve the above problem

Sample Responses: Here are six responses given by different students

(i) $13.5 \div 0.25$

$$0.25\overline{)13.5}\,(41.4$$
100
35
25
100
100

(ii)

$$0.25\overline{)13.5}\,(.5$$
125
10

(iii)

$$0.25\overline{)13.5}\,(5.4$$
125
100
100
X

or .54
or .0.54

(iv) $13.5 \div 0.25$

$= 1.35 \div 0.25$

$$0.25\overline{)1.35}\,(5.4$$
125
100
100
×

(v) $13.5 \div 25$

$= 13.50 \div 0.25$

$= 1350 \div 25$

$= 50$

$$25\overline{)1350}\,(50$$
125
000
000
×

(vi) $13.5 \div 0.25$

$= 13.50 \div 0.25$

$= 1350 \div 25$

$= 54$

$$25\overline{)1350}\,(54$$
125
100
100
X

Proficiency Level : On the basis of the above six responses, following proficiency levels have been identified by a teacher.

Stage I : Response (i) indicates that the child does not know the division algorithm and is, therefore, placed at Stage I.

Stage II : Child knows very little about division algorithm [Response (ii)].

Stage III : Child knows division algorithm but is ignoring decimal point of the divisor or decimal point of dividend or is counting the number of digits to the right of decimal points of both the numbers [(Responses (iii) and (iv)].

Stage IV : Child knows the division algorithm but does careless mistake in computing [(Response (v)].

Stage V : Child knows the division algorithm and solved the given question correctly [Response (vi)].

Remedial Measures

Child can be asked to compare 13.5 and 0.25 i.e., how many 0.25 will make 13.5 or child can relate 13.5 as ₹ 13 and 50 paisa which is equal to 1,350 paisa

and 0.25 as 25 paisa. Now, child can divide 1,350 paisa by 25 paisa. Similar such measures can be thought of according to the situation.

4. Summative Evaluation

Summative Evaluation generally comes at the end of a course (or unit) of instruction. It is designed to determine the extent to which the learning indicators have been achieved and is used primarily for assigning course grades or for certifying pupil's mastery of the intended learning outcomes. The summative evaluation is usually done by the teacher made achievement test which may include paper-pen test of objective and subjective questions only. However, it may also include various other components/techniques such as laboratory work, project work, reflective prompts, peer assessment, open ended questions etc.

In CCE, the evaluation starts from the very moment the child enters a class, with placement evaluation and continues throughout the year in the form of diagnostic and formative evaluation, until summative evaluation is conducted at the end of the instructional programme.

1.5 Characteristics of Continuous and Comprehensive Evaluation

The main characteristics of CCE as listed in the Position Paper by National Focus Group on Examination Reforms (NCF-2005) are as follows:

- CCE refers to a system of school-based evaluation of students that covers all aspects of students' development.
- The 'continuous' aspect of CCE takes care 'continual' and 'periodicity' of evaluation.
- Continual means assessment of students in the beginning of instruction (placement evaluation) and assessment during the instructional process (formative evaluation) done informally using multiple techniques of evaluation.

The concept of periodicity should not be viewed in negative sense i.e., giving more and more tests. It must not at any cost burden the student and the teacher.

- The 'comprehensive' component of CCE takes care of assessment of alround development of the child's personality. It includes assessment in scholastic as well as co-scholastic aspects of pupil's growth.
- Scholastic aspects include curricular areas or subject-specific areas, whereas co-scholastic aspects include co-curricular and personal/social qualities, interests, attitudes and values.

- Assessment in scholastic areas is done both informally and formally using multiple techniques of evaluation continually and periodically. The diagnostic evaluation takes place at the end of unit/term. The causes of poor performance in some units are diagnosed using diagnostic tests. These are purposefully re-mediated by giving interventions followed by retesting.
- Assessment in co-scholastic areas is done using multiple techniques on the basis of identified criteria, while assessment in social/personal qualities is done using behaviour indicators for various interests, values, attitudes etc.

1.5.1 Basis of Continuous and Comprehensive Evaluation

1. Written Examination

Written examination includes questions of essay type, short answer type and objective type.

2. Mental/Oral Tests

Oral tests are used for testing those skills which are not tested by written type questions. These tests include speed and accuracy in reading of data and their interpretation by performing mental calculations, language comprehension etc.

3. Sessional Work Done by a Student

Apart from written and oral tests, sessional work should also be assessed. Assessment of sessional work includes the following:

- Home assignments/research assignment/project work
- Class work discussion
- Use of library
- Practical work done in Mathematics laboratory
- Craft work.

The basis of assessment may also include personality and character development, physical development etc.

1.5.2 Benefits of Continuous and Comprehensive Evaluation

CCE helps in reducing stress of learners by:

- identifying learning progress of learners at regular time intervals on small portions of content.
- employing a variety of remedial measures of teaching based on learning needs and potential of different students.
- desisting from using negative comments on the learner's performance.
- encouraging learning through employment of a variety of teaching aids and techniques.

- involving learners actively in the learning process.
- recognising and encouraging specific abilities of learners, who do not excel in academics but perform well in other co-scholastic curricular areas.

Misconceptions about Comprehensive and Continuous Evaluation (CCE)

There are some misconceptions about CCE which are listed below along with comments on them:

- *CCE is nothing but converting marks into grades.* In fact CCE is about the overall development of the child and destresses and discourages rote learning and unhealthy competition.
- *Students undergoing continuous assessment may feel that they are continually under surveillance, and that every error that they make along the way can count against them. This can create stress among the students.* On the contrary, in CCE it is expected that at no point of time the students should realise that they are under surveillance. More particularly they should be made aware of the error that they made and be told that this is nothing but a stepping stone of learning.
- *Teachers feel that each one of them has to give some home assignment every day to assess child's learning. This increases load and stress on both the teacher and the student.* The assignment should be properly planned and coordinated by the teachers as per the requirement of CCE.
- *There is a wastage of time of students when they do their projects/activities at home.* On the contrary these activities enhance understanding of the concepts.
- *There is more testing and less teaching.* Infact in CCE assessment is a part of teaching and learning. There is a series of tests and mini examinations.

1.6 Role of Teachers in Continuous and Comprehensive Evaluation

A teacher plays a crucial role during the process of assessment of students learning. It is expected as well as desirable that the teacher should use the best understanding (in the light of CCE) about the methods and procedures of assessment. In view of the above, a teacher is expected to know the following:

- Assessment process includes the assessment of (a) previous knowledge; (b) understanding level; and (c) learning process. This assessment will help in improving the teaching-learning process.
- It is not necessary to take frequent tests and examinations.
- A variety of tools and techniques can be used to assess conceptual understanding without always depending on traditional paper-pencil test.

These tools (oral, projects, group activities, presentation, discussions etc.) give every student a chance to exhibit her/his understanding, different abilities, learning styles and skills.

- Students are to be encouraged to come forward with alternatives (or unconventional methods) of answering the questions.
- A friendly atmosphere of the classroom reduces the fear of assessment. Rather teacher may encourage each of her/his students to assess herself /himself. Group activities and peer assessment generate a cordial atmosphere.
- Incorrect answers may be used as a tool to analyse a child's level of understanding and thinking process, which can help in gradually building up correct concept.
- It is not a good idea to judge the child's capability by the final outcome only rather judgement should be process-oriented.
- Sharing of assessment criteria with learners and parents help them to understand what is expected of them. Such an approach can provide an opportunity to students to reflect and improve on the assigned task.
- Assessment outcomes are conveyed in a suitable manner for involvement and motivation of students and guardians.
- Conventional labeling of students as slow learner, poor performer or intelligent child are avoided and it should also not be gender, caste, religion or age biased. Instead, positive and suggestive remarks in a simple language can help in improving performance of the students.
- It is better to device tasks which facilitate in constructing concepts than those which assess only computational competencies or mathematical facts.
- Coordination between all teachers teaching particular class improves by frequent meetings among themselves to chalk out various projects/ activities/home assignments given to the students so that they are not overloaded.

Thus keeping in mind the aim of learning mathematics teacher can make assessment continuous, interactive, child-friendly and part of the process of learning rather than rigid and static.

1.7 Why the Source Book

The Source Book attempts to answer some of the queries mentioned below that teachers frequently raise:

- Why should children be assessed?
- What should be assessed?

- When should assessment be done?
- How should assessment be done?
- How can assessment information be used?

Reflection on these questions will provide the necessary direction to guide the process of assessment that is to be undertaken. As can be seen, these questions focus on the child who is central to teaching-learning that goes on in a classroom.

All these aspects have been discussed in Chapter 1. Chapter 2 deals with the indicators of learning in mathematics at Upper Primary Stage along with various tools of assessment. Chapter 3 deals with the various tasks (methods) that can be followed to make the assessment more exhaustive, objective and cause-oriented. For the suggested tasks, examples are drawn from the textbooks and other sources. Chapter 4 provides the details regarding how to record and report the assessment of an individual child over the entire session.

Chapter 5 lists a few case studies and success stories of practising teachers so that apprehensions regarding the feasibility and practicality of the suggested assessment methods do not crop up time and again.

In the light of RTE 2009, the role of teacher has become more challenging with reference to assessment in mathematics learning at Upper Primary Stage. Thus, the teacher should keep the key features of RTE in mind while designing and implementing assessments for children joining the main stream. For this purpose a brief outline of RTE has been provided in the Source Book.

Right to Education Act — 2009–2010

The Right of Children to Free and Compulsory Education Act or Right to Education Act (RTE), which was passed by the Indian Parliament on 4 August, 2009, describes the modalities of the provision of free and compulsory education for children between the age group of 6–14 years in India under Article 21A of the Indian Constitution.

Key Features of Right to Education (RTE)

- Free and compulsory education to all children between the age group of 6-14 years.
- This Act has come into force from, 1 April, 2010.
- No Fee will be charged and no Entrance test will be there.
- The right to education will be accorded the same legal status as the right to life as provided by Article 21A of the Indian Constitution.
- Every child in the age group of 6-14 years will be provided 3 years of elementary education in neighbourhood school.

1.8 Some Frequently Asked Questions (FAQ) on Continuous and Comprehensive Evaluation (CCE)

1. Does CCE mean frequent tests and assignments?

The term continuous in CCE refers to periodicity and regularity in assessment. It does not mean that tests and assignments have to be conducted or given frequently. On the contrary, the scheme of CCE discourages mechanical testing. It envisages employment of variety of tools and techniques for assessment in informal and formal settings which are more interesting, relevant and meaningful and involve learners for greater participation and learning.

2. How would CCE help in reducing stress of students?

CCE helps in reducing stress of students by:

- identifying learning progress of students at regular time intervals **on small portions of content.**
- employing a variety of remedial measures of teaching based on learning needs and potential of different students.
- desisting from using negative comments on the learner's performance.
- encouraging learning through employment of a variety of teaching aids and techniques.
- involving learners actively in the learning process.
- recognising and encouraging specific abilities of students, who do not excel in academics but perform well in other co-curricular areas.

3. How would CCE help in the improvement of student's performance?

CCE helps in improving student's performance by identifying her/his learning difficulties at regular time intervals right from the beginning of the academic session and employing suitable remedial measures for enhancing her/his learning performance.

4. It is observed that sometimes the projects are bought from the market and submitted for assessment. Is there any check on such unfair practices?

The purpose of project work is to enable the learners to apply and extend classroom learning to life outside the school. The Board is proposing and promoting group project work. Every individual is required to contribute in the completion of the project and may be awarded marks/grades in proportion to her/his contribution.

However, it is essential that in order to make project work meaningful and learning-oriented, the identification and selection of project work may be done with utmost care. Sufficient guidance may be provided to the group or individual for carrying out the project effectively. Suitable timeframe may also be decided in advance for its completion. The subject teacher may ensure and certify that the project has been completed by the group or the individual as desired. Preferring group projects over individual projects will also promote social skills and lessen teacher's correction workload.

CHAPTER 2

INSTRUMENTS FOR EVALUATION

2.1 INTRODUCTION

Mathematics plays an important role in our life. It not only helps in day-to-day situations but also develops logical reasoning, abstract thinking and imagination. It enriches life and provides new dimensions to thinking. The struggle to learn abstract principles develops the power to formulate and understand arguments and the capacity to see interrelations among concepts. The enriched understanding helps us deal with abstract ideas in other subjects as well. It also helps us understand and make better patterns, maps, appreciate area and volume and see similarities between shapes and sizes. The scope of mathematics includes many aspects of our life and our environment. This relationship needs to be brought out at all possible places.

The main goal of mathematics education in schools is the mathematisation of the child's thinking. Clarity of thought and pursuing assumptions to logical conclusions is central to the mathematical enterprise. There are many ways of thinking, and the kind of thinking one learns in mathematics is an ability to handle abstractions, and an approach to problem-solving.

Apart from the above qualities of mathematics, it is also perceived as a subject of which society at large, often talks about fear and failure. **This problem may be addressed by designing the classroom situations in a way which may not evoke panic, as does the examination.** The educationists are of the opinion that most of the problems like fear and failure are related to tyranny of procedures of assessment which is based on memorisation of formulas in school mathematics. The central cause for this faulty procedure is the faulty ways of assessment. More specifically, summative evaluation has been made a central part of total evaluative scheme. In this process of evaluation, unfortunately, the conceptual learning has been replaced by procedural memory only. Those who fail to do it experience panic and suffer the most. The fear of failure becomes more striking in the total process of learning mathematics. The irony is that while mathematics may be a major ground for learning formal problem-solving in schools, it has become an arena where children see little room for learning problem-solving related to life skills. Such outdated practices, like crude method of assessment need to be thoroughly reformed if any basic change is to be

brought about in the light of true philosophy of Continuous and Comprehensive Evaluation (CCE). School based CCE is needed to:

(i) reduce stress and fear of failure among students.

(ii) make evaluation comprehensive and regular **(regular does not mean conducting tests every day).**

(iii) provide space for the teacher to be creative in thinking and teaching.

(iv) provide a tool for diagnosis and equipping students with mathematical life skills.

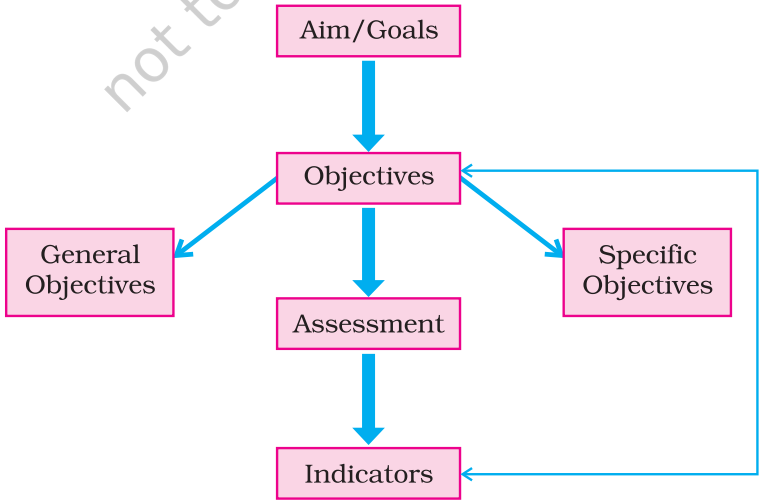
The two basic philosophies of CCE are following:

1. Assessment during the instructional process (formative assessment) done informally using multiple techniques of assessment.

   More emphasis on formative assessment rather than on summative assessment.

2. No mode of assessment should make the child aware that she/he is being assessed which may cause stress, anxiety and fear of failure.

Ideally, the present mode of summative assessment for learning should be replaced by formative assessment, i.e., but for all practical purposes some structural changes have to be designed in such a manner which may serve the purpose of summative assessment also. Till date most of assessment tools and techniques that have been devised and used by teachers and schools at large are to ascertain whether preidentified/predetermined objectives (which are generally perceived as vague in terms of clearcut indicators) have been achieved or not? Thus, the need is to make our assessment 'indicators based'.



2.1.1 What are Indicators?

One of the major aims of devising a task on the basis of indicators is to make sense of the information collected during and after assessment programme. In other words, by using the available information to arrive at an understanding

of what has been collected and recorded and what is intended to report. More specifically, what this result is and how this may be used in drawing conclusions about how a child is learning progressively. In a subtle sense, the necessary element is to identify where the student is at present and what needs to be done to help the student to turn her/his misconceptions/difficulties (if any) into right concept formation. This requires individual as well as group analysis on the part of the teacher as well as the students and review of the record and reflection of relevant information through assessment. Thus, it can be asserted that proper interpretation of the information would require identification of well defined indicators to facilitate not only the process of assessment but teaching-learning also. Now, an important question arises – what is an indicator? and how the concept of an indicator has been conceived in totality of whole mathematical assessment schemes?

In a traditional mode of assessment, most of the assessment tools and techniques have been devised to find out whether preidentified learning objectives have been achieved or not without taking care of specific individual need of a student in a particular domain of mathematical knowledge. For example:

Question

"2030 plants are to be planted in a garden in such a way that each row contains as many plants as the number of rows. Find the number of rows. How many plants would be left out in this arrangement?"

Sample Responses

(i) Total number of plants = 2030

number of rows = $\sqrt{2030}$

$2030 = 2 \times 5 \times 203$

$\frac{0}{3}$

(ii) Total number of plants = 2030

number of rows = number of plants in a row

$\therefore$ number of rows = $\sqrt{2030}$

$$\begin{array}{r|l} & 4 \\ 5 & \overline{2030} \\ & 20 \\ \hline & 30 \end{array}$$

$\frac{0}{3}$

(iii) Total number of plants = 2030

let number of rows = x

number of rows = number of plants in a row

Hence, number of plants in a row = x

∴ Total number of plants = x^2

∴ x = $\sqrt{2030}$

```
      42
   4 | 2030
       16
   ----------
  162 | 430
       324
   ----------
       106
```

$\frac{1}{3}$

number of rows = 42

(iv) Total number of plants = 2030

let number of rows = x

number of rows = number of plants in a row

Hence, number of plants in a row = x

∴ Total number of plants = x^2

∴ x = $\sqrt{2030}$

```
      45
   4 | 2030
       16
   ----------
   86 | 430
       430
   ----------
       000
```

$\frac{2}{3}$

number of rows = 45

number of plants left = 0

(v) Total number of plants = 2030

let number of rows = x

number of rows = number of plants in a row

Hence, number of plants in a row = x

∴ Total number of plants = x^2

∴ x = $\sqrt{2030}$

```
    45
  ------
4 | 2050
    16
  ------
85  450
    425
  ------
     25
```

$\frac{2}{3}$

So number of rows = 45

and number of plants left = 25

(vi) Total number of plants = 2030

let number of rows = x

number of rows = number of plants in a row

Hence, number of plants in a row = x

∴ Total number of plants = x^2

∴ x = $\sqrt{2030}$

```
    45
  ------
4 | 2030
    16
  ------
85  430
    425
  ------
      5
```

$\frac{3}{3}$

So number of rows = 45

and number of plants left = 5

Let us analyse the above traditional method of assessment. In response (i) and (ii) although both the students are getting the same marks (zero) but their levels of understanding about square root are different.

Also for responses (iv) and (v), students are getting same marks (2 out of 3). But again their levels of understanding are different.

Here marks allotted to both the responses never intimate what mistakes they are committing or what they need to learn further. Therefore, there is a need to change the method of assessment.

Following alternate way of assessment is being suggested for knowing the level of understanding of a child.

Proficiency Level

Stage I : Child is not able to understand the problem or she/he is not able to apply the appropriate algorithm for finding square root (response i) [conceptual error].

Stage II : Child is not able to compute properly (response ii) [computational error].

Stage III : Child is able to do initial step by selecting correct square which is less than the first pair (group) on the left side. However, she/he is not able to proceed further (responses iii and iv).

Stage IV : Child can apply proper algorithm but does careless mistake (response v).

Stage V : Child can apply proper algorithm (response vi).

However, the need of the hour is to take care of a specific learning need which may clearly underline the conceptual and procedural knowledge (not procedural memory only) and processing skills that are required by an individual to grow as per her/his specific needs, past experiences and learning styles, keeping in mind the basic nature of the discipline – mathematics. Thus, we all need to devise a suitable strategy to make learning objectives (for assessment purpose) more concrete by formulating a kind of task which may provide meaningful feedback to teacher for her/his own use and to students for better learning and self-analysis.

Here comes an important role of indicators which are **viewed as learning objectives in a broader sense.** But, for all practical purposes, indicators may be viewed as subsets of learning objectives which are specific in nature. These can also be viewed in terms of learning competencies that help and equip students to become competent enough for further mathematical knowledge construction and generation. These can also be identified as ascertaining the

levels of students which they are expected to achieve after the teaching-learning has taken place. Some people are of the view that indicators are aims of a particular task out of the series of tasks formulated to cover a particular area of a concept in mathematics learning. These may also be stated as the process of concretisation of learning objectives for the sole purpose of assessment.

In brief, the indicators help the teachers in a number of ways by:

- focusing and understanding student's learning.
- suggesting a reference point for childrens' learning which can be communicated to the child, parents and others to gauge the progress of every child.
- providing a framework for monitoring and recording the progress of a child and providing necessary feedback.

The list of indicators has been provided in different topics in Chapter 3 of this book.

However, indicators are only provided as a suggestive frame of reference. One may adapt them as per their syllabi needs, in general, and level requirements, in particular.

2.1.2 Why Indicators-based Assessment?

Today we are using a number of tests, largely objective (closed) type and very few subjective (open) type tests, primarily for assessing students learning of the declarative knowledge of a subject. They are not adequate for assessing conceptual knowledge, process skills and the higher order thinking that mathematicians and mathematics educators consider most important. Since current efforts to improve curricula are beginning to concentrate on these skills, new tests and other assessment devices are needed to serve as natural indicators of student-learning in mathematics. The assessment tasks should include exercises that imply free response techniques – not only pencil and paper problems but also hands on mathematical experiments, computer simulations and question formulation. The assessment process should also include some more innovative techniques and strategies such as reflective prompts and journal writing.

Reflective Prompts

Reflective Prompts is a technique that provides a set of flexible questions by a teacher to the students that prompts her/him to reflect on their own learning.

In this technique, each student is asked to note down and answer some questions such as given below after completion of a lesson/unit by the teacher.

- What did you enjoy about the lesson/unit?

- What was difficult or easy in the lesson/unit?
- Can you explore the ideas of the lesson/unit further?
- What help you want from me?

The teacher will now use this feedback for improving her/his teaching and enhance students' learning.

Journal Writing

In journal writing, students are motivated to keep a record in the form of a continual documentation of their expressions, feelings and experiences regarding mathematics learning. It is an excellent form of non-traditional assessment. Students can reflect on their thoughts about new concepts without feeling that they are being assessed.

Teacher can use journaling as a kind of window to look into how students are thinking about what they are learning. Students journal can be an important source of information about learning difficulties, misconceptions, strengths and weaknesses and metacognition i.e., learning about learning. The act of transferring thoughts, ideas and feelings into written words also encourages students to examine their own thought processes. It is a private record of students' thoughts that provides a safe way of communicating with the teacher, giving teachers insights into those thoughts.

In order to set clear expectations and procedures for journal writing, teachers must plan how often students will write in their journals, when, for how long, and for what purpose.

The teacher must provide feedback in the form of a written conversation, questions, notes in the margin, or some notation that lets students know you are reading and thinking about their entries.

Journaling in mathematics helps students use more precise mathematical language to express their ideas. It encourages students to reflect on their own knowledge and their own ways to solve problems. For example, teach students the rules for rounding to estimate a product or sum. Let students explain the steps in the process of estimating and how they will remember the steps. Encourage them to write the explanation so that other students could understand.

When introducing journalising, start by asking open-ended questions to encourage students to write about how they feel or their opinions about mathematics. For example:

- I learned that to .....
- I was surprised to find zero is not a natural number.....

- I was happy about.....
- I wish I knew more about.....

As they become familiar with journalising, ask students to write about mathematics process that they already know, as a way to review mathematics content.

For example

- Explain how to add two numbers.....
- Explain how improper fractions can be changed to mixed fractions and remain the same amount.
- Explain how to use a ruler to measure an object.

Then ask students to explain their understanding of new mathematics concepts.

For example

- What is the most important thing to know about place value?
- What have you learned bout decimals today?
- How could you use percentage in shopping?

Encourage students to use diagrams or drawings to explain their thinking, if appropriate, and make them write about problem-solving experiences including the guesses they made and how they form their answers.

Indicators-based assessment plans should be based on the following considerations:

- No single measure of assessment in mathematics is sufficient, because mathematics education involves dimensions involving a complex set of characteristics. The indicators to be used should be matched to the models of mathematics education and learning style of the students.
- Indicators should be constructed keeping in view the fact that there is no single, absolute level of mathematics learning and that various levels of attainment are possible in different student groups.
- Any measure used to generate indicators should be supplemented by research and field trials to validate what is actually being measured.
- Indicators may be expressed in terms of descriptive patterns of problem-solving and other non-numerical ways.
- The indicators should also probe the students' understanding of the nature of mathematics and its role in our life and society.
- Indicators of mathematics learning would also be useful for individual school as a means of monitoring change in the levels of accomplishment over a period of time.

- Indicators-based assessment will help us to assess the level of conceptual knowledge and processing skills in mathematics.
- Indicators-based assessment will help us to address students' processing speed which is typically in terms of her/his reaction time in performing a particular task.
- Indicators-based assessment will help us to assess the pattern recognition ability among students. Pattern recognition is important in many activities and good measures of this skill reduces the load on 'working memory'.
- Indicators-based assessment will help us to assess the students' knowledge organisation ability i.e., how mathematical knowledge is organised in the cognitive structure of a student and how it changes with practice.
- Indicators-based assessment will help us in terms of identifying the skill in retrieving information and internal representations of problem.
- The indicators will help all the students to have knowledge and reasoning skills that good mathematics education provides. Not only should students leave school mathematically literate, but they should also have acquired the mental tools with which they can renew that literacy throughout their lives.

2.2 Tools and Techniques

Today we have a number of tools and techniques which are used for the assessment purpose such as Observation, Home Assignments, Projects, Portfolios, Checklists, Rating Scales, Anecdotal Records etc.(For details, refer pages12-13, *Source Book on Assessment* for Classes I-V, NCERT, 2008). Apart from these there are a number of other tools and techniques which are used for the assessment purpose in mathematics teaching and learning such as Interview (Structured and Unstructured), Pencil-Paper Tests, Problem Solving, Journal Writing, Reflective Prompts, Self Questioning etc.

2.3 Making Good Tasks for Assessment

A good task is that which solves the purpose for which it has been designed. Thus a good task must be based on (a) some specific content area and also on; (b) well identified indicator.

A good task must also admit one and only one interpretation as far as final product/answer is concerned. At the same time, it should also admit more than one interpretations as far as process of arriving at the result is concerned.

The language of a good task must be simple and within the comprehension level of the students for whom it has been designed. There are different forms of tasks and each form is suitable for a situation. The task designer must take care of content, language and its form. Difficulty level is a very important

characteristic of a task. In different situations, tasks of different difficulty level are needed.

Psychological Perspectives in Assessment of Mathematical Learning at Upper Primary Level

1. Assessment is one aspect among many in the process of learning and not the end product of it. This needs to be internalised by the evaluator (i.e., the subject teacher) so that accordingly he evaluates the learners. Equally important is the need to communicate the same to the learner as well. When the teacher first highlights a learner's positive efforts and all that he has accomplished and then points out where he has made errors, learner gets the message that there are some portions which he has done 'right' and yet he has to improve upon his errors. This then motivates him to make more efforts to learn. Further more this effort of not only highlighting the errors of a learner but also his 'correct' responses/steps in solving mathematical problems, may help to reduce in him the anxiety and fear of, performance, failure, ridicule and being labelled.
2. Assessing how far a learner is able to transfer her mathematical learning to other related disciplines of learning (such as science, social science, sports and games, art and craft etc.) and also in their daily living situations (at home, neighbourhood, school) can be an indicator of their internalisation and application of mathematical learning.
3. Finally, orderliness, rational thinking, logical reasoning, accuracy, estimating, planning etc., are some of the attitudes, skills, values and aesthetics which a student is expected to develop alongwith mathematical learning. Hence, need is also felt to assess the development of these in the learner as she learns mathematics.

Therefore, a dynamic assessment (in lieu of a rigid and static assessment) of the learner may be encouraged for assessing a student's mathematical learning.

2.4 General Observations

Activity

Most children love mathematics in the beginning but something during the school years kills that joy of learning mathematics amongst many. Engaging students in mathematics-related activities can be an important tool in retaining interest of students in this subject.

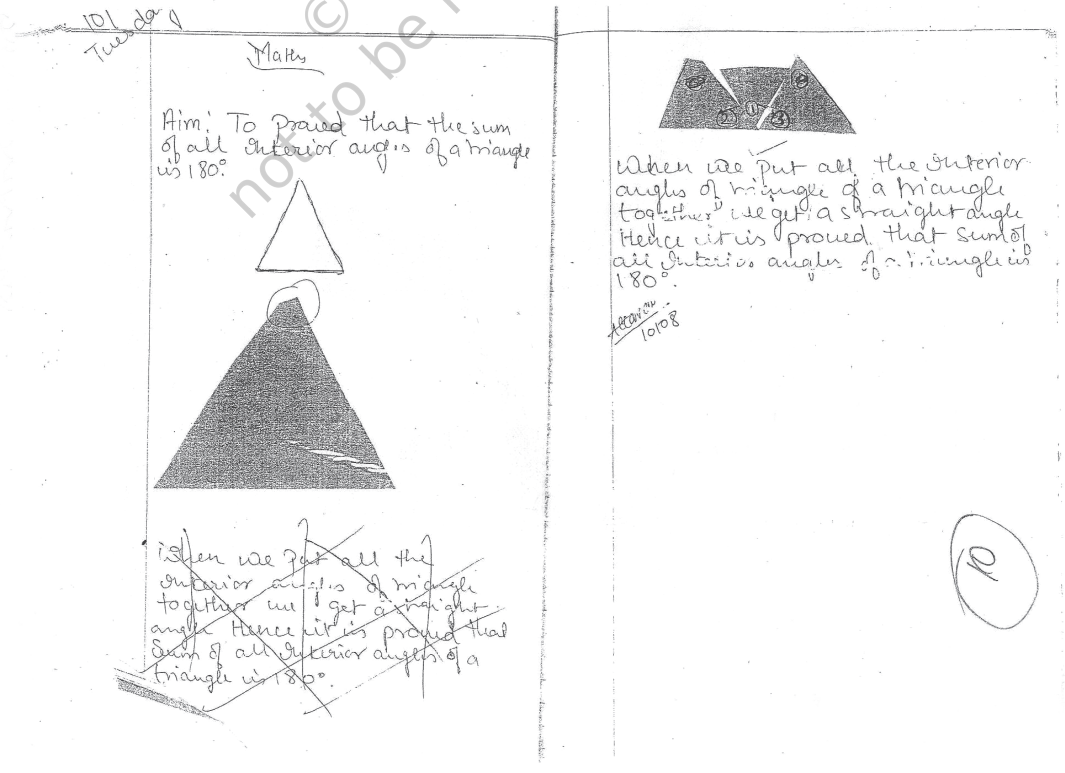
Various activities related to mathematics can be conducted in the classroom. These activities should not be given to students as their home work. Instead, the students should be encouraged to complete the assigned work in the classroom in collaboration with their peers under the supervision of the teacher. This will lead to a better understanding of concepts and foster a spirit of self-evaluation amongst the students. It may be noted that while most of these activities will be related to the development of the concept, some of these will also need to be administered for the reinforcement of this concept.

It is always a good practice to execute these activities in a planned manner. The teacher should clearly communicate to the students the expectation and the outcome of such activities. The students too, should be mentally prepared to undertake these activities. Clear communication of the outcomes of the activities being administered greatly aids the measurement of individual result of such activity. Gaps in the outcomes of these activities will provide important information to the teacher to aid the development plan for a particular student.

Have a look at this Anecdote.

Anecdote

The students were asked to verify the angle – sum property of a triangle through an activity. One of the students performed the activity and wrote about it this way:



The following observations can be made on the student's response to the activity:

- The aim given by the teacher for this activity is not complemented by an approach required to complete this activity.
- Most probably the teacher has passed oral instruction in explaining the method to complete this activity.
- It is evident that the student has made a mistake in the execution of this activity with regards to its approach. There is an ambiguity in the instructions provided by the teacher.

Project Work

Project involves collection and analysis of data. Projects are useful in theme-based tasks to be completed as class-work and/or home work in groups. It should be related to the child's environment/culture/lifestyle/community-based social programmes.

Checklist for Projects in Maths

1.	Is the approach towards the project original?	YES/NO
2.	Is the content and information authentic and relevant?	YES/NO
3.	Is the presentation of the project done aesthetically?	YES/NO
4.	Is the project supported with proper write-up?	YES/NO
5.	Has the result been interpreted appropriately?	YES/NO
6.	Does the project help in clarity of accurate?	YES/NO
7.	Is the content of the project accurate?	YES/NO
8.	Is the project based on experimentation or survey?	YES/NO

Project work may be represented in any of the following forms:

(a) Written project reports

(b) Charts/Models

(c) Power Point presentation

(d) Survey analysis

Projects can be evaluated on the following:

- Rationale of the project
- Inquisitiveness, observation skill, thinking skill, analytical ability
- Application of knowledge
- Drawing conclusion
- Presentation in style.

Observations in Maths Using a Checklist

1.	Does the child have good understanding of the concepts?	YES/NO
2.	Has the child worked with precision, neatness and accuracy?	YES/NO
3.	Can the child interpret word problems into mathematical form?	YES/NO
4.	Can the child interpret data?	YES/NO
5.	Is the child able to solve problems?	YES/NO
6.	Can the child think logically and rationally?	YES/NO
7.	Does the child solve problems accurately?	YES/NO

Quiz

1. This can be played with four teams. The number of teams will vary as per the situation.
2. Each team can comprise of 5 students.
3. Two successive periods may be engaged for this purpose.
4. There may be five rounds for asking questions. Each team will be asked one question in each round. The number of rounds will vary as per the need.
5. After the question is asked by the teacher/anchor to a team, sufficient time should be given for the team members to decide the answer. One member of the team will answer the question. In every round, different members

will answer the question so that every member of the team will get an opportunity to speak.

6. Five marks may be awarded for correct response.

7. If a team is not able to answer it correctly, the question will be passed on to the successive teams. If it is answered correctly by another team, 3 bonus marks should be awarded to the team. If none of the teams answer it correctly, it may be thrown open for the other students of the class and a small discussion may also be generated keeping the time constraint in mind.

8. The teacher may observe whether every student of a team is participating in the discussion.

9. The teacher may decide the stage at which a team is placed on the basis of examples of tasks given earlier. It will be the stage for every member of that team. Some questions from Mensuration and their answers are given here as exemplar.

10. After the quiz, answers of the questions may be discussed.

Sl. No.	Questions	Answers
1.	What do you mean by perimeter?	The sum of the lengths of all sides of a rectilinear figure is called its perimeter.
2.	What is the name given to a closed plane figure formed by line segments?	Rectilinear figure
3.	The magnitude of the region enclosed by a plane figure is called as what?	Area
4.	Give the formula to calculate area of a rectangle.	Length Breadth
5.	Is the formula to calculate perimeter of a square and rhombus same?	Yes, P = 4 side
6.	How many pairs of identical rectangular faces are there in a cuboidal box?	3 pairs
7.	In a cubical box, how many faces are squares?	6 faces
8.	What is the formula to calculate area of 4 walls?	2h (l + b)

9.	$2\pi rh$ is the formula to calculate lateral surface area of a cylinder. True/False.	True
10.	Give the formula to calculate volume of cuboid.	Area of base height = $l \times b \times h$

2.5 Suggestions for Projects and their Assessment

Students of Class VI may be given the following project in Mathematics:

Project on "Numbers"

- To collect information about different types of numbers by referring books and surfing the internet.
- Present the information in a folder with pictures, illustrations, tables etc.
- The folder should be submitted for evaluation within 15 days.
- The folders will be evaluated on the following criteria:
  - Content, neatness of presentation and illustration.
  - Students should complete the task individually and submit the folders by the given time.

Teacher grades the work of the students as per the assessment criteria.

The teacher may think on the following questions regarding the project

- Is it a good formative task?
- How are the students helped by the teacher and peer groups in doing the task?
- What are the objectives of giving a project?
  - To assess the student's ability to collect information and present them?

Or

  - To enable the students to deepen their learning?

If the purpose is to help the learners acquire a deeper understanding of the topic of the project then the project should be organised differently.

- Teacher should discuss the project with the learners.
- Student will explore ways in which information could be gathered, understood and adapted.

- Provide scope for group work so that learners study the topic collaboratively and help and support each other.
- Teacher monitors the entire process at regular intervals, giving feedback for correction, modification and refinement.
- Besides submitting a folder, the learners are also required to make a presentation to the class. During presentation discussion can be generated.
- Assessment is done by involving the learners in peer assessment.
- The information gathered by the teacher and the learners is used to improve the teaching-learning process.

One major concern with regard to such projects and assignments is that the teacher has very little scope to ensure that they are done by the students themselves. It is now common knowledge that projects and assignments can be 'bought' from shops. Instances of parents doing the projects are also not uncommon. Furthermore, downloading information from the internet also leads to very little learning.

Hence, to use projects and assignments as effective tools of assessment, the teacher should take certain precautions:

— Make the learners do the task as far as possible in the school itself under the direct supervision of the teacher.

— Discuss the project with the learners and monitor their progress at every stage.

— Involve them in the assessment process through self and peer assessment.

— Give descriptive feedback as instructional strategy to move students forward in their learning.

— Help students link their classroom learning with task and their experience.

— Follow it up with activities like revisiting some of the concepts explanations etc.

CHAPTER 3

THEME BASED EVALUATION

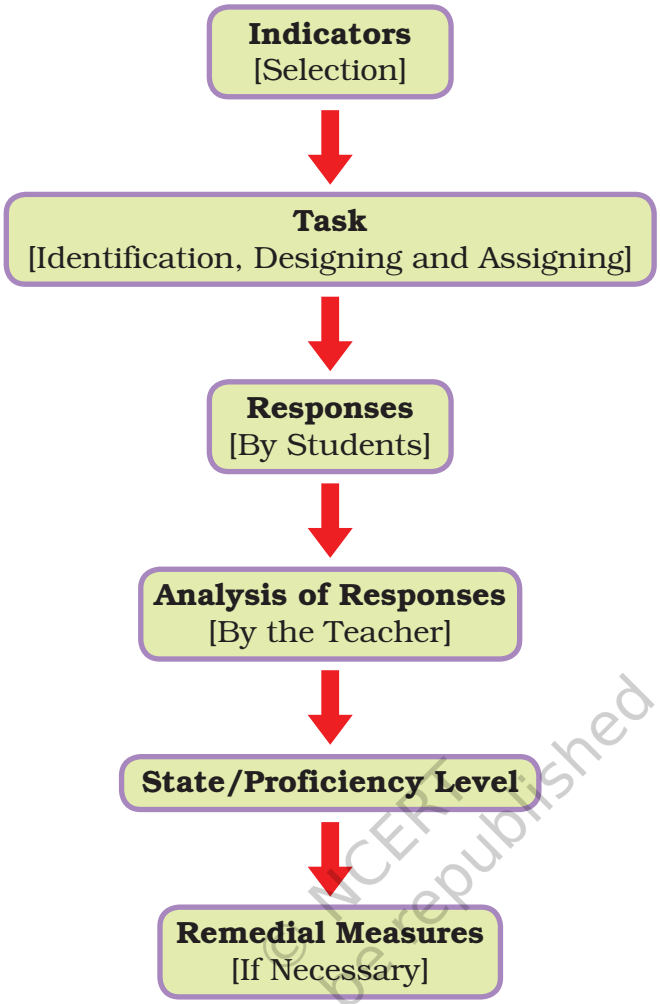
In Chapter 1, we have discussed needs and ways of assessing a child, in general, and mathematics, in particular, and also importance of the Continuous and Comprehensive Evaluation (CCE). In Chapter 2, we further discussed various tools of assessment and emphasised that assessment should be indicator-based and also explained about indicators.

We also explained that marks obtained by a child on a task based on certain indicators do not reflect her/his true level of knowledge and performance in CCE, it is necessary to know child's true performance regarding her/his conceptual knowledge and processing skills in mathematics in order to provide her/him appropriate remedial measures. The present chapter deals with some sample tasks based on certain indicators taken from the list of indicators given in Chapter 2.

Responses of children on a task have been categorised into five stages as shown below:

Stage I	:	Child is not able to understand the concept.
Stage II	:	Child is not able to compute properly.
Stage III	:	Child is able to do initial step but is not able to apply the concept correctly.
Stage IV	:	Child has understood the concept but does careless mistake.
Stage V	:	Child has understood the concept.

Some remedial measures have been suggested to overcome weaknesses of the children. Steps of this process can be displayed as under.

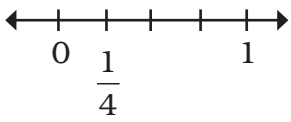


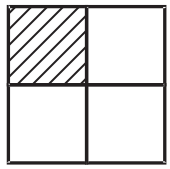
The examples given in this chapter are suggestive and not exhaustive. Teacher can construct her/his own task based on a given indicator in her/his own way depending on the requirement of the child. The examples suggested in the chapter are for a specific purpose mainly to assess a child's performance level. Therefore, the other forms of task such as multiple choice type/matching type/fill in the blanks type have not been included. However, if teacher feels she/he may include such types of tasks also for the purpose of assessment in mathematics.

3.1 Number System

At primary stage, learning of number system mainly focuses on 'number sense', 'number representation', 'relation between numbers' and 'number operations' (addition, subtraction, multiplication and division) upto 3 digit numbers. At upper primary stage also, these four aspects are focal points of learning alongwith the discussion of properties of number system (whole numbers, integers, rational numbers).

Number Sense involves the ability to acknowledge the value assigned to the given number. For example, value of $\frac{5}{4}$ is between 1 and 2 and value of –3 is three less than zero.

Number Representation is knowing number names and their symbolic representation in the form of numerals (or representation on number line or pictorial representation). For example, one fourth is $\frac{1}{4}$. Its representation on number line is 

and pictorial representation is 

Relation between Numbers is understanding the interconnection between numbers, ordering or sequencing, concept and properties of number system. For example, the numbers 11, 0, –15, 9, –2 when arranged in ascending order is –15, –2, 0, 9, 11. This also shows that negative numbers are smaller than zero. Also when students are exposed to different properties of numbers, they understand the interconnection of numbers and realise the need of different number systems. For example, whole numbers are not closed under subtraction, hence, need arises for integers.

Knowledge of Basic Operations is required when a child deals with numbers to encounter various daily life situations. Hence, understanding and learning of algorithms of basic operations is essential.

In all, we can say, that these four aspects form a basic structure of study of various number systems.

Hence, while constructing the knowledge of different number systems, it is mandatory for a teacher to keep the hierarchy and importance of these aspects in mind, since this helps in framing indicators (learning objectives) before delivering a lesson.

At the upper primary stage, under number system following topics have been discussed:

(a) Whole numbers

(b) Integers

(c) Fractions

(d) Decimals

(e) Rational Numbers

(f) Exponents and Powers

(g) Square Roots and Cube Roots.

For each of these sub-topics, a suggestive list of indicators is given below. These indicators help in assessing and reporting the progress of a child.

Indicators of Learning

1. Knowing our Numbers

- Child can form the largest and the smallest numbers using given digits (upto 5 digits).
- Child can arrange the given collection of numbers in ascending and descending order:
  - (a) Child can read large number (6 digits).
  - (b) Child can write numbers in words/in numeral.
- Child can acknowledge the value assigned to the number.
- Child can do operations (+/–/×/÷) with large numbers.
- Child can understand and can apply proper operation in solving the word problems.
- Child can understand and can apply the use of brackets in various operations.
- Child can round off the given numbers to the nearest ten, hundred, thousand, ten thousand or lakh.
- Child can estimate the sum, difference or product of two numbers (at most 3 digits only) by suitable approximation.

2. Playing with Numbers

- Child can write multiples of a given number (two digit numbers).
- Child can write factors of a given number (limited upto 3 digits).
- Child can distinguish between prime and composite number.
- Child can identify even and odd numbers.
- Child can observe pattern in multiples of 2, 5, 10, 4 and 8 and hence can apply tests of divisibility (2, 5, 10, 4 and 8) for large numbers.
- Child can apply test of divisibility (3, 6, 9) by adding digits of a given number.
- Child can identify pairs of co-primes from a given collection of numbers.
- Child can apply other divisibility rules (e.g. If a number is divisible by two co-prime numbers then it is divisible by their product also).
- Child can find prime factors of given numbers and can find HCF of two or more numbers, using prime factorisation.

- Child can find common factors and hence can find HCF of two or three numbers (limited to 3 digit numbers).
- Child can find common multiples and hence LCM of two or three numbers (limited to 2 digit numbers).
- Child can solve and make daily life problems using HCF or LCM.
- Child can reason out logically with reference to mathematical operations.

3. Whole Numbers (W) and Integers (I)

- Child can write predecessor and successor of given numbers.
- Child can represent number on a number line.
- Child can arrange the given collection of numbers in ascending and descending order.
- Child can find relation between numbers with regard to < or >.
- Child can represent and interpret information given in the form of integers.
- Child can write all the numbers between given pairs of numbers.
- Child can show operations (addition/subtraction/multiplication) on numbers on a number line.
- Child can write additive inverse of a given integer.
- Child can understand the closure property of numbers under addition and multiplication (W and I) and, under subtraction for integers.
- Child can apply knowledge of following properties in simplifying expressions:
  - Commutativity of addition
  - Commutativity of multiplication
  - Associativity of addition
  - Associativity of multiplication
  - Distributivity of multiplication over addition
  - Additive identity
  - Multiplicative identity
  - Division by zero.
- Child can perform all operations (+/–/×/÷) and solve word problems
  - with a negative integer and a whole number

— with two negative integers

- Child can make word problems.

4. Fractions

- Child can explain what a fractional number is.
- Child can define denominator and can identify it in a fractional number.
- Child can represent a fractional number pictorially and on number line.
- Child can make equivalent fractions and can choose equivalent fraction in a given collection.
- Child can write a fraction into its simplest form.
- Child can choose the biggest and the smallest fraction in a given collection.
- Child can explain fractional number as an operation of division and as a part of a whole. For example, Half apple, one-fourth of bread etc.
  - Child can convert improper fraction into a mixed fraction and vice-versa.
  - Child understands that for comparing, addition or subtraction of fractions; she/he needs to change to equivalent fractions with the same denominator [like fractions].
  - Child can multiply two fractional numbers to get another fractional number.
  - Child can write reciprocal of a fraction.
- Child can divide
  - a fraction by (non-zero) whole number or a whole number by a fractional number.
  - a fractional number by a fractional number.
- Child can understand, solve and make word problems involving various operations on fractions.

5. Decimals

- Child understands some situations where use of decimal is required. For example, Money, Height etc.
- Child can read and write a decimal number according to its place value.
- Child can represent decimals on number line.

- Child can write a fraction (p/q) in decimal form.
- Child can write a decimal in fractional form.
- Child can compare two decimals.
- Child can represent metric measures – kilometre, metre, kilogram, gram, rupee and paise in decimal form.
- Child can add and subtract decimals (atmost 3 digits only).
- Child can multiply decimals
  - by 10, 100 and 1000.
  - by decimals.
- Child can divide
  - decimals by 10, 100 and 1000.
  - decimal by non-zero whole number.
  - whole number by decimal.
  - decimal by decimal.
- Child can understand, solve and make word problems, involving decimals and their operations.

6. Rational Numbers

- Child can distinguish between integers, fractions and rational numbers.
- Child can identify common properties between whole numbers, integers and rational numbers (closure property, commutativity, associativity etc.).
- Child can understand representation of rational numbers on a number line.
- Child can write equivalent rational numbers and write a rational number in its standard form.
- Child can compare two rational numbers.
- Child can write rational numbers between two rational numbers.
- Child can perform operations on rational numbers. Child can add, subtract, multiply and divide rational numbers.

7. Exponents

- Child can write a repeated multiplication with one number in the form of powers/exponential notation e.g. $4 \times 4 \times 4 = 4^3$.

- Child can geometrically interpret/calculate the square of a given number (2 digit number).
- Child can identify perfect squares (maximum 3 digits).
- Child can observe some patterns in the sequence of perfect squares and can write the next number using the pattern.
- Child can identify/construct Pythagorean triplets.
- Child can find square root of a number
  - — through prime factorisation.
  - — through division method.
- Child can estimate the value of a square root to the nearest whole number like square root of 250 is approximately 16 (limited to 4 digits).
- Child can geometrically interpret/identify the perfect cubes.
- Child can observe pattern in the sequence of perfect cubes and write the next perfect cube.
- Child can find the cube of a given whole number (maximum upto 2 digits).
- Child can compute the cube root of a perfect cube.
- Child can solve daily life problems involving concept of square, square root, cube and cube root.

  e.g. 1. To find the area of a given square.
  2. To find the side of a square of given area.
  3. To find the volume of a box in the form of a cube of given side.
  4. To find the side of a cube of given volume.
- Child can write large numbers in a shorter form using exponential form (e.g. $10000 = 10^4$, $256 = 2^8$).
- Child can expand a number given in exponential form

  e.g. $10^3 = 1000$, $15^2 = 225$, $2^5 = 32$
- Child can verify/construct the following laws for integral exponents m and n and for integers x and y and use them to simplify large expressions

  (i) $x^m \times x^n = x^{m+n}$

  (ii) $x^m \div x^n = x^{m-n}$

  (iii) $(x^m)^n = x^{mn}$

  (iv) $x^m \times y^m = (xy)^m$

(v) $\frac{x^m}{y^m} = \left(\frac{x}{y}\right)^m$

(vi) $x^{-m} = \frac{1}{x^m}$

(vii) $x^\circ = 1$

- Child can write very large and very small numbers in the standard form using exponents

  e.g. (i) $15600000 = 1.56 \times 10^7$ (ii) $0.00015 = 1.5 \times 10^{-4}$

- Child can name the contexts where we need to write the numbers in the standard form e.g. the approximate distance between the Sun and the Earth is 149,600,000,000 metres = 1.496×10^{11} metres.

To achieve these indicators, teacher has to design good tasks for students. Through these tasks, she/he can assess students' responses and can judge her/his progress of learning.

Here some examples of tasks are given with sample responses. On the basis of responses, different stages of learning are indicated. These stages help both teacher and students to take required remedial measures, if any.

Task 1

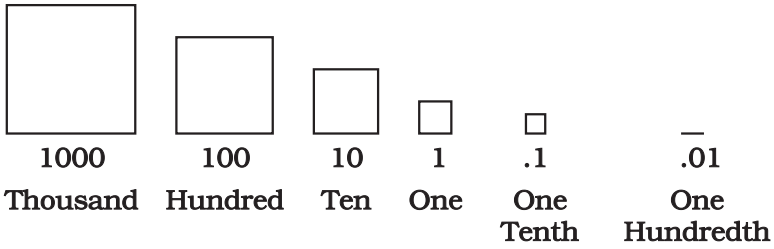
Conceptual Area: Whole numbers/Decimal numbers

Indicators

(i) Child can recognise the place value of a given decimal number.

(ii) Child can read a decimal number according to its place value.

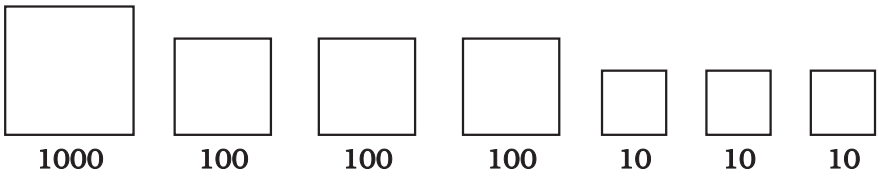
Cut-outs of different sizes of squares are given to the child.



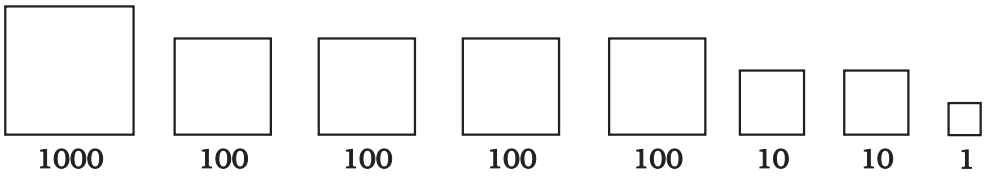
Task : Child is asked to represent a decimal number (3421.53) using these paper cut-outs.

Sample Responses

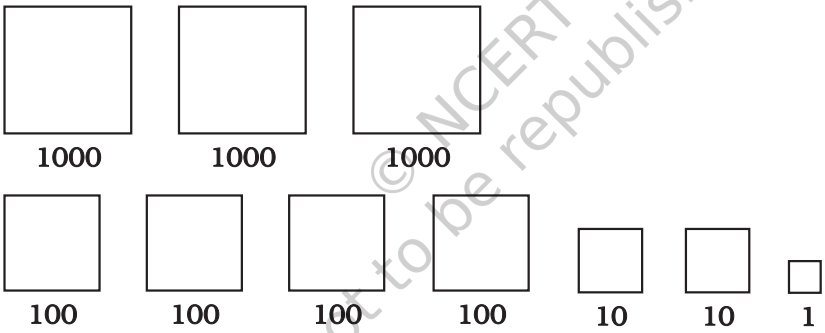
(i) Three thousand and three hundred twenty.



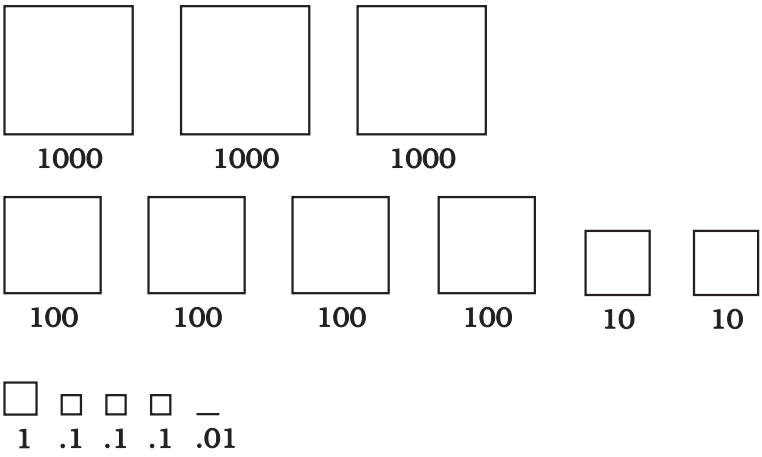
(ii) Thousand four hundred twenty one.



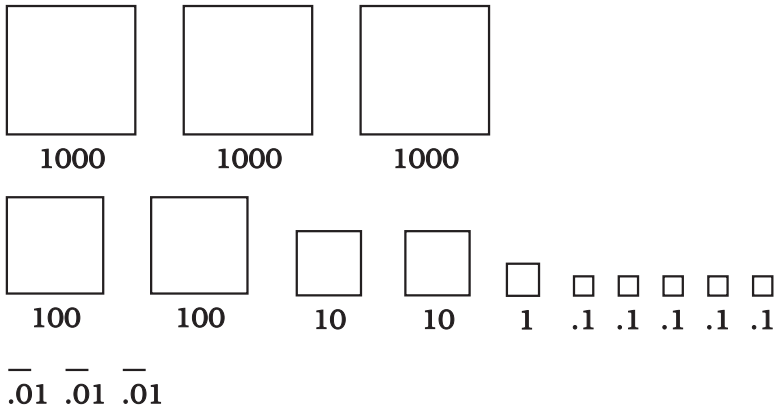
(iii) Three thousand four hundred twenty one.



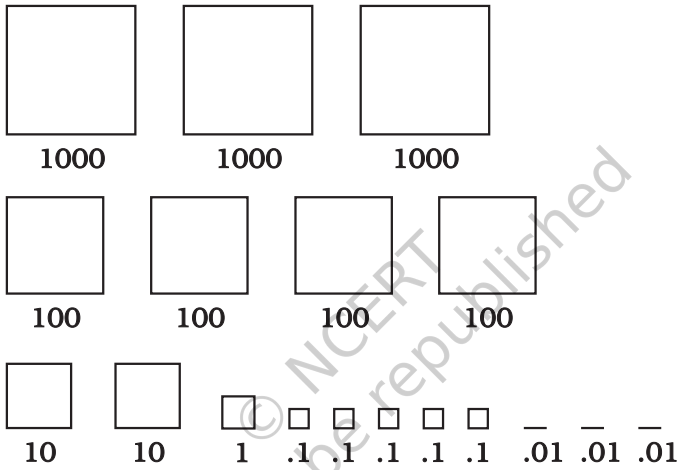
(iv) Three thousand four hundred twenty one and three.



(v) Three thousand four hundred twenty one point five three



(vi) Three thousand four hundred twenty one point five three.



Reflection on Responses

- In response (i), the student is either not able to understand the activity or does not know the place value concept. She is at Stage I.
- In response (ii), the student knows the place value upto 100 place but is not able to read number correctly. She is at Stage II.
- In responses (iii) and (iv), the student knows the place value upto 1000 place and is not able to read place values after decimal point. She is at Stage III.
- In response (v), the student knows the concept of place value and can read the number correctly but makes careless mistake while representing it. She is at Stage IV.
- In response (vi), the student knows the concept of place value from 1000 to hundredth place and is able to read the number correctly. She is at Stage V.

Remedial Measures

Child may be asked to perform this activity initially with three or four digit whole numbers and then can move gradually towards tenth place and hundredth place.

Remarks: This activity can be used to distinguish between tenth, hundredth and thousandth place by visualising the size of cut-outs.

Task 2

Indicator

Child can arrange the given collection of integers in decreasing order.

Task

Child is asked to arrange the numbers:

–100, –80, 90, 0, 25 in decreasing order

Sample Responses

(i) –100, –80, 90, 0, 25

(ii) 90, 25, –100, –80, 0

(iii) 90, 25, 0, –100, –80

(iv) –100, –800, 25, 90

(v) 90, 25, 0, –80, –100

Reflection on Responses

- In response (i), the child has not understood the activity and so is at Stage I.
- In responses (ii) and (iii), the child does not know about ordering of negative integers and so is at Stage III.
- In response (iv), the child has understood the concept but has done careless mistake by arranging the numbers in increasing order instead of decreasing order and so is at Stage IV.
- In response (v), the child has understood the concept and has arranged numbers as asked to do and so is at Stage V.

Remedial Measures

- Teacher can divide the class into heterogeneous groups to discuss among themselves about generating patterns.
- The concept of ordering of integers may be revisited. Number line can be used.

Remarks: Child can be asked to generate patterns with the multiples of integers or rational numbers.

Task 3

Indicators

(i) Child can multiply a fraction with a whole number.

(ii) Child can convert mixed fraction into improper fraction.

Task

Child is asked to cut $\frac{3}{4}$ part of a few circular cut-outs. The child is asked to multiply the fraction $\frac{3}{4}$ by 2, 3, 4 and 5 by using identical cut-out of $\frac{3}{4}$. Child is also asked to write the outcomes both in improper and mixed fractions.

Sample Responses

(i)

$\frac{3}{4} \times 1 =$  $= 1$

$\frac{3}{4} \times 2 =$  $= 2$

$\frac{3}{4} \times 3 =$  $= 3$

$\frac{3}{4} \times 4 =$  $= 4$

$\frac{3}{4} \times 5 =$  $= 5$

(ii)

$\frac{3}{4} \times 1 =$  $= \frac{3}{4}$

$\frac{3}{4} \times 2 =$  $= \frac{6}{4}$

$\frac{3}{4} \times 3 =$  $= \frac{9}{4}$

$\frac{3}{4} \times 4 =$  $= \frac{12}{4}$

$\frac{3}{4} \times 5 =$  $= \frac{15}{4}$

(iii)

$\frac{3}{4} \times 1 =$   $= \frac{3}{4}$

$\frac{3}{4} \times 2 =$   $= 1\frac{1}{2}$

$\frac{3}{4} \times 3 =$   $= 2\frac{1}{4}$

$\frac{3}{4} \times 4 =$   $= 3$

$\frac{3}{4} \times 5 =$   $= 3\frac{3}{4}$

(iv)

$\frac{3}{4} \times 1 =$   $= \frac{3}{4} = \frac{3}{4}$

$\frac{3}{4} \times 2 =$   $= 1\frac{1}{2} = \frac{3}{4}$

$\frac{3}{4} \times 3 =$   $= 2\frac{1}{4} = \frac{9}{4}$

$\frac{3}{4} \times 4 =$   $= 3 = \frac{12}{4}$

$\frac{3}{4} \times 5 =$   $= 3\frac{3}{4} = \frac{14}{4}$

(v)

$\frac{3}{4} \times 1 =$   $= \frac{3}{4} = \frac{3}{4}$

$\frac{3}{4} \times 2 =$   $= 1\frac{1}{2} = \frac{6}{4}$

$\frac{3}{4} \times 3 =$   $= 2\frac{1}{4} = \frac{9}{4}$

$\frac{3}{4} \times 4 =$   $= 3 = \frac{12}{4}$

$\frac{3}{4} \times 5 =$   $= 3\frac{3}{4} = \frac{15}{4}$

Reflection on Responses

- In response (i), the child is not able to understand the given task. She/he is at Stage I.
- In response (ii), child can multiply fractions but is not able to represent pictorially and in mixed fraction form. She/he is at Stage II.
- In response (iii), child can multiply fractions, can represent pictorially but is not able to write (or forgot to write) in improper fractions. She/he is at stage III.
- In response (iv), child understands and is able to complete the task but commits careless mistakes. She/he is at Stage IV.
- In response (v), child understands and is able to complete the task. She/he is at Stage V.

Remedial Measures

This task can be divided into small tasks as pictorial representation, addition of fractions, conversion of mixed fractions into improper fractions through pictorial representation and vice-versa. Practise should be given on these smaller tasks as per requirement.

Task 4

Indicators

(i) Child can add fractions.

(ii) Child can multiply fractions.

Task: To compute $\left(\frac{3}{4}\times\frac{2}{5}\right)+\frac{1}{5}$

Sample Responses

(i) $\left(\frac{3}{4}\times\frac{2}{5}\right)+\frac{1}{5}$

$= \frac{5}{9}+\frac{1}{5}$

$= \frac{6}{14}$

(ii) $\left(\frac{3}{4}\times\frac{2}{5}\right)+\frac{1}{5}$

$= \frac{6}{20}+\frac{1}{5}$

$= \frac{7}{14}$

(iii) $\left(\frac{3}{4}\times\frac{2}{5}\right)+\frac{1}{5}$

$= \frac{15+8}{20}+\frac{1}{5}$

$= \frac{23}{20}+\frac{1}{5}$

$= \frac{23+4}{20}$

$= \frac{27}{20}$

(iv) $\left(\frac{3}{4}\times\frac{2}{5}\right)+\frac{1}{5}$

$= \frac{6}{20}+\frac{1}{5}$

$= \frac{6}{100}$

$= \frac{9}{20}$

(v) $\left(\frac{3}{4}\times\frac{2}{5}\right)+\frac{1}{5}$

$= \frac{6}{20}+\frac{1}{5}$

$= \frac{6+1}{20} = \frac{7}{20}$

(vi) $\left(\frac{3}{4}\times\frac{2}{5}\right)+\frac{1}{5}$

$= \frac{6}{20}+\frac{1}{5}$

$= \frac{6+4}{20} = \frac{10}{20} = \frac{1}{2}$

or $\left(\frac{3}{4}\times\frac{2}{5}\right)+\frac{1}{5}$

$= \frac{3}{10}+\frac{1}{5}$

$= \frac{3+2}{10} = \frac{5}{10} = \frac{1}{2}$

Reflection on Responses

- In response (i), child is not able to multiply and add fractions and hence is at Stage I.
- In response (ii), child knows the algorithm of multiplication of fraction only so is at Stage II.
- In responses (iii) and (iv), child is not able to distinguish between multiplication and addition. Hence, she is applying algorithm of either addition or multiplication only so she is at Stage III.
- In response (v), child is able to compute but does careless mistakes so she is at Stage IV.

- In response (vi), child is able to compute correctly and hence is at Stage V.

Remedial Measures

- Child can be made to revisit the concept of addition of fractions initially. Then she can be asked to attempt some multiplication problems also. After that she can compare the algorithms of both the operations. She can then be asked to attempt problems having both operations.

Task 5

Indicator

Child can apply the laws $x^m \div x^n = x^{m-n}$ and $x^{-m} = \frac{1}{x^m}$

Task

Child is asked to solve $3^2 \div 3^4$ by applying laws of exponents.

Sample Responses

(i) $3^2 \div 3^4 = 3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$

(ii) $3^2 \div 3^4 = 3^2 = 3 \times 3 = 9$

(iii) $3^2 \div 3^4 = 3^{-2} = 3 \times 3 = 9$

(iv) $3^2 \div 3^4 = 3^{-2} = \frac{1}{3^2} = 9$

(v) $3^2 \div 3^4 = 3^{-2} = \frac{1}{3^2} = \frac{1}{9}$

Reflection on Responses

- In response (i), child does not have any understanding of the Laws of Exponents for division of powers. He is at Stage I.
- In response (ii), child is not able to apply the law [$a^m \div a^n = a^{m-n}$] properly. She/he is at Stage II.
- In response (iii), child has applied the law for division of powers but is not able to apply the law $a^{-n} = \frac{1}{a^n}$. He is at Stage III.
- In response (iv), child can apply the above Laws of Exponents but does careless mistake. He is at Stage IV.
- In response (v), child can also apply the required knowledge of Laws of Exponents.

Remedial Measures

(i) Students can be asked to revisit the Laws of Exponents.

(ii) This problem can also be solved as

$$3^2 \div 3^4 = \frac{3^2}{3^4} = \frac{3\times3}{3\times3\times3\times3} = \frac{1}{3\times3} = \frac{1}{9}$$

Task 6

Indicators

(i) Child can identify a perfect cube.

(ii) Child can compute the cube root of a perfect cube.

Task

Child is asked to find a number which is to be multiplied by 2025 to make it a perfect cube.

Sample Responses

(i) Prime factorisation of 2025 =

5	2025
5	45
9	9
	1

$= 5 \times 5 \times 9$

It is not a perfect cube. To make it a perfect cube, 2025 should be multiplied by 5 and 9. So, 5 and 9 is to be multiplied by 2025 to make it a perfect cube.

(ii) Prime factorisation of 2025 =

5	2025
5	45
3	9
3	3
	1

So, $2025 = 5 \times 5 \times 3 \times 3$

$2025 \times 5 \times 3 = 5 \times 5 \times 3 \times 3 \times 5 \times 3$

Hence 2025 is to be multiplied by 15 (i.e., 5×3).

(iii) Prime factorisation of 2025 =

5	2025
5	405
3	81
3	27
3	9
3	3
	1

$2025 = 5 \times 5 \times 3 \times 3 \times 3 \times 3$

$2025 \times 5 = 5 \times 5 \times 3 \times 3 \times 3 \times 3 \times 5$

$10125 = 5 \times 5 \times 5 \times 3 \times 3 \times 3 \times 3$

Hence, 2025 is to be multiplied by 5 to make it a perfect cube.

(iv) Prime factorisation of $2025 = 5 \times 5 \times 3 \times 3 \times 3 \times 3$

$2025 \times 5 \times 3 \times 3 = 5 \times 5 \times 3 \times 3 \times 3 \times 3 \times 5 \times 3 \times 3$

Hence, 2025 is to be multiplied by $5 \times 3 \times 3$ i.e., 30 to make it a perfect cube.

(v) Prime factorisation of $2025 = 5 \times 5 \times 3 \times 3 \times 3 \times 3$

$\therefore 2025 \times 5 \times 3 \times 3 = 5 \times 5 \times 3 \times 3 \times 3 \times 3 \times 5 \times 3 \times 3$

Hence, 2025 is to be multiplied by $5 \times 3 \times 3$ i.e., 45 to make it a perfect cube.

Reflection on Responses

- In response (i), child is not able to factorise and so is at Stage I.
- In response (ii), child is not able to factorise but could form triplets to make a perfect cube and so is at Stage II.
- In response (iii), child is able to factorise the given number could form triplets of 5 but did not form triplets of 3 to make a perfect cube and so is at Stage III.
- In response (iv), child can compute and can make perfect cube but does some silly mistakes and so is at Stage IV.
- In response (v), child can compute and can make perfect cube and so is at Stage V.

Remedial Measures

- Some practice of factorisation of numbers can be given.
- Child can be asked to factorise perfect cubes [e.g. 64, 729] where he can observe that prime factors of a perfect cube appear in triplets.

Task 7

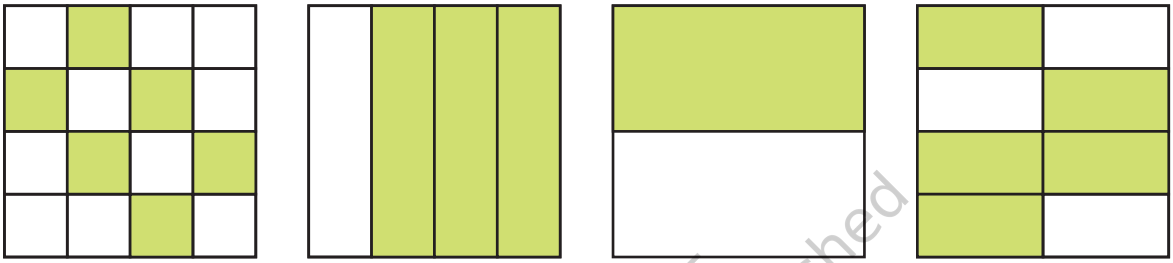
Indicators

(i) Child can write a fraction by observing the shaded portion of a picture.

(ii) Child can compare fractions.

Task

Teacher displays four pictures shown below with some shaded portion representing fractions. Child is expected to write the corresponding fractions for these pictures and arrange the fractions in an increasing order.



a b c d

Sample Responses

(i) $a = \frac{6}{16}$; $b = \frac{3}{16}$; $c = \frac{5}{16}$; $d = \frac{5}{16}$
Increasing order: a, b, c, d

(ii) $a = \frac{6}{16}$; $b = \frac{4}{3}$; $c = \frac{2}{1}$; $d = \frac{8}{5}$
Increasing order: c, b, d, a

(iii) $a = \frac{6}{16}$; $b = \frac{3}{4}$; $c = \frac{1}{2}$; $d = \frac{5}{8}$
Increasing order: c, b, d, a

(iv) $a = \frac{6}{16}$; $b = \frac{3}{4}$; $c = \frac{1}{2}$; $d = \frac{5}{8}$
Increasing order: b, d, c, a

(v) $a = \frac{6}{16}$; $b = \frac{3}{4}$; $c = \frac{1}{2}$; $d = \frac{5}{8}$
Increasing order: a, c, d, b

Reflection on Responses

- In responses (i) and (ii), child is not able to write proper fractions by observing pictures and is also not able to compare fractions. She is at Stage I.
- In response (iii), child is able to write proper fractions by observing pictures but is not able to compare fractions. She is at Stage III.
- In response (iv), child is able to complete task correctly but does careless mistake. She is at Stage IV.
- In response (v), child could complete the task correctly. She is at Stage V.

Remedial Measures

- Child may be asked to revisit the concept of fractions.
- Some practice of writing equivalent fractions of a given number can be given. Through equivalent fractions she can be asked to select like fractions and compare.

Task 8

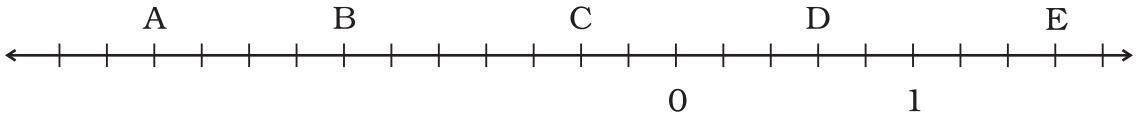
Indicator

Child understands representation of rational numbers on a number line.

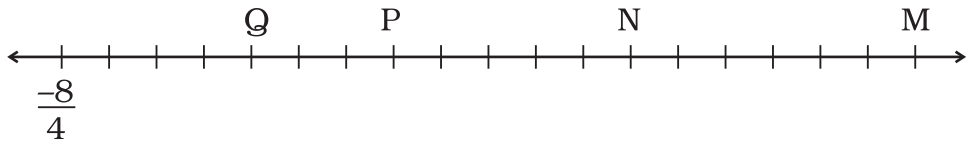
Task

Child is asked to answer the given question after observing the respective number lines and write the rational numbers for the points marked on it.

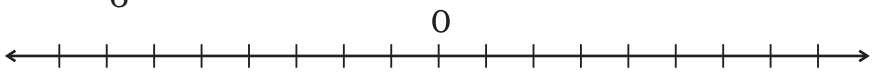
(a)



(b)

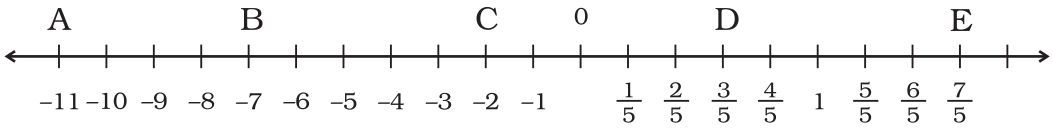


(c) Show $\frac{-7}{6}$ on the following number line.

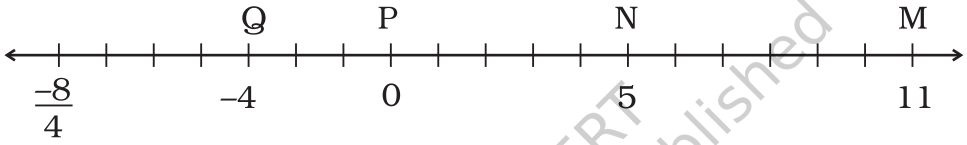


Sample Responses

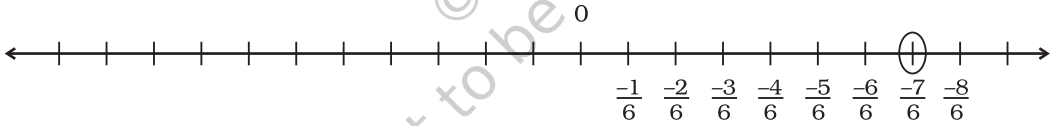
(i) (a)



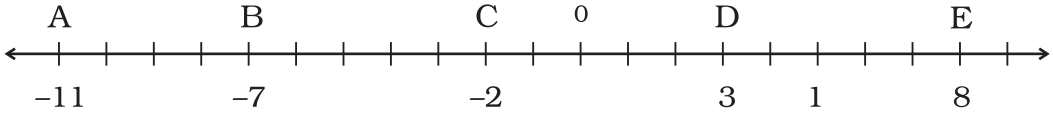
(b)



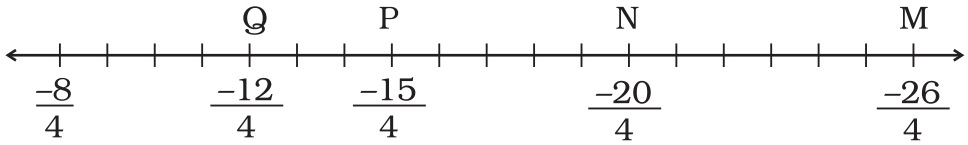
(c)



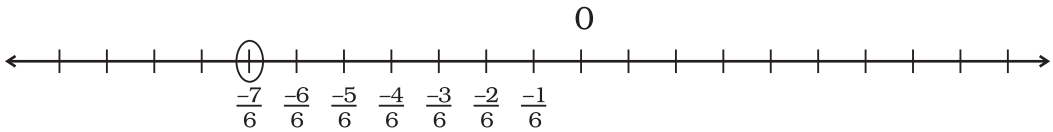
(ii) (a)



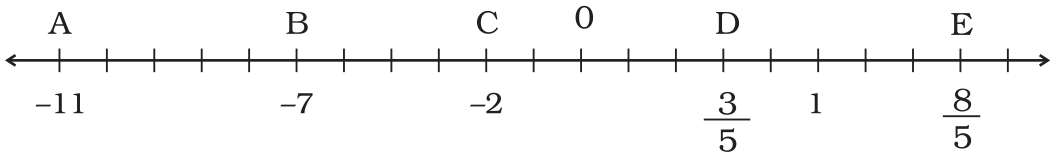
(b)



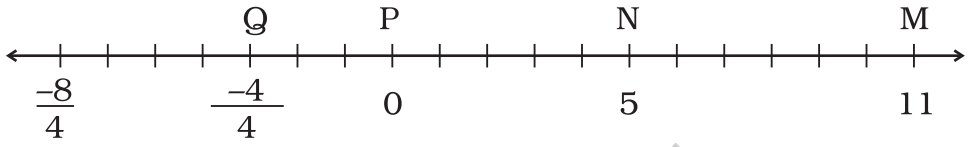
(c)



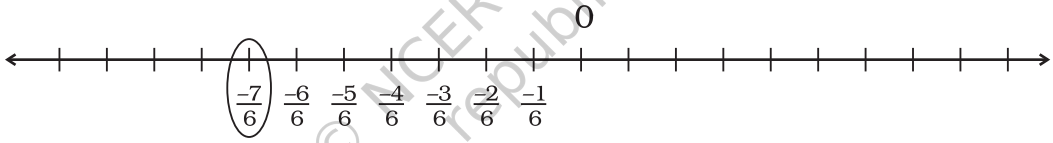
(iii) (a)



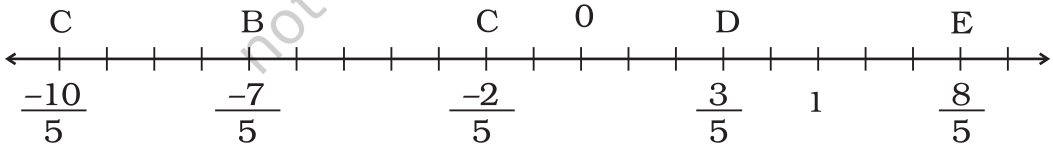
(b)



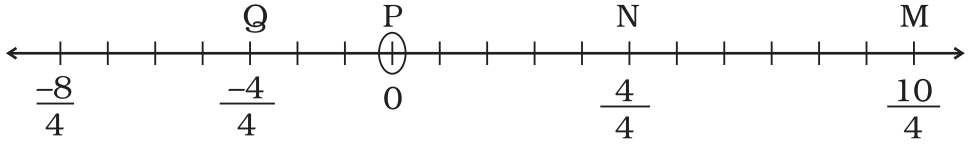
(c)



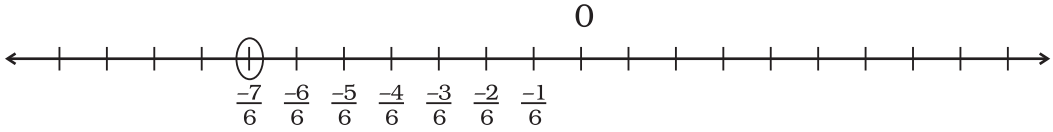
(iv) (a)



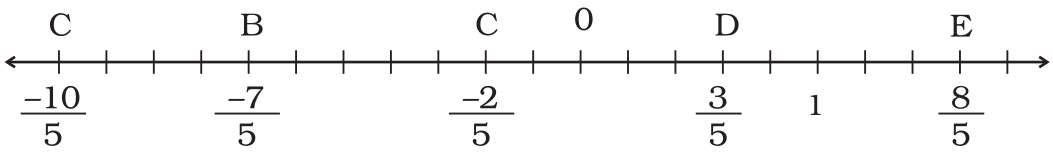
(b)



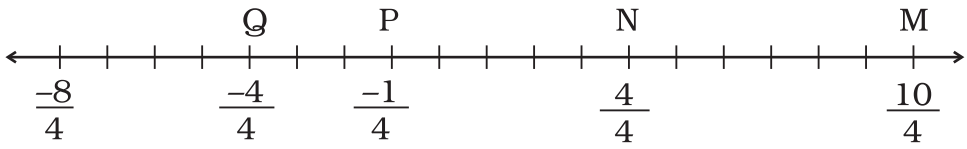
(c)



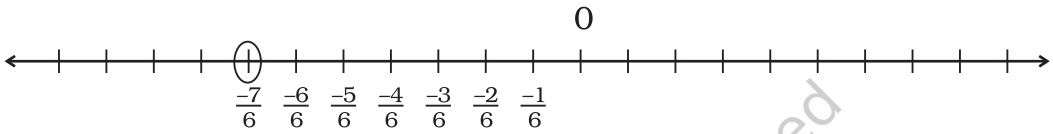
(v) (a)



(b)



(c)



Reflection of Responses

- In response (i), child is not able to mark rational numbers on a number line. He is at Stage I.
- In response (ii), child is not able to identify the marked points but is able to mark rational numbers on a number line. He is at Stage II.
- In response (iii), child is able to mark rational numbers on a number line and is able to write appropriate rational numbers for some of the marked points. He is at Stage III.
- In response (iv), child is able to mark rational numbers and is also able to identify rational number corresponding to marked points but commits some careless mistakes. He is at Stage IV.
- In response (v), child is able to mark rational numbers and is also able to identify rational numbers corresponding to the marked points. He is at Stage V.

Task 9

Indicator

Child can reason out logically with reference to mathematical operations (addition, subtraction and multiplication).

Task

Child is expected to read article 16.4 given in Class VIII, NCERT textbook, page 253 (child can also consult other books on this topic or visit any related website). Then she/he is expected to frame at least two questions based on the concept given in the article and present it in the classroom. Other students will also try to answer the questions.

Sample Presentation

Question (i)

$$\begin{array}{r} 3\,A \\ \times\, 2\,A \\ \hline 1\,B\,A \\ B\,0\,\times \\ \hline 8\,B\,A \end{array}$$

(ii)

$$\begin{array}{r} 4\,B\,C \\ \times\, C\,A \\ \hline B\,0\,C\,A \\ 4\,B\,C\,\times \\ \hline A\,B\,2\,A \end{array}$$

Solution for (i) A = 5, B = 7

Here, A can be 1, 5 or 6 because 1 × 1 = 1, 5 × 5 = 25 and 6 × 6 = 36 (these numbers when multiplied by itself give the number with same digit at its unit place).

In the second row 2 × A gives B 0 (unit's place is zero). So A = 5 [since 1 × 2 = 2, 6 × 2 = 12 and 5 × 2 = 10]

Hence problem becomes

$$\begin{array}{r} 3\,5 \\ \times\, 2\,5 \\ \hline 1\,B\,5 \\ B\,0\,\times \\ \hline 8\,B\,5 \end{array}$$

Now B = 7 since 5 × 35 = 175 and 35 × 2 = 70

solution for (ii) A = 7, B = 3 , C = 1

Here C = 1 only because any number multiplied by 1 gives the same number and first row of solution shows A × C is the number with unit digit A.

Now problem becomes

$$\begin{array}{r} 4\,B\,1 \\ \times\, 1\,A \\ \hline B\,0\,I\,A \\ 4\,B\,1\,\times \\ \hline A\,B\,2\,A \end{array}$$

Here third row of solution shows A = B + 4

But B ≠ 1 (as C=1). So, 4B1 × A = B 0 1 A (first row of solution) shows that A × B is an odd number. Hence A and B are odd numbers. So either A = 7 and B = 3 or A = 9 and B = 5. But 5 × 9 = 45 (does not go with first row of solution). Therefore, A = 7 and B = 3.

Reflection on Responses

- If the child is not able to frame questions or not able to give right solution of the question framed by herself, she is at Stage I.
- If the child is able to frame questions but not able to give proper explanation on how to arrive at the right solution of the problem, then she is at Stage III.
- If the child is able to frame questions and can explain how to arrive at the right solution of the problem, then she is at Stage V.

In this sample presentation, the child will be placed at Stage V.

Task 10

Indicator

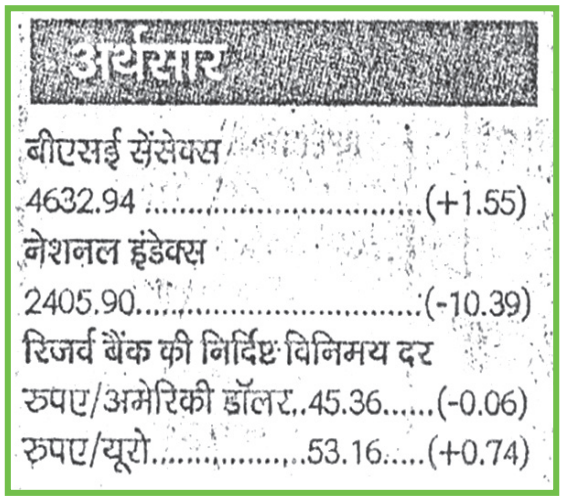
Child can interpret information given in integers.

Task

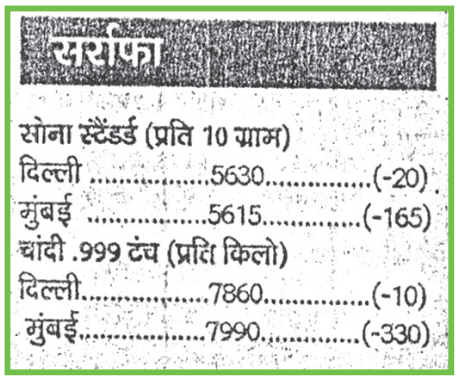
Child is asked to collect and interpret information from various sources like magazines or newspapers, in which negative integers are used.

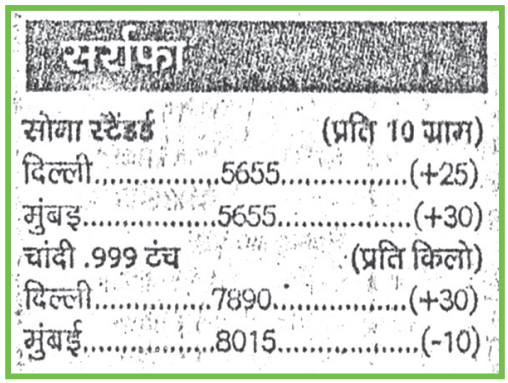
Sample Responses

(i)



(ii)





(iii) **Interpretation:** Numbers in bracket are integers.

MONTH	AVERAGE SUNLIGHT PER DAY (Hours)	TEMP IN DEGREES CELSIUS			
		Average of past 10 years		Expected this winter	
		Max	Min	Max	Min
October	9	39	18	34	11
November	10	34	9	29	5
December	9	28	5	23	1
January	7	29	-1	21	-2
February	9	32	0	24	-5

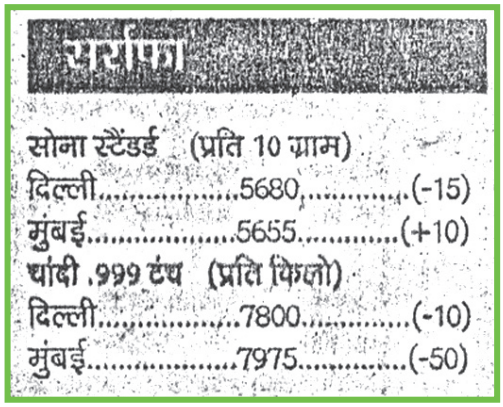
(a)

(b)

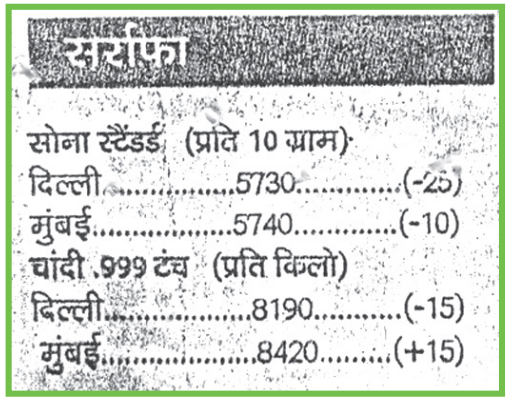
Interpretation : (a) Numbers in Maximum temperature and Minimum temperature in winter column are positive integers which first decrease and then increase. Number in Average sunlight column first increase then decrease and then again increase.

(b) Numbers in bracket are negative integers.

(iv)



(a)



(b)

Interpretation : (a) For Gold numbers in bracket are integers which are increasing. For Silver numbers in bracket are integers which are decreasing.

(b) For Gold and Silver both, integers in bracket are increasing.

Footnote

(i) अर्थसार — Arthasaar
(ii) सर्राफा — Sarrafa
(iii) सोना स्टैंडर्ड — Gold Standard
(iv) चाँदी — Silver
(v) प्रति 10 ग्राम — Per 10 gram
(vi) प्रति किलो — Per Kilogram
(vii) बीएसई सेंसेक्स — BSE Sensex

Reflection on Responses

- Child is not able to collect the information. He is at Stage I.
- In response (i), the information collected by the child involves decimal numbers and so is irrelevant. He has also not given any interpretation. He is at Stage II.
- In response (ii), child has collected some material but is not able to interpret it properly. He is at Stage III.
- In response (iii), child is able to interpret information with positive integers only. He is at Stage IV.
- In response (iv), child is able to interpret the material collected by him. He is at Stage V.

3.2 Algebra

Indicators of Learning

Child can

1. construct an algebraic expression in a given situation and vice-versa.
2. identify terms of an expression, factors and coefficients of a term.
3. identify like and unlike terms.
4. classify expressions as monomial, binomial, trinomial.
5. add, subtract and multiply algebraic expressions.
6. find the value of an algebraic expression for given values of the variable.
7. develop standard identities and apply them in solving problems.

8. factorise a given algebraic expression.
9. divide a polynomial by another polynomial by factorisation.
10. differentiate between an equation and an identity.
11. construct linear equations in one variable in a given context and vice-versa.
12. solve equations and verify the solutions.
13. apply equations in solving real life problems.
14. solve equations reducible to linear form.

Task 1

Indicator

Child can construct a linear equation and solve it.

Task

Child is asked to make an **E** and using them to make patterns as in the figure, with the help of matchsticks

E, EE, EEE,.....

For that teacher asks them

(a) How many matchsticks are needed to make an E?

(b) How many matchsticks will be needed to make ten E?

(c) How many matchsticks are needed to make an E?

(d) You have only 45 matchsticks. How many E can be formed? Write this in the form of an equation.

(e) How many E will be formed with 45 matchsticks?

Sample Responses

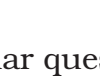
	(a)	(b)	(c)	(d)	(e)
(i)	5	-	-	-	-
(ii)	5	10 × 5	-	-	-
(iii)	5	10 × 5	5n	-	-
(iv)	5	10 × 5	5n	5n = 45	7
(v)	5	10 × 5	5n	5n = 45	9

Reflections on Responses

- In response (i), the student has responded only for part (a). For the remaining parts, he either gives wrong answer or is not able to reply. He has no understanding of the concept. He is on Stage I of proficiency level.
- In response (ii), the student could respond only for 10 E's and could not generalise. He also does not give answer for other parts. He has only little understanding of the concept and so is on Stage II.
- In response (iii), the student could make an algebraic expression but is not able to form an equation. He has understood the concept but is not able to apply it. He is on Stage III.
- In response (iv), the student could form an equation but makes careless mistake in solving it. He is on Stage IV.
- In response (v), the student is able to form an equation and also solves it correctly. He is on Stage V.

Remedial Measures

Remediation can be provided by asking similar questions for making **L, A, V** etc.

For reinforcement, ask similar questions for making  , etc.

Task 2

Indicator

Child can construct an algebraic expression in a given situation and vice-versa.

Task

Teacher asks children to write an expression for 'n is multiplied by –2 and then the result obtained is subtracted from 1'.

Sample Responses

(i) $n - 2 - 1$

(ii) $n \times -2 - 1$

(iii) $1 - 2 \times n = 1 - 2\ n$

(iv) $1 - (-2\ n) = 1 - 2\ n$

(v) $1 - (-2\ n) = 1 + 2\ n$

Reflection on Responses

- In response (i), the child has not at all understood the concept. So he is on Stage I.
- In response (ii), the child has little knowledge of the concept. Also he is putting two operations together and so is on Stage II.
- In response (iii), the child has understood but is not able to apply the concept correctly and so is on Stage III.
- In response (iv), the child has understood the concept but is making careless mistake and so is on Stage IV.
- In response (v), the child has understood the concept and is applying it correctly and so is on Stage V.

Remedial Measures

Remediation can be provided by giving more practice of similar questions starting with two terms involving one operation. Focus must be laid on making the child understand the language of the problem.

Task 3

Indicator

Child can identify like and unlike terms.

Task

Teacher asks students to identify the following as like and unlike terms in each pair:

$(2x, 2x^2)$, $(3y, -7y)$, $(7xy, -2xy)$, $(7x, 7xy)$, $(4mn^2, 4m^2n)$, $(x^2, -x^2)$, $(2xy, -4yx)$, $(1, 5)$

Sample Responses

	Like Terms	Unlike Terms
(i)	$(2x, 2x^2)$ $(3y, -7y)$ $(4mn^2, 4m^2n)$ $(x^2, -x^2)$	$(7xy, -2xy)$ $(7x, 7xy)$ $(2xy, -4yx)$ $(1, 5)$
(ii)	$(3y, -7y)$ $(7xy, -2xy)$ $(x^2, -x^2)$	$(2x, 2x^2)$ $(7x, 7xy)$ $(4mn^2, 4m^2n)$ $(2xy, -4yx)$ $(1, 5)$

(iii)	$(3y, -7y)$ $(7xy, -2xy)$ $(x^2, -x^2)$	$(2x, 2x^2)$ $(7x, 7xy$ $(4mn^2, 4m^2n)$ $(2xy, -4yx)$ $(1, 5)$

Reflection on Responses

- In response (i), the child has little understanding of the concept and so is at Stage II.
- In response (ii), the child has understood the concept but is not able to understand that xy and yx are like terms and so is at Stage III.
- In response (iii), the child has understood the concept and is able to apply it correctly in all the situations and so is on Stage V.

Remedial Measures

Remediation can be provided by giving more practice of such questions. Questions can also be asked to give two like terms or two unlike terms for a given term. The use of commutativity can also be recapitulated.

Task 4

Indicator

Child can subtract algebraic expressions.

Task

Teacher asks students to subtract

$2x^2 + x^4 + 3x$ from $3x^4 + 7x^2 + 8$

Sample Responses

(i) $3x^4 + 7x^2 + 8$

$\underline{x^4 + 2x^2 \quad + 3x}$

$4x^4 + 9x^2 + 8 + 3x$

(ii) $3x^4 + 7x^2 + 8$

$\underline{\_\,x^4 + \_\,2x^2 + \_\,3x}$

$2x^4 + 5x^2 + 5x$

(iii) $3x^4 + 7x^2 + 8 - 2x^2 + x^4 + 3x$

$= 4x^4 + 5x^2 + 8 + 3x$

(iv) $3x^4 + 7x^2 + 8$

$\underline{\_\,2x^2 \_\,+ x^4 \_\,+ 3x}$

$x^4 + 6x^2 + 8 - 3x$

(v) $x^4 + 2x^2 + 3x$

$\underline{\_3x^4 + \_7x^2 + \_8}$

$-2x^4 + 5x^2 + 3 - 8$

(vi) $3x^4 + 7x^2 + 8$

$\underline{\_x^4 + \_2x^2 + \_3x}$

$3x^4 + 5x^2 + 8 - 3x$

(vii) $3x^4 + 7x^2 + 8$

$\underline{\_x^4 + \_2x^2 \_+ \_3x}$

$2x^4 + 5x^2 - 8 - 3x$

(viii) $3x^4 + 7x^2 + 8$

$\underline{\_x^4 + \_2x^2 + \_3x}$

$2x^4 + 5x^2 + 8 - 3x$

(ix) $3x^4 + 7x^2 - (2x^2 + x^4 + 3x)$

$= 3x^4 + 7x^2 - 2x^2 - x^4 - 3x$

$= 2x^4 + 5x^2 + 8 - 3x$

Reflection on Responses

- Teacher noticed that the students are facing problem in language comprehension, identification and operations on like and unlike terms and the concept of subtraction of algebraic expressions.
- In responses (i) and (ii), the child is not aware of the concept of subtraction of algebraic expressions and so he is on Stage I.
- In responses (iii) and (iv), the child knows the concept of subtraction but does not have complete understanding of like-unlike terms and is therefore, on Stage II.
- In response (v), the child knows the concept but is not able to apply it due to language comprehension problem and so is at Stage III.
- In responses (vi) and (vii), he understands the concepts involved but is making careless mistakes. He is on Stage IV.
- In responses (viii) and (ix), he understands the concepts and is able to apply them correctly. He is on Stage V.

Remedial Measures

- Remediation in language comprehension problem can be provided by giving questions involving numbers. For example:

  Subtract 3 from –1

  Subtract – 4 from 9

  From 3 subtract –2 etc.

These can be given on worksheets. Questions can be in the form of true/false, fill in the blanks or choose the correct answer.

- Remediation in problems of like-unlike terms can be given by related questions like:

  Identify pairs of like-unlike terms from

  $(x^2, -x^2)$, $(5x, 5x^2)$, $(2xy, -4yx)$, $(1, 5)$, $(4mn^2, 4m^2n)$

  They can also be asked to give three like terms of some given term say $-5x^3$.

- Remediation in concept of subtraction of algebraic expressions can be provided by giving more practice of questions ranging from simple to complex.
- Remediation in all the above type of problems can be provided by conducting quiz in the class.
- For reinforcement, the teacher can ask questions such as:

  What should be added in $2x^3 + x - 1$ to obtain $x^2 + 2x - 7$?

Task 5

Indicator

Child can add and subtract algebraic expressions.

Task

Teacher asks students "what should be added to $2x^2 + 3xy$ to obtain $x^2 + 2xy + y^2$."

Sample Responses

(i) $2x^2 + 3xy$

$$\frac{x^2 + 2xy + y^2}{3x^2 + 5xy + y^2}$$

(ii) $x^2 + 2xy + y^2$

$$\frac{\_\,2x^2 + \_\,3xy\,\_}{-x^2 - xy - y^2}$$

(iii) $2x^2 + 3xy$

$$\frac{\_\,x^2 + \_\,2xy + \_\,y^2}{x^2 + xy - y^2}$$

(iv) $x^2 + 2xy + y^2$

$$\frac{\_\,2x^2 + \_\,3xy}{-2x^2 - xy + y^2}$$

(v) $x^2 + 2xy + y^2$

$\underline{\_2x^2 + \_3xy}$

$-x^2 - xy + y^2$

So, $-x^2 - xy + y^2$ should be added.

(vi) $x^2 + 2xy + y^2 - (2x^2 + 3xy)$

$= x^2 + 2xy + y^2 - 2x^2 - 3xy$

$= -x^2 - xy + y^2$

So, $-x^2 - xy + y^2$ should be added.

Reflections on Responses

- In response (i), the child has no understanding of the concept and so, she is on Stage I.
- In response (ii), she has little understanding of the concept that is about what she has to subtract to achieve the answer and so, she is on Stage II.
- In response (iii), she has the understanding of the concept but is not able to apply it correctly. So, she is on Stage III.
- In response (iv), she has the understanding of the concept but is making careless mistake and so, she is on Stage IV.
- In responses (v) and (vi), she has understanding of the concept and is able to apply it correctly and so, she is on Stage V.

Remedial Measures

Remediation can be provided by giving more practice of such questions by giving numbers in place of algebraic expressions. More questions can he given for practice, initially involving like terms and then switching over to both like and unlike terms.

Task 6

Indicator

Child can find the value of an algebraic expression for given value of the variable.

Task

Teacher asks students to find the value of $x^4 - x^2 - 3x + 1$ at $x = -2$

Sample Responses

(i) $-2\ -2\ -3\ -2 + 1$

(ii) $-2^4\ -2^2 - 3 \times 2 + 1 = -8 - 4 - 6 + 1 = -17$

(iii) $(-2)^4\ -2^2 - 3\ (-2) + 1 = -16 - 4 - 6 + 1 = 27$

(iv) $(-2)^4\ -(-2)^2 + 6 + 1 = -8\ -\ 4 + 6 + 1 = 3$

(v) $(-2)^4 - (-2)^2 - 3\ (-2) + 1 = 16 - 4 - 6 + 1 = 7$

(vi) $(-2)^4 - (-2)^2 - 3\ (-2) + 1 = 16 - (-4) + 6 + 1 = 16 + 4 + 6 + 1 = 27$

(vii) $(-2)^4 - (-2)^2 - 3\ (-2) + 1 = 16 + 2^2 + 6 + 1 = 16 + 4 + 6 + 1 = 27$

(viii) $-2^4 - (-2)^2 - 3\ (-2) + 1 = -16 - 4 + 6 + 1 = -13$

(ix) $(-2)^4 - (-2)^2 - 3\ (-2) + 1 = 16 - (4) - 5 + 1 = 8$

(x) $(-2)^4 - (-2)^2 + 6 + 1 = -8 - (4) + 6 + 1 = -5$

(xi) $(-2)^4 - (-2)^2 - 3\ (-2) + 1 = 16 - 4 + 6 + 1 = 19$

Reflection on Responses

- In response (i), the child has no understanding of the concept of finding the value of algebraic expression involving exponential terms when the base is a negative integer. So she is on Stage I.
- In responses (ii) and (iii), the child is not able to find the value of powers. So she is on Stage II.
- In responses (iv), (v) and (vi), she has the understanding of exponents but is not able to compute values correctly. So she is on Stage III.
- In responses (vii), (viii) and (ix) she has the understanding of the concept but is making careless mistakes. So, she is on Stage IV.
- In response (x), she is able to find value of the expression correctly so, she is on Stage V.

Remedial Measures

Teacher noticed that the students are facing problem in writing the exponent when the base is negative, multiplying two negative numbers, simplification of brackets, calculation of exponents and subtraction of numbers.

Some problems involving multiplication of negative integers such as listed below may be given to the children for practice.

$-2\ (-4)$

$-10\ (-1)$

$-1 \times (-3)$

$-(-2)3$

$-(-4)2$

$-(-1)10$

$2 - 4 + 3$

$-7 + 5 - 4 + 12$ etc.

Then he may be asked to compute values of an algebraic expression by giving different expressions as listed below:

At $x = 1$, the value of the algebraic expression $x^3 - x - 2$ is .....

At $x = -3$, the value of the algebraic expression $3x^3 + x^2 + 1$ is .....

At $x = 0$, the value of the algebraic expression $2x^4 - x^3 + 2x^2 - 3$ is .....

Task 7

Indicator

Child can construct an algebraic expression in a given situation and vice-versa.

Task

Teacher plays a game with the students. She writes on the board the instructions to be followed and asks to write all their steps in notebooks:

Step Number

1. Think of a number x
2. Add 2 to it
3. Multiply the result by 2
4. Add 3 to the result obtained in step 3
5. Now subtract the original number from the result in step 4
6. Add 13 to the result in step 5
7. Subtract the original number from the result.

What number do you get finally? Is it 20?

Sample Responses

	Steps						
Response	1	2	3	4	5	6	7
(i)	x	—	—	—	—	—	—
(ii)	x	x+2	—	—	—	—	—
(iii)	x	x+2	2x+4	2x+7	—	—	—
(iv)	x	x+2	2x+4	2x+7	x+7	x+19	19
(v)	x	x+2	2x+4	2x+7	x+7	x+20	20

The number so obtained is 20.

Reflection on Responses

- In response (i), the student is able to respond only upto Step 1. So she is on Stage I.
- In response (ii), she is able to respond upto Step 2 i.e., can add only. She is not able to multiply the expression. So is on Stage II.
- In response (iii), she is able to respond only upto Step 4 i.e., can add and multiply also. She is not able to subtract two algebraic expressions. So she is on Stage III.
- In response (iv), she is able to respond correctly upto last step but she is making careless mistakes and hence is not able to get the correct answer. She is on Stage IV.
- In response (v), she is able to respond correctly in all the steps and so is on Stage V.

Remedial Measures

- Initially, child may be asked to take a particular number in place of x and repeat all the steps. More practice of addition, subtraction and multiplication of algebraic expressions (monomials and binomials) be given to him. After sufficient practice, he may be asked to replace the given number by a variable.
- Remediation can also be provided by giving questions such as:
  - 7 added to 2x
  - y is multiplied by –8
  - Subtract 11 from 3t.

Task 8

Indicator
Child can factorise a given algebraic expression.

Task
Teacher asks students to factorise $x^3 - 4x^2 + 4x$

Sample Responses

(i) $x^3 - 4x^2 + 4x$
$= x^3 - 4\,(x^2 + 1)$
$= (x^3 - 4)\,(x^2 + 4)$

(ii) $x^3 - 4x^2 + 4x$
$= x\,(x^2 - 4\,x + 4)$

(iii) $x^3 - 4x^2 + 4x$
$= x\,[x^2 - 4\,x + 4]$
$= x\,[\,x^2 - 2x - 2x + 4]$
$= x\,[x\,(x - 2) - 2\,(x - 2)]$
$= x\,(x - 2) - (x - 2)$

(iv) $x^3 - 4x^2 + 4x$
$= x\,[x^2 - 4\,x + 4]$
$= x\,[\,x^2 - 2x - 2x + 4]$
$= x\,[x\,(x - 2) - 2\,(x - 2)]$
$= x\,[(x - 2)\,(x + 2)]$

(v) $x^3 - 4x^2 + 4x$
$= x\,[x^2 - 4\,x + 4]$
$= x\,[x^2 - 2 \times 2x + 2^2]$
$= x\,(x + 2)^2$

(vi) $x^3 - 4x^2 + 4x$
$= x\,[x^2 - 4\,x + 4]$
$= x\,[\,x^2 - 2x - 2x + 4]$
$= x\,[x\,(x - 2) - 2\,(x - 2)]$
$= x\,(x - 2)\;\;(x - 2)$

(vii) $x^3 - 4x^2 + 4x$
$= x\,[x^2 - 4\,x + 4]$
$= x\,[x^2 - 2 \times 2x + 2^2]$
$= x\,(x - 2)^2$

Reflections on Responses

- In response (i), the child is making conceptual error i.e., not able to take x as a common factor. So, she is on Stage I.
- In response (ii), the child is able to find common factor x but is not able to factorise $x^2 - 4x + 4$. So, she is on Stage II.

- In response (iii), the child is able to split the middle term ($-4x$) correctly for the purpose of factorisation of $x^2 - 4x + 4$ but is not able to factorise further. So, she is on Stage III.
- In responses (iv) and (v), children are making careless mistakes in factorisation. They are at Stage IV.
- In responses (vi) and (vii), the children are giving correct answer and so they are on Stage V.

Remedial Measures

The students may be given questions in the increasing order of complexity to identify and take out common factors from the given expressions.

$6p - 12q$

$14ab + 35abc$

$x^2yz + xy^2z + xyz^2$

$10y^2 - 6y + 2y - 3$

$x^2 + 7x + 10$

$x^2 + 2x - 3$

$3x^2 - 30x + 63$

Task 9

Indicator

Child can solve equations reducible to linear form.

Task

Teacher asks children to solve the following equation:

$$\frac{4x}{4x-3} - 1 = \frac{2}{3}$$

Sample Responses

(i) $\frac{4x}{4x-3} - 1 = \frac{2}{3} \Rightarrow \frac{4x}{4x-3} - 1 = \frac{2}{3} = \frac{x}{x-3} = \frac{2}{3} - 1$

(ii) $\frac{4x}{4x-3} = \frac{2}{3} + 1 \Rightarrow \frac{4x}{4x-3} = \frac{5}{3} \Rightarrow \frac{4x \times 3 = 5 \times (4x-3)}{3(4x-3)}$

$$\Rightarrow \frac{12x = 20x + 15}{3(4x-3)} \quad \Rightarrow \quad \frac{12x - 20x = 15}{3(4x-3)} \Rightarrow \frac{-8x = 15}{3(4x-3)} \Rightarrow x = \frac{-15}{8}$$

(iii) $\frac{4x}{4x-3} = \frac{2}{3} + 1 \quad \Rightarrow \quad \frac{4x}{4x-3} = \frac{4}{3} \Rightarrow 12x = 16x - 12$

$\Rightarrow 12 = 4x \Rightarrow x = 3$

(iv) $\frac{4x}{4x-3} = \frac{2}{3} + 1 \quad \Rightarrow \quad \frac{4x}{4x-3} = \frac{5}{3} \Rightarrow 12x = 20x - 15$

$\Rightarrow 8x = 15 \Rightarrow x = \frac{8}{15}$

(v) $\frac{4x}{4x-3} = \frac{2}{3} + 1 \quad \Rightarrow \quad \frac{4x}{4x-3} = \frac{5}{3} \Rightarrow 12x = 20x - 15$

$\Rightarrow 8x = 15 \Rightarrow x = \frac{15}{8}$

Reflections on Responses

- In response (i), the child is making conceptual and computational mistakes. So, she is on Stage I.
- In response (ii), the child is making conceptual mistake at Step 2 by writing the equations in an incorrect way. So, she is on Stage II.
- In response (iii), the child understood the concept and is able to apply it correctly. But makes computational mistake at second step. So she is on Stage III.
- In response (iv), the child understands the concept, solves the equation but does careless mistake at the last step. So, she is on Stage IV.
- In response (v), the child is able to solve the equation correctly. So, she is on Stage V.

Remedial Measures

Teacher noticed that the students are facing problem in multiplication and division of algebraic expressions and in applying the rule of transpositions in solving the equation.

- For multiplication and division of algebraic expressions, teacher may give questions such as:
  - Simplify: $-3(x-4)$, $2x(3x+5)$, $5(3-t)$, $a^2(2ab-5c)$
  - True/False type questions such as:

$\frac{-x+7}{-3} = \frac{x+7}{3}$ (True/False)

$\frac{4x+1}{4} = x + 1$ (True/False)

$\frac{3x+6}{3} = x + 2$ (True/False)

$\frac{2x+5}{5} = 2x + 1$ (True/False)

- For applying the rule of transpositions, ask the students to solve questions of the following types:

  – $x + 4 = 2$, $5x + \frac{3}{2} = 14$, $6 = z + 2$

    $\frac{t}{2} = 5$, $\frac{2x}{3} = 12$

  – Write True or False

    $5x + \frac{3}{2} = 14 \Rightarrow 10x = 14 - 3$ (True/False)

    $\frac{x}{3} + 2 = \frac{5}{7} \Rightarrow 7x = 15 - 2$ (True/False)

    $\frac{x}{3} + 2 = \frac{5}{7} \Rightarrow \frac{x}{3} = \frac{5}{7} - 2$ (True/False)

    $\frac{x}{3} + 2 = \frac{5}{7} \Rightarrow 7(x + 2) = 15$ (True/False)

Task 10

Indicator

Child can apply equations in solving real life problems.

Task

Teacher asks students to find the present ages of Suhail and Sanya when their present ages are in the ratio 4 : 5 and eight years from now the ratio of their ages will be 5 : 6 (using equations).

Sample Responses

(i) Let the present ages be 4 and 5

So, $\frac{4+8}{5} = \frac{5}{6}$

(ii) Let their present ages be $4x$ and $5x$, respectively

So, $\frac{4x+8}{5x} = \frac{5}{6}$

$\frac{4x}{5x} = \frac{5}{6} - 8$

$\frac{4x}{5x} = \frac{-43}{6}$

$\frac{4}{5} = \frac{-43}{6}$

(iii) Let their present ages be $4x$ and $5x$

So, $\frac{4x+8x}{5x+8x} = \frac{5}{6}$

$\frac{12x}{13x} = \frac{5}{6}$

(iv) Let their present ages be $4x$ and $5x$

So, $\frac{4x+8}{5x} = \frac{5}{6}$

$6(4x + 8) = 5 \times 5x$

$24x + 8 = 25x$

$x = 8$

So, present age of Suhail is 32 years and

present age of Sanya is 40 years.

(v) Let their present ages be $4x$ and $5x$, respectively

$\frac{4x+8}{5x+8} = \frac{5}{6}$

$20x + 40 = 30x + 48$

$20x - 30x = 48 - 40$

$10x = 8$

$x = \frac{8}{10}$

(vi) Let their present ages be $4x$ and $5x$, respectively

$$\frac{4x+8}{5x+8} = \frac{5}{6}$$

$24x + 48 = 25x + 8$

$x = 40$

So, the present age of Suahil is $4 \times 40 = 160$ years

and present age of Sanya is $5 \times 40 = 200$ years

(vii) Let their present ages be $4x$ and $5x$, respectively

$$\frac{4x+8}{5x+8} = \frac{5}{6}$$

$24x + 48 = 25x + 40$

$x = 8$

(viii) Let the present ages be $4x$ and $5x$, respectively

So, $\frac{4x+8}{5x+8} = \frac{5}{6}$

$24x + 48 = 25x + 40$

$x = 8$

$\therefore$ Present age of Sanya = 32 years

Present age of Suhail = 40 years

(ix) Let the present ages be $4x$ and $5x$, respectively

So, $\frac{4x+8}{5x+8} = \frac{5}{6}$

$24x + 48 = 25x + 40$

$x = 8$

$\therefore$ Present age of Suhail = 32 years

Present age of Sanya = 40 years

She verifies that 32 : 40 = 4 : 5 and 32 + 8: 40 + 8 = 5 : 6

Reflections on Responses

- In response (i), child is not even able to form a proper equation. So, she is on Stage I.
- In responses (ii), (iii) and (iv), the child understands the concept of ratio but is neither able to form an equation nor understands how to form an equation and makes mistakes in computing ratios. She also makes mistakes in subtraction of algebraic expressions. So, she is on Stage II.
- In responses (v) and (vi), the child is able to form correct equation but is not able to solve due to computational error. So, she is on Stage III.
- In responses (vii) and (viii), she makes careless mistakes. So, she is on Stage IV.
- In response (ix), the child is able to solve the question correctly using the concept of equations. So she is on Stage V.

Remedial Measures

- For remediation of these errors, teacher may ask simple questions such as:
  - Convert into fractions 4 : 3, 1 : 2 etc.
  - Simplify: $2x - 3x$, $5t - 4t$, $4y - y$ etc.
  - Your age is 13 years. After 4 years what will be your age? How have you calculated this? If your age now is 13x, what will be your age after 4 years?
  - If your sister's age is x years and your mother's age is 3x
    (i) What will be their ages after 4 years?
    (ii) Write their ages after 4 years in the form of ratio.
  - Solve: $2(x + 3) = 16$, $-4(2 + x) = -8 + x$

3.3 Geometry

Geometry gives ample opportunity for exact reasoning, for real induction applied to very simple data, correlation with other works like architecture, geography, and the physical sciences as well as with algebra, for exercise of the space intuition, for practical applications, for drills in numerical computation, for training in habit of neatness and exactitude and for cultivation of the powers of precise thoughts and accurate expressions. Development of the space intuition and training for active and careful reasoning might be regarded as the main function of the teaching of geometry.

The reason to teach geometry is to give children certain ideas about the nature of proofs. The purpose of teaching geometry is not just to acquaint children with geometric facts on theorems but to teach the nature of deductive

proof and to equip children with the model for logical thinking and reasoning. Problem of geometry teacher is not only to bring her children to an understanding of the nature of proof but also to stimulate them to make actual use of this concept in their lives outside of mathematics classrooms. It is of little value to an individual to understand thoroughly what is necessary to prove a proposition unless the knowledge leads him to clarify his own thinking and to be critical of new ideas presented to him.

Indicators of Learning

Lines, Rays and Angles

Child can

1. distinguish between rays and line segment.
2. measure the angle.
3. differentiate between different types of angles.
4. discuss adjacent angles, linear pair, vertically opposite angles.
5. explain definition of collinear and non-collinear points.
6. list different properties of different angles (i.e., complementary, supplementary, adjacent, linear pair etc.).
7. state and prove that "If two lines intersect then the vertically opposite angles are equal".
8. identify pairs of angles made by a transversal with two lines.
9. verifies the properties related to pairs of angles such as corresponding angles, alternate interior angles etc., when a transversal intersects two parallel lines.

Quadrilateral and Polygon

Quadrilateral: Shapes and Properties

Child can

1. differentiate between simple closed curve, a closed curve that is not simple, simple curve that is not closed and a curve which is not simple.
2. differentiate between convex and concave polygon.
3. recognise regular and irregular polygon.
4. identify diagonals of given polygon.
5. differentiate between interior and exterior angles.

6. find a rule for determining the number of triangles that a polygon with n sides can be decomposed into by drawing diagonals from a single vertex.
7. compute the sum of measures of the interior angles of a polygon.
8. compute the measure of exterior angle of a regular polygon of n sides.
9. recognise the curves that are polygons and curves that are not polygons.
10. explain and differentiate properties of parallelogram and some special parallelograms.
11. compare the properties of rhombus and rectangle.
12. conclude that square has the properties of a parallelogram, rhombus and a rectangle.
13. conclude that in a rectangle, each angle is a right angle.
14. recall the sum of angles of a quadrilateral.
15. explain that diagonal of a parallelogram divides it into two congruent triangles.

Triangles: Shapes and Properties

Child can

1. identify the type of triangle.
2. state and verify properties of triangles, such as angle sum property, exterior angle property, sum of any two sides is greater than third side etc.
3. evaluate the third angle by using given two angles.
4. recognise congruent objects/figures.
5. differentiate between congruent and similar figures.
6. discriminate between congruent and non-congruent triangles.
7. understands SSS, SAS, ASA, AAS, RHS congruence rules.
8. assess congruence of triangles using congruence rule.
9. identify corresponding sites and corresponding angles in two congruent triangles.

Circle: Shapes and Properties

Child can

1. list various circular objects.
2. construct a circle with compasses.
3. observe that longest chord is diameter.

Practical Geometry

Child can

1. perform the following elementary constructions:
   - (i) line segment equal to a given line segment.
   - (ii) perpendicular bisector of a given line segment.
   - (iii) angle equal to a given angle.
   - (iv) perpendicular to a line from a given point.
   - (v) line parallel to a given line through a point not lying on the line.
   - (vi) bisector of a given angle.
   - (vii) angles of measures 60 , 30 , 45 , 90 etc.
2. draw a triangle in the following cases:
   - (i) when three sides are given (SSS).
   - (ii) when two sides and included angle are given (SAS).
   - (iii) when two angles and included side are given (ASA).
   - (iv) a right triangle when hypotenuse and a side are given (RHS).
3. construct quadrilaterals with the given dimensions.
4. judge whether a quadrilateral can be made with any five measurements or not.

Symmetry

Child can

1. identify objects/figures having line symmetry.
2. find and count lines of symmetry in given objects/figures.
3. identify rotational symmetry.

Task 1

Indicator

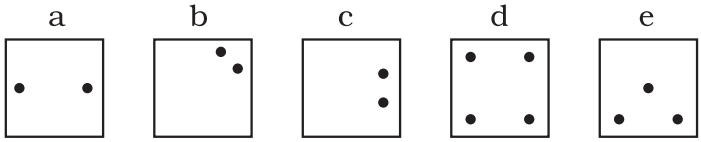
Child can find the line of symmetry in given figures.

Task

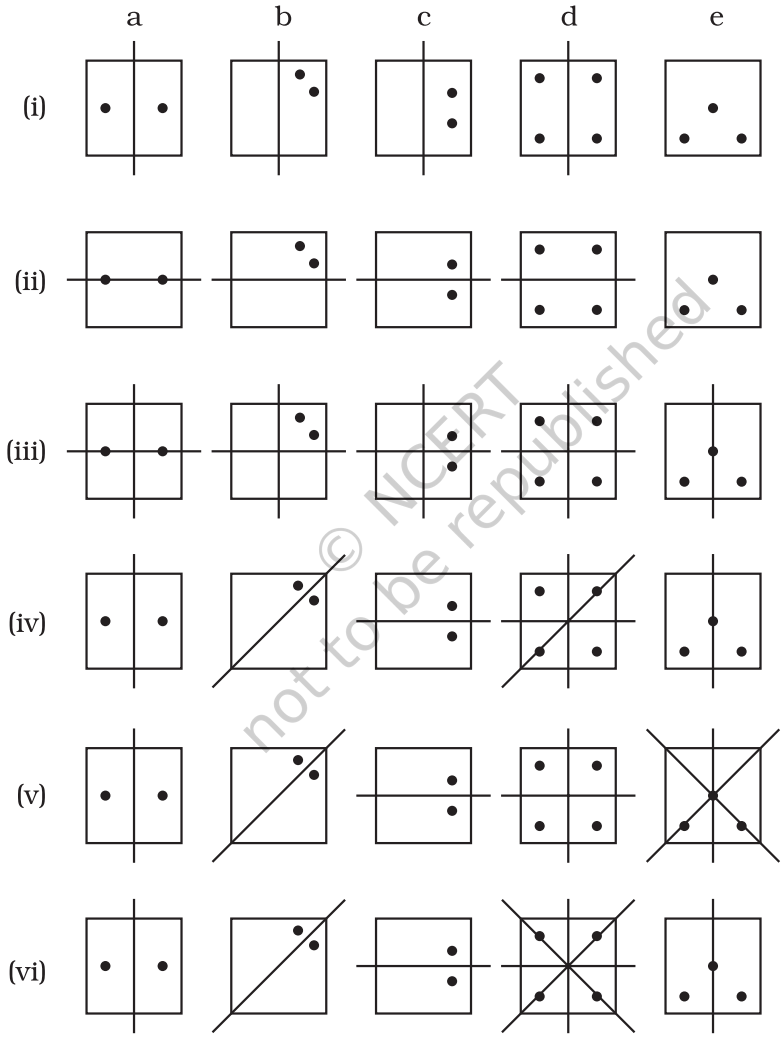
(i) Teacher draws five squares as given below on the board and then asks the students to find their lines of symmetry.

or

(ii) Teacher can provide five square paper cards to the students with dots as punch in the figures shown below. Teacher can now instruct students to find the lines of symmetry in each of these cards.



Sample Responses **Correct Responses**



(i) a, d

(ii) a, d

(iii) a, c, d, e

(iv) a, b, c, d, e

(v) a, b, c

(vi) Correct answer

Reflections on Responses

- In responses (i), (ii) and (iii), students have little understanding of the concept. They are on Stage II.
- In response (v), it is seen that student can understand the concept but cannot apply it completely. She is on Stage III.

- In response (iv), it can be seen that the student has an understanding of the concept but committed careless mistake. She is on Stage IV.
- In response (vi), the student has understood the concept of symmetry and answered all parts correctly. She is on Stage V.

Remedial Measures

- Students at Stage I need to be made to revisit the concept, through simple examples.
- The students at Stage II can be given more questions based on lines of symmetry.
- Students at Stage III need more practice of multiple lines of symmetry.
- Students at Stage IV need more concentration in doing the activity. Hence, they need more practice.

Task 2

Indicator

Child can recall and use the following criterion of congruency of two triangles

(i) Side-Side-Side (SSS)

(ii) Side-Angle-Side (SAS)

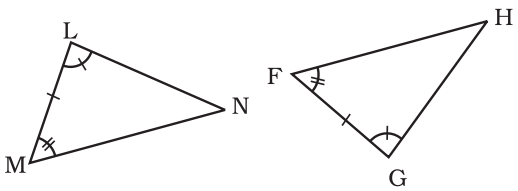
(iii) Angle-Side-Angle (ASA)

(iv) Right Angle-Hypotenuse-Side (RHS)

Task

Teacher provides a sheet of paper to the students on which triangles are drawn with marking on equal parts and asked them to state the congruency property related to the set of given triangles.

(a) (b)



(c) (d)

Sample Responses

	(i)	(ii)	(iii)	(iv)	(v)
(a)	No answer	SSS	SSS	SSS	SSS
(b)		SSA	SAS	SAS	SAS
(c)		AAS	ASA	ASS	ASA
(d)		SSA	SSA	RHS	RHS
Number of Correct Responses	$\frac{0}{4}$	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$	$\frac{4}{4}$

Reflections on Responses

- In response (i), child does not understood the concept. She is on Stage I.
- In response (ii), the child has a little understanding of the concept. She is on Stage II.
- In response (iii), the child is unable to apply the RHS criterion and confused it with SSA. She is on Stage III.
- In response (iv), the child committed careless mistake as she did not observe congruency criterion in all the figures carefully. She is on Stage IV.
- In response (v), child succeeded in understanding the concept of congruency. She is on Stage V.

Note: The response of figure (c) may be ASA as well as AAS as some students may think beyond the content. So both are correct responses.

Remedial Measures

- Students in the Stage I need to revisit the concept of congruency of triangles.
- Students at other stages need to be explained about the congruence criterion which they are not aware of.

Task 3

Indicator

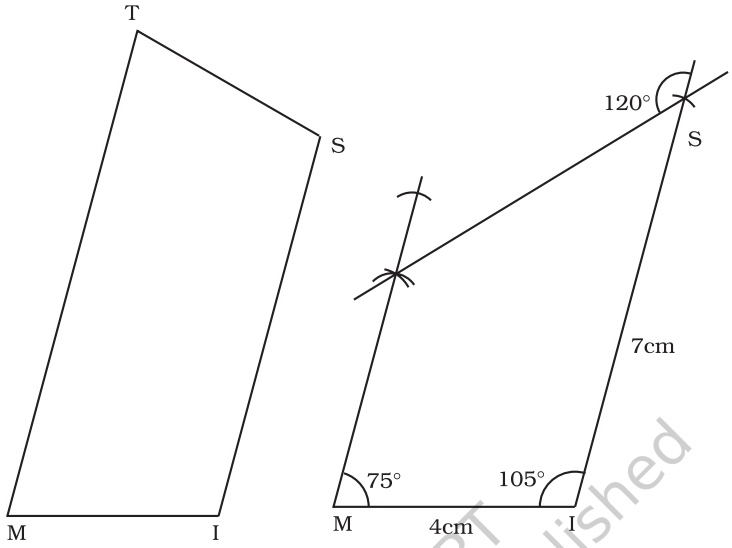
Student can construct a quadrilateral using given measurements.

Task

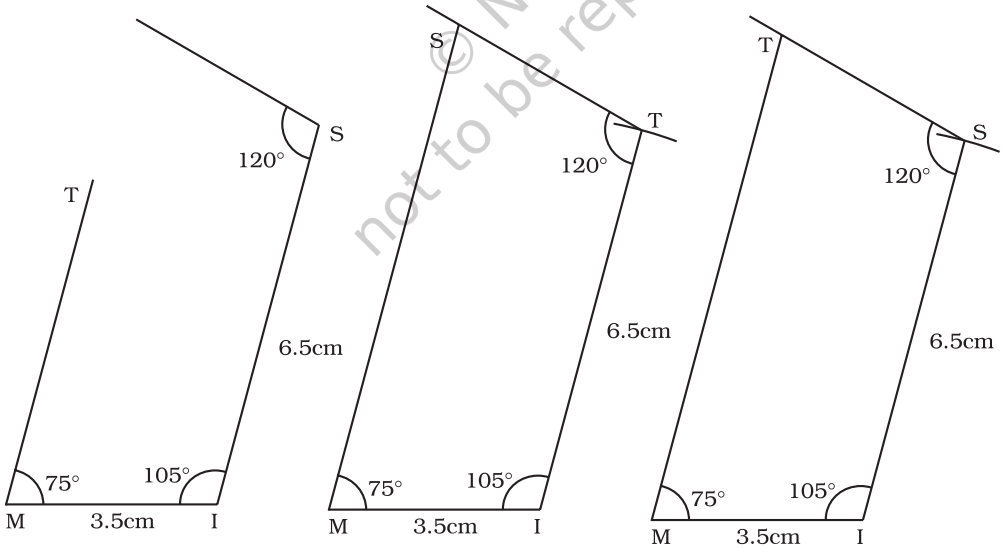
Student is asked to construct a quadrilateral MIST with the following measurements:

MI = 3.5 cm; IS = 6.5 cm; ∠I = 105 ; ∠M = 75 ; ∠S = 120

Sample Responses



(i) (ii)



(iii) (iv) (v)

Reflections on Responses

- In response (i), the student is not capable of handling geometric instruments. He has simply drawn a quadrilateral without understanding the given task. He is on Stage I.

- In response (ii), the student has rounded off the lengths and marked few arcs unnecessarily which are not expected in the construction. But he has constructed base angles with protractor correctly though ∠S is not correctly drawn. He has a little understanding of concept. He is on Stage II.
- In response (iii), the student is unable to apply the concept as he has constructed ∠S =120 at S incorrectly. He is on Stage III
- In response (iv), the student is good enough to construct the quadrilateral with the given measurements but because of carelessness he put T instead of S as a vertex. He is on Stage IV.
- In response (v), the student has understood the concept of construction of a quadrilateral with the given measurements well. He is on Stage V.

Remedial Measures

- The student in Stage I needs to revisit the concept of construction of quadrilateral using the given measurement with geometrical instruments.
- Student in Stage II has to understand that it is not required to round off the given lengths, as the constructions becomes inaccurate. Hence, they need more practice on them.

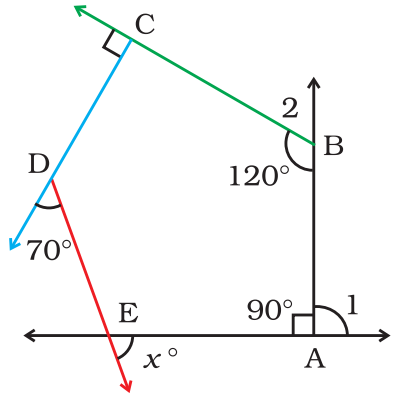
Task 4

Indicator

Child can apply the concept that the sum of all exterior angles of a polygon is 360 to solve a problem.

Task

Students were asked to solve the problem: Find ∠x in the given polygon.



Sample Responses

(i) $x = 90$ alternate interior angle

(ii) $\angle 1 + \angle 2 + \angle C + \angle D + \angle x = 540$

$\angle 1 + \angle 2 + \angle 90 + \angle 70 + \angle x = 540$

$\angle 1 + \angle 2 + \angle 160 + \angle x = 540$

$\angle 1 = 90$

$\angle 2 + \angle 120 = 180$

$\angle 2 = 60$

(iii) In given figure

$\angle 1 + \angle 2 + \angle C + \angle D + \angle x = 540$

$90 + 60 + 90 + 70 + x = 540$

$x = 540 - 180 - 130$

$x = 540 - 310$

$x = 230$

(iv) $\angle 1 + \angle 2 + \angle C + \angle D + \angle x = 360$

$90 + 60 + 90 + 70 + x = 360$

$x = 360 - 180 - 120$

$x = 360 - 300$

$x = 60$

(v) $\angle 1 + \angle 2 + \angle C + \angle D + \angle x = 360$

$90 + 60 + 90 + 70 + x = 360$

$x = 360 - 180 - 130$

$x = 360 - 310$

$x = 50$

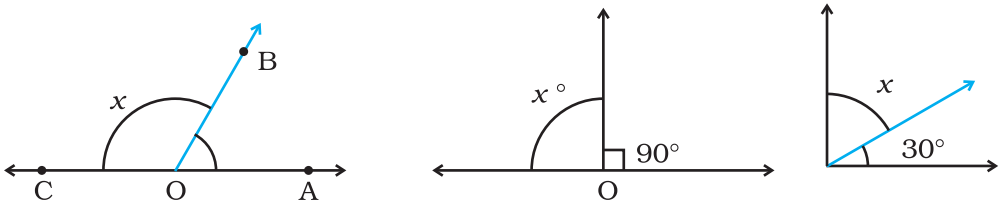
Reflections on Responses

- In response (i), the student does not know the concept of exterior angle property of polygon. She simply answers $x = 90$ without any proper reason. She is on Stage I.
- In response (ii), the student has a little understanding about sum of exterior angles of a polygon. She is on Stage II.
- In response (iii), the student is able to understand how to solve the problem, but is unable to apply the concept of sum of exterior angles of a polygon. She is on Stage III.

- In response (iv), student is careless in calculation of the value x. She is on Stage IV.
- In response (v), student understands the concept of sum of exterior angles of a polygon. She is on Stage V.

Remedial Measures

- Students on Stage I need to revisit the concept.
- Student on Stage II is confused in writing the sum of exterior angles. As she has little understanding of the concept, she needs to be exposed more to the adjacent angles, linear pair etc.



- Student on Stage III knows the concept of sum of exterior angles but unable to apply. She needs more practice with different polygons.

Task 5

Indicator

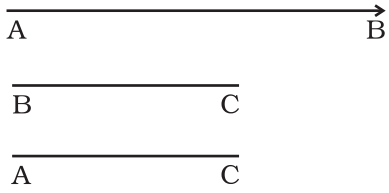
Child can draw line segment of a given measurement and find relationship between given points.

Task

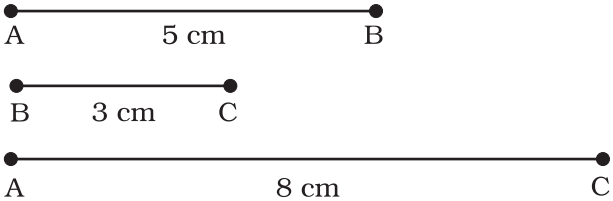
If A, B, C are three points on a line such that AB = 5 cm, BC = 3 cm and AC = 8 cm, which point lies between the other two.

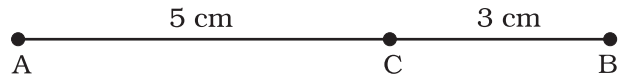
Sample Responses

(i) A B C or

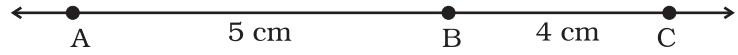


(ii)

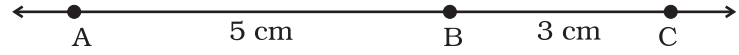


(iii) 

C lies between A and B

(iv) 

B lies between A and C

(v) 

B lies between A and C

Reflections on Responses

- In response (i), student is unable to understand the concept of lines, line segment or she has not read the problem properly. She is on Stage I.
- In response (ii), student knows only a little bit of concept regarding line segment and the lengths, are not drawn as required. She is on Stage II.
- In response (iii), student understands the concept of drawing line segments as required but is unable to apply betweenness concept properly. She is on Stage III.
- In response (iv), student has drawn the line segment as required but makes a careless mistake. She is on Stage IV.
- In response (v), student is good in understanding the concept of line segment and about which point comes between the other two. She is on Stage V.

Remedial Measures

- Student in Stage I needs to revisit the concept of drawing line segment using proper instruments.
- Student on Stage II should understand the concept of collinearity, which needs more reinforcement by giving more examples.
- Student in Stage III should be given more questions of similar type for practice to understand the concept.
- Student in Stage IV has made careless mistake. She has to read the problem carefully and needs more practice.

Task 6

Indicator

Child can apply angle sum property of a triangle in solving problem.

Task

Teacher asks the students to solve the following problem:
In a given triangle ABC, $\angle A = 85$ and $\angle B = 45$, find $\angle C$.

Sample Responses

(i) No answer

(ii) $\angle A + \angle B + \angle C = 100$

(iii) $85 + 45 + \angle C = 100$

(iv) $\angle A + \angle B + \angle C = 180$

$85 + 45^{\circ} + \angle C = 180$

$\angle C = 180 - 85 - 45$

$= 30$

(v) $\angle A + \angle B + \angle C = 180$

$85 + 45 + \angle C = 180$

$\angle C = 180 - 85 - 45$

$= 180 - 130$

$\angle C = 50$

Reflections on Responses

- In response (i), child does not know anything about angle sum property of a triangle. Child is on Stage I
- In response (ii), child knows little about angle sum property. Child is on Stage II.
- In response (iii), child knows angle sum property but is not able to apply the property. Child is on Stage III.
- In response (iv), child knows angle sum property and tried to apply it also but commits careless mistake. Child is on Stage IV.
- In response (v), child knows the angle sum property and applies it correctly. Child is on Stage V.

Remedial Measures

- Child needs to revisit the concept.
- Child needs more explanation of the angle sum property.
- Child should be given some questions for applying angle sum property.

Task 7 Project-Based

Indicator

(i) Child can give examples of symmetric and asymmetric figures.

(ii) Child can examine the symmetry of known figures about a line.

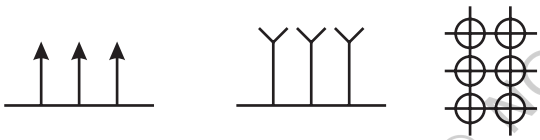
(iii) Child can identify different figures and shapes (quadrilateral, circle etc.) having a symmetry about a line

Task

Teacher asks the students to collect different designs of grills used in boundary walls, roof, balcony, windows and doors etc., in houses, offices and then in these designs identify different figures, shapes having symmetry about a line.

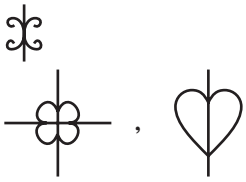
Sample Responses

(i) No sample collection

(ii) 

(iii) 

– Grills are symmetric
– Grills have different design and shapes

(iv) 

Circle ,

Square 

(v) Able to identify closed, open figures. Able to identify symmetrical design and line of symmetry.

Reflections on Responses

- In response (i), student is not able to collect the sample. She is unable to understand the project. She is on Stage I.
- In response (ii), student is able to collect samples but cannot identify figures or shapes. Student has little understanding of concept. She is on Stage II.
- In response (iii), student is able to collect and identify few samples, which implies that she has understanding of the concept but is not able to apply it properly. She is on Stage III.
- In response (iv), student has collected samples and identified different figures and shapes. But in some cases she has committed careless mistake in identification of symmetry. She is on Stage IV.
- In response (v), student has collected samples and also identified all samples correctly. Student is on Stage V.

Remedial Measures

- Student needs revisiting the concept.
- On Stage II, student needs to be told about symmetric and asymmetric objects, closed and open figures. More practice can be given by asking them to draw above mentioned different types of figures.
- Concepts of lines of symmetry need to be reinforced.

Task 8

Indicator

Child can construct the following properties of triangle:

(i) Sum of two sides of a triangle is greater than the third side.

(ii) In a right triangle, the square of the hypotenuse is equal to the sum of squares of other two sides (Pythagoras Property).

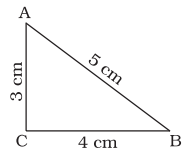
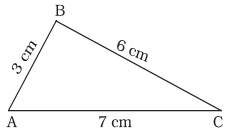
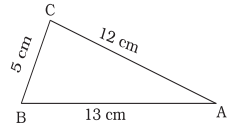
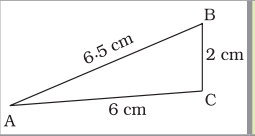
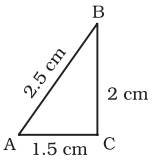
Task

Teacher asked students to fill up the following worksheet and after filling it answer these questions:

(i) What type of triangle is it when $a^2 + b^2 = c^2$?

(ii) Is there any relation between the lengths of 3 sides in a triangle?

Note: In triangle ABC, the side opposite to vertex A is a, the side opposite to vertex B is b and the side opposite to vertex C is c.

	a	a^2	b	b^2	c	c^2	$a^2 + b^2$	Is $a + b > c$? (Yes/No)	Is $a + c > b$? (Yes/No)	Is $b + c > a$? (Yes/No)	Is $a^2 + b^2 = c^2$? (Yes/No)
											
											
											
											
											

Sample Responses

(i) Child could fill up columns of a, b, c only. Did not know how to proceed further.

(ii) Child filled up the columns of a, b, c, a^2, b^2, c^2 and $a^2 + b^2$ but could not proceed further.

(iii) Child filled up all the columns correctly but could not answer the questions or answered them incorrectly/partially.

(iv) Child filled up many of the columns correctly and made computational errors in a few, but could answer the questions correctly.

(v) Child filled up the columns correctly and answered the questions correctly.

Reflections on Responses

- In response (i), child did not understand the instructions or the concept to be explored. She is on Stage I.
- In response (ii), child has little understanding of the task to be carried out. She is on Stage II.
- In response (iii), child has not drawn any inference or proper inference. She is on Stage III.
- In response (iv), child has understood the problem but does computational errors. She is on Stage IV.

- In response (v), child has completely understood the problem. She is on Stage V.

Remedial Measure

Students need revisiting the concept.

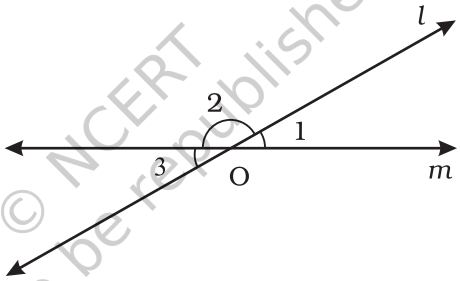
Task 9

Indicator

Student can discriminate between linear pair, vertically opposite angles and compute the unknown angles.

Task

In the given figure if $\angle 1 = 30$, find $\angle 2$ and $\angle 3$ without measuring with instrument (protractor)



Sample Responses

(i) $\angle 1 = 30$ (given), $\angle 2 = 150$, $\angle 3 = 30$ [Using protractor]

(ii) $\angle 1 = \angle 3$ (linear pair)

Therefore, $\angle 3 = 30$

$\angle 1 + \angle 2 = 180$ (Vertically opposite angles)

$30 + \angle 2 = 180$

$\angle 2 = 150$

(iii) $\angle 1 = \angle 3$ (Vertically opposite angles)

$\angle 1 + \angle 2 = 180$ (Linear pair)

(iv) $\angle 1 = \angle 3$ (Vertically opposite angles)

$\angle 3 = 30$

$\angle 1 + \angle 2 = 180$ (Linear pair)

$30 + \angle 2 = 180$

$\angle 2 = 180 + 30 = 210$

(v) $\angle 1 = \angle 3$ (Vertically opposite angles)

$\angle 3 = 30$

$\angle 1 + \angle 2 = 180$ (Linear pair)

$30 + \angle 2 = 180$

$\angle 2 = 180 - 30$

$= 150$

Hence $\angle 2 = 150$

$\angle 3 = 30$

Reflections on Responses

- In response (i), student has used protractor to find the angles. He has not applied the property of vertically opposite angles and the linear pair concept. This shows that his concept is not clear. He is on Stage I.
- In response (ii), student has little understanding of concept. He is on Stage II.
- In response (iii), student knows the concept but cannot apply it to solve the problem. He is on Stage III.
- In response (iv), student understands the concept of angles but commits careless mistakes. He is on Stage IV.
- In response (v), student is able to solve the problem. He is on Stage V.

Remedial Measures

- Students are advised to revisit the concept.
- Students are suggested to practice more questions based on linear pair and vertically opposite angles separately.

Task 10

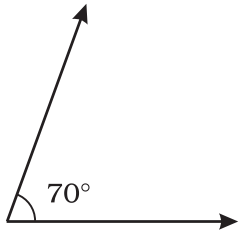
Indicator

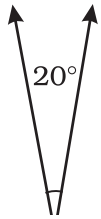
Child can identify the pair of complementary angles.

Task

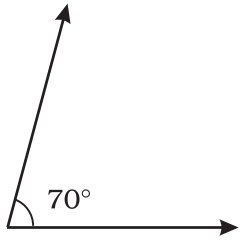
Which pairs of following angles are complementary ? why?

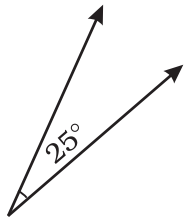
(a)



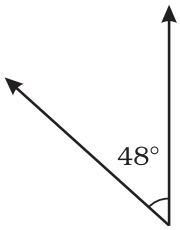


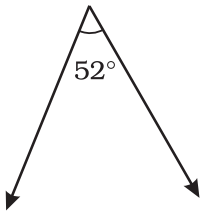
(b)



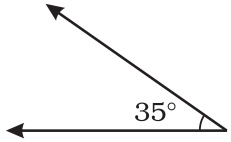


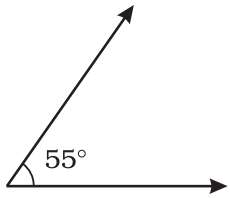
(c)



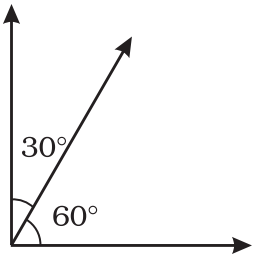


(d)





(e)



Sample Responses

(i) None of them form complementary angles because none are of 90 .

(ii) None of these form complementary angles, because only 45 and 45 make complementary angles as their sum is 90 .

(iii) Only in (e), complementary angles are formed, because others are not adjacent.

(iv) In (d) and (e), complementary angles are formed because they lie on same line and their sum is 90 . But in (a) angles are not complementary because direction of one of them is upward.

(v) (a) 70 + 20 = 90

(b) 75 + 15 = 90

(c) 48 + 52 = 100

(d) 35 + 55 = 90

(e) 30 + 60 = 90

Angles are complementary in (a), (b), (d) and (e) as their sum is 90 .

(vi) Angles in (a), (d) and (e) are complementary because their sum is 90 .

Reflections on Responses

- In response (i), concept of complementary angles is not clear to the student. Student is on Stage I.

- In responses (ii) and (iii), student has little understanding of the concept. Student is on Stage II.
- In response (iv), student understands the concept but is not able to apply it. Student is on Stage III.
- In response (v), student commits careless mistake. Student is on Stage IV.
- In response (vi), student is able to identify complementary angles successfully. Student is on Stage V.

Remedial Measure

- Students needs revisiting the concept.
- Students may be provided practice for identifying complementary angles having their opening in different direction.

Task 11

Indicator

Child can judge whether a quadrilateral can be constructed with the given measurements.

Task

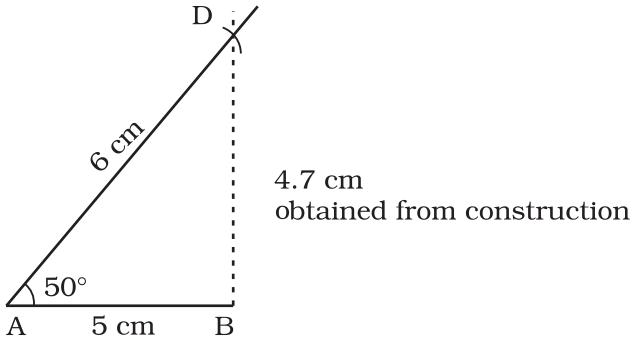
Arshad has five measurements of a quadrilateral ABCD. These are AB = 5cm, $\angle A = 50$, AC = 4cm, BD = 5cm and AD = 6cm. Can he construct a quadrilateral?

Give reasons for your answer.

Sample Responses

(i) Yes, he can construct a quadrilateral because five measurements are given, which are required for making a quadrilateral.

(ii) No, construction is not possible because actual construction measurements are not appropriate and unique triangle ABD cannot be constructed.



Reflections on Responses

- In response (i), the student has not drawn a rough diagram which would have helped him in getting correct answer. Student has just rote memorised that five measurements can determine a quadrilateral uniquely. Student is on Stage II.
- In response (ii), student has attempted the question by actually constructing a quadrilateral with given dimensions. Student is on Stage V.

Remedial Measure

Students need revisiting the concept.

Task 12

Indicator

Child can predict the construction of a quadrilateral with given measurements after suitable computation.

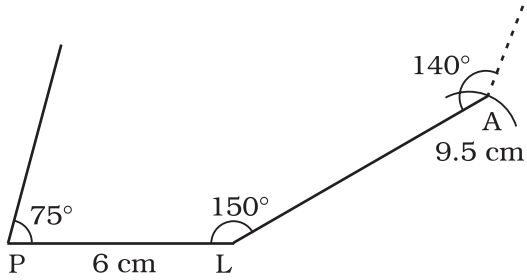
Task

Can you draw a quadrilateral PLAT if PL = 6cm, LA = 9.5cm, $\angle P = 75$, $\angle L = 150$, $\angle A = 140$. If yes construct, if no why not?

Sample Responses

(i) Yes, because five measurements of quadrilateral are given.

(ii) Student may try to construct quadrilateral of given measurement and then write that quadrilateral cannot be constructed.



(iii) Quadrilateral cannot be drawn using these measurements because sum of given angles: $75 + 140 + 150 = 365 > 360$.

Reflections on Responses

- In response (i), child has rote memorised the concept and is unaware of the important properties of triangles and quadrilaterals. Child is on Stage II.

- In response (ii) child understands the concept but is unable to apply properties of triangles and quadrilaterals. Child is on Stage III.
- In response (iii) child understands the concept fully and gives reason. Child is on Stage V.

Remedial Measures

- Child on Stage I needs revisiting the concept.
- Children need more practice of questions on properties of triangles and quadrilaterals.

Task 13

Indicator

(i) Child can construct a circle.

(ii) Child can identify centre, radius, diameter and chord of a given circle.

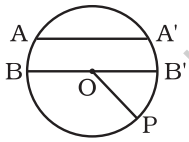
Task

Draw a circle using compasses and mark its centre, radius, diameter and chord.

Sample Responses

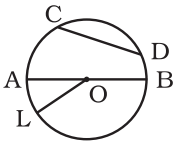
(i)

(ii)



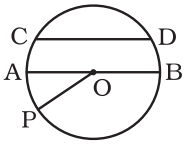
O – Centre
BB' –Radius
OP – Chord
AA' –Diameter

(iii)



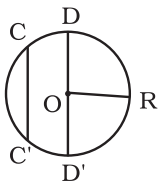
OL –Radius
O – Centre
AB – Chord
CD –Diameter

(iv)



OP – Radius
CD – Chord
AB – Diameter

(v)



O – Centre
OR – Radius
CC' – Chord
DD' – Diameter

Reflections on Responses

- In response (i), student is not able to construct a circle with centre. Student is on Stage I.
- In response (ii), student has developed little understanding of the concept.

Student is able to construct circle with centre but has little idea about radius, chord and diameter. Student is on Stage II.

- In response (iii), student has understood the concept but he is not able to apply it. Student is confused between chord and diameter. Student is on Stage III.
- In response (iv), student has shown everything in the figure but by mistake he forgot to write the centre. Student is on Stage IV.
- In response (v), student has marked different parts correctly. Student is on Stage V.

Remedial Measures

- Student needs revisiting the concept as well as practice drawing a circle with compasses.
- Student needs to be introduced to radius, diameter and chord again.
- Student needs the clarification that the longest chord is called diameter.

Task 14

Indicator

Child can give examples of parallel lines from letters of English alphabet.

Task

Teacher can ask students to identify letters of English alphabet which have parallel lines, display parallel lines with the help of matchstick or colour strips.

This task can be done as **group activity** or as an **individual activity** in the classroom.

Sample Responses

They can be displayed in classroom as group work. Evaluation can be done in the form of peer evaluation. Individual work of similar type can be kept in portfolio for individual work assessment.

Further projects can be carried out on similar track. Students can be encouraged to present them in classroom individually or in a group. Some of the examples are:

1. Identify intersecting lines in English alphabet.
2. Identify altitudes or lines intersecting at 90 in English alphabet.

3.4 Mensuration

Indicators of Learning

Child can

1. understand and compute areas of plane objects:
   - (i) calculate the area and perimeter of a given square, rectangle, triangle, parallelogram, circle and trapezium.
   - (ii) calculate the area of figures by combining the figures or parts of them mentioned above.
   - (iii) compute the area of a quadrilateral by dividing it into two triangles by joining a diagonal and dropping perpendiculars on it.
   - (iv) compute the area of a polygon by splitting into triangles, rectangles and trapezium etc.
   - (iv) compute the area enclosed between
     - two concentric circles
     - two rectangles.
2. understand and compute surface areas and volumes of solids:
   - (i) observe the number of faces in a solid.
   - (ii) identify the congruent faces of a cuboid, cube and cylinder etc.
   - (iii) calculate the lateral/total surface area total surface area of a cuboid, a cube and a cylinder.
   - (iv) compute the volume of a cuboid, a cube and cylinder when the requisite measurements are given.
   - (v) compare the volumes of two given objects.
   - (vi) convert units of volume from m^3 to cm^3 etc.

Task 1

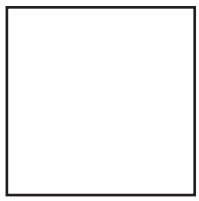
Indicator

Child can calculate the area and perimeter of Square and Rectangle.

Task

A wire is in the shape of a square of side 10 cm. If the wire is rebent into a

rectangle of length 12 cm, find its breadth. Which encloses more area, the square or the rectangle?



Sample Responses

(i) Side of square = 10 cm

Length = 12 cm

Area = 12 10 cm^2 = 120 cm^2

(ii) Side of square = 10 cm

Length of wire = Perimeter of square = 4 × side

= 4 × 10 cm

= 40 cm

Length of rectangle = 12 cm

Breadth = $\frac{\cancel{40}^{10}}{\cancel{12}_{3}}$ = 3.33 cm

(iii) Side of square = 10 cm

Length of wire = Perimeter of square = 4 × side

= 4 × 10 cm

= 40 cm

Length of rectangle = 12 cm

Let b be the breadth of the rectangle

P = 2 (l + b)

40 = 2 (l + b)

40 – 2 = l + b

38 = l + b

38 – 12 = b

b = 26 cm

∴ Area = 26 × 12 cm^2 = 312 cm^2

Area of square = 100 cm^2

(iv) Side of square = 10 cm

Length of square = 4 × side = 40 cm

Length = 12 cm

b = ?

P = 2 (l + b)

$\frac{40}{20}$ = 12 + b

$20 - 12 = b$

$b = 8$ cm

Area of square = $10^2 = 100$

Area of rectangle = $l \times b = 12 \times 8 = 96$ cm^2

Child forgets to compare the area of square and rectangle.

(v) Child finds the areas of square and rectangle as in (iv) and finds that area of square is greater than the area of the rectangle.

Reflections on Responses

- In response (i), the child does not understand the language of the word problem. Child is at Stage I.
- In response (ii), the child does not know the correct formula of the perimeter of rectangle. Child is at Stage II.
- In response (iii), the child is able to understand the question, child knows the correct formula but does computational error. Child is on Stage III.
- In response (iv), child has correctly found the area of rectangle and square but forgot to compare the two areas. Child is on Stage IV.
- In response (v), the child knows the concept very well. Child is on Stage V.

Remedial Measure

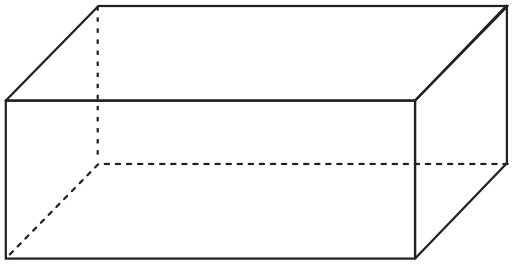
Child needs revisiting the concept.

Task 2

Indicators

(i) Child can apply the understanding of volume of cuboid in solving daily life problems.

(ii) The child can calculate volume and time required to fill the reservoir.



Task

Water is pouring into a cuboidal reservoir at the rate of 60 litres per minute. If the volume of reservoir is 108 m^3, find the number of hours it will take to fill the reservoir?

Sample Responses

(i) Volume of the reservoir = 108 m^3

No. of hours = $\frac{\cancel{108}^{18}}{\cancel{60}^{10}}$ = 1.8 hours

(ii) Volume of the reservoir = 108 m^3

1 m^3 = 100 litres

So, Capacity = 108 × 100 L

= 10800 L

Amount poured in one hour = 60 L

No. of hours = $\frac{10800}{60}$ = 180 hours

(iii) Volume of reservoir = 108 m^3

1 m^3 = 1000 l

Capacity = 108 × 1000 l = 10800 L

Amount poured in one minute = 60 L

Amount poured in one hour = 60 × 60 L

∴ No. of hours = $\frac{108000}{60\times60} = \frac{130}{6} = \frac{56}{3}$ hours

(iv) Volume of reservoir = 108 m^3

1 m^3 = 1000 L

Capacity = 108 × 1000 l = 10800 L

Amount poured in one minute = 60 L

Amount poured in one hour = 1080000 × 60

= 6480000 hours

(v) Volume of the reservoir = 108 m^3

1 m^3 = 1000 litres

Capacity of the reservoir = 108 × 1000 litres

= 108000 litres

Amount poured in one minute = 60 litres

∴Amount poured in one hour = 60 × 60 litres

Thus number of hours required to fill the reservoir = $\frac{108000}{60\times60} = 30$

∴ The required number of hours = 30

Reflections on Responses

- In response (i), the child is unable to understand the language of the problem. Child is on Stage I.
- In response (ii), the child understands the question but does not apply the correct formula for converting m^3 into litres. Child is on Stage II.
- In response (iii), the child is able to understand the language of the problem, child could convert m^3 into litres but could not proceed further correctly. Child is on Stage III.
- In response (iv), the child understood the problem, child proceeds in a correct manner but does careless error. Child is on Stage IV.
- In response (v), child gives correct answer. Child is on Stage V.

Remedial Measures

- The child needs revisiting the concept of volume as child lacks language comprehension. Child is unable to understand the meaning of the question.
- Child needs practice on conversion of unit. Questions on unit conversion and unitary method may be given for practice.
- Child needs more drill and practice to improve concentration in order to avoid careless mistakes.

Task 3

Indicator

The child can calculate the lateral surface area of a cylinder and apply it in solving daily life problems.

Task

Teacher asked the students to solve the following question:

A road roller makes 250 complete revolutions to move once over to level a road. Find the area of the road levelled if the diameter of the road roller is 84 cm and its length is 1 m.

Sample Responses

(i) Diameter = 84 cm; Radius = 42 cm

Area of road = 84×250

= 2100 cm^2

(ii) Radius = $\frac{84}{2}$ cm = 42 cm

Length = 1 m = 100 cm

Area of road = L × B

= 42 × 100 cm²

= 4200 cm²

(iii) Radius = $\frac{84}{2}$ cm = 42 cm

Length (h) = 1 m = 100 cm

LSA of cylinder = $2\pi rh = 2 \times \frac{22}{7} \times 42^{8}$ 100 cm²

= 44 × 8 × 100 cm²

= 35200 cm²

(iv) (1m = 100 cm)

Radius = $\frac{84}{2}$ cm = 42 cm

Length = 1 m = 100 cm

LSA = $2\pi rh = 2 \times \frac{22}{7} \times 42 \times 100$ cm²

Area levelled in 250 revolution = $\frac{26400}{250}$ cm²

= $\frac{528}{5}$ cm²

(v) A road roller is a cylinder, such that

Radius = $\frac{84}{2}$ cm = 42 cm

Length (h) = 1 m = 100 cm

As lateral surface area of a cylinder = $2\pi rh$

∴Lateral surface area of the road roller

= $2 \times \frac{22}{7} \times 42 \times 100$ cm²

= 2 × 22 × 6 × 100 cm²

$\therefore$ Area of road levelled in 1 revolution = 26400 cm^2

$\therefore$ Area of road levelled in 250 revolutions $= 250 \times 26400\ cm^2$

$$= \frac{250 \times 26400}{100 \times 100}\ m^2$$

$= 660\ m^2$

Reflections on Responses

- In response (i), the child is not able to understand the language of the problem. Child is on Stage I.
- In response (ii), the child does not know the correct formula for calculating lateral surface area of cylinder. Child is on Stage II.
- In response (iii), child understands the concept but commits computational error and is also not able to complete the solution. Child is on Stage III.
- In response (iv), child understands the concept and is also able to apply it but commits careless mistake. Child is on Stage IV.
- In response (v), the child has understood and could apply the concept correctly. Child is on Stage V.

Remedial Measure

Child needs revisiting the concept of lateral surface area of a cylinder.

Task 4

Indicators

Child can explore the relationship between:

(i) Length (in cm) of the side and Perimeter (in cm) of a square.

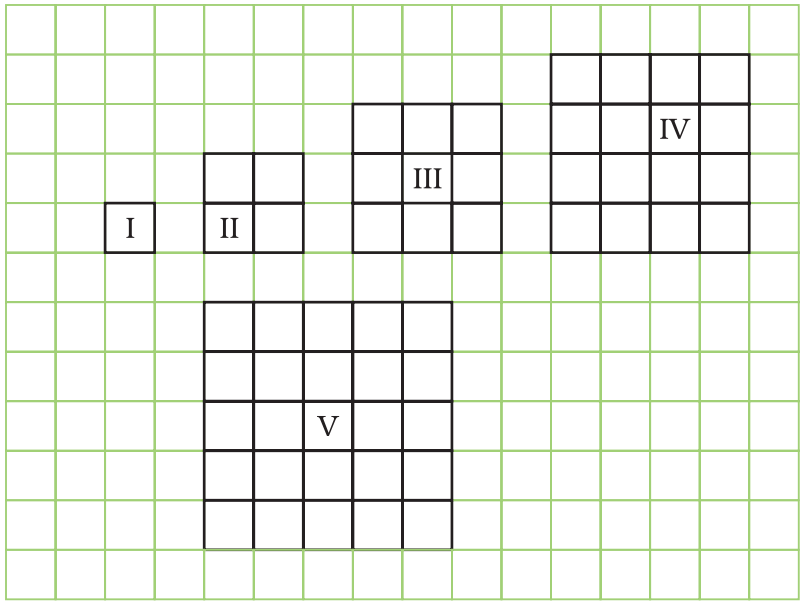
(ii) Length (in cm) of the side and area (in cm^2) of a square.

Task

Teacher asked the students to complete the worksheet and fill the table after observing the following graph sheet and asked them to explore the relationship between:

1. Length of side and Perimeter of square

2. Length of side and area of square



Sample Responses

(i)

Worksheet

Square	Square I	Square II	Square III	Square IV	Square V
Length (L) of a side of a square (in cm)	1	2	3	4	5
Perimeter (P) (in cm)	4	8	12	16	20
Area (A) (in cm^2)					

(ii)

Worksheet

Square	Square I	Square II	Square III	Square IV	Square V
Length (L) of a side of a square (in cm)	1	2	3	4	5
Perimeter (P) (in cm)	4	8	12	16	20
Area (A) (in cm^2)	1	4	9	16	25

Table

Square	Square I	Square II	Square III	Square IV	Square V
$\frac{P}{L}$	4	4	4	4	4
$\frac{A}{L}$					

or

Square	Square I	Square II	Square III	Square IV	Square V
$\frac{P}{L}$	$\frac{1}{4}$	$\frac{2}{8}$	$\frac{3}{12}$	$\frac{4}{16}$	$\frac{5}{20}$
$\frac{A}{L}$	$\frac{1}{1}$	$\frac{2}{4}$	$\frac{3}{9}$	$\frac{4}{16}$	$\frac{5}{20}$

(iii)

Worksheet

Square	Square I	Square II	Square III	Square IV	Square V
Length (L) of a side of a square (in cm)	1	2	3	4	5
Perimeter (P) (in cm)	4	8	12	16	20
Area (A) (in cm^2)	1	4	9	16	25

Table

Square	Square I	Square II	Square III	Square IV	Square V
$\frac{P}{L}$	4	4	4	4	4
$\frac{A}{L}$	1	2	3	4	5

(iv)

Worksheet

Square	Square I	Square II	Square III	Square IV	Square V
Length (L) of a side of a square (in cm)	1	2	3	4	5
Perimeter (P) (in cm)	4	8	12	16	20
Area (A) (in cm^2)	1	4	9	16	25

Table

Square	Square I	Square II	Square III	Square IV	Square V
$\frac{P}{L}$	4	4	4	4	4
$\frac{A}{L}$	1	2	3	4	5

$\frac{P}{L}$ = 4, so, length = 4 × perimeter, $\frac{A}{L}$ = L

(v)

Worksheet

Square	Square I	Square II	Square III	Square IV	Square V
Length (L) of a side of a square (in cm)	1	2	3	4	5
Perimeter (P) (in cm)	4	8	12	16	20
Area (A) (in cm^2)	1	4	9	16	25

Table

Square	Square I	Square II	Square III	Square IV	Square V
$\frac{P}{L}$	4	4	4	4	4
$\frac{A}{L}$	1	2	3	4	5

From the table, in every square, $\frac{P}{L} = 4$. So, Perimeter of a square = 4 × Length

From the table, for every square, $\frac{A}{L}$ is equal to L. So, $\frac{\text{Area}}{\text{Length}}$ = Length

∴ Perimeter of a square = 4 × Length

Area of a square = Length × Length

Reflections on Responses

- In response (i), the child could not fill up the worksheet. Child is on Stage I.
- In response (ii), the child fills the worksheet but is not able to fill the given table completely/correctly. So child is on Stage II.
- In response (iii), the child is able to fill the worksheet, fills the table correctly but is not able to explore the relationship between length and perimeter of square and/or length and area of square. Child is on Stage III.
- In response (iv), the child does careless error while exploring the relationship. Child is on Stage IV.
- In response (v), the child is able to perform the activity correctly and is able to explore the relationship between area and perimeter of a square. Child is on Stage V.

Remedial Measures

- Child needs revisiting the concept.
- Child needs practice for generalising such situations to develop.

3.5 Comparing Quantities

Indicators of Learning

Child can

1. express some daily life situations in the form of ratios.

2. understand the meaning of proportion.
3. check whether given four numbers (quantities) are in proportion or not.
4. solve daily life problems by using ratio or proportion.
5. write ratios equivalent to the given ratio.
6. compare any two given ratios.
7. solve daily life problems using unitary method.
8. understand the meaning of percentage.
9. compare quantities using percentage.
10. convert percentage into a ratio (fraction) and a ratio (fraction) into percentage.
11. convert percentage into decimal fraction and a decimal fraction into percentage.
12. find percentage of a given quantity and quantity (how many) of given percentage.
13. understand use of percentage in discount, simple interest, compound interest, profit and loss, sales tax, VAT etc., by
    (i) simply applying definition of the concept involved.
    (ii) applying formula.
14. solve problems relating to direct and inverse proportion such as time and work, time and distances etc.

Task 1

Indicator

Child can check whether given few numbers (quantities) are in proportion or not.

Task

Teacher asks the child if ₹ 15, ₹ 45, 40 paise, 120 paise are in proportion or not?

Sample Responses

(i) 15 : 45 : 40 : 120

(ii) 15 : 45 : : 40 : 120

$$\frac{₹15}{120\text{ paise}} = \frac{1}{8}$$

Also $\frac{₹45}{40 \text{ paise}} = \frac{4500}{40} = 112.5$

So, ₹ 15, ₹ 45, 40 paise, 120 paise are not in proportion.

(iii) 15 : 45 : : 40 : 120

$\frac{15}{40}, \frac{40}{120}$

So, ₹ 15, ₹ 45, 40 paise, 120 paise are not in proportion.

(iv) 15 : 45 : : 40 : 120

₹ 15 = 1500 paise

₹ 45 = 4500 paise

And $\frac{1500}{4500} = \frac{1}{3}$

$\frac{40}{120} = \frac{1}{3}$

So, ₹ 15, ₹ 45, 40 paise, 120 paise are not in proportion.

(v) 15 : 45 : : 40 : 120

$\frac{15}{45} = \frac{1}{3}$, so 15 : 45 = 1 : 3

$\frac{40}{120} = \frac{1}{3}$, so 40 : 120 = 1 : 3

So ₹ 15, ₹ 45, 40 paise, 120 paise are in proportion.

Reflections on Responses

- In response (i), child has not understood the concept of proportion. Child is on Stage I.
- In response (ii), child has compared the ratio of first term to the last term and second term to the third term. Child has little understanding of the concept. Child is on Stage II.
- In response (iii), child is comparing first to the second term and third term to the fourth term but child is not able to reduce the fractions in its lowest form. Child is on Stage III.
- In response (iv), child is able to solve the question but does careless mistake. Child is on Stage IV.

- In response (v), child is able to solve the question correctly. Child is on Stage V.

Remedial Measures

- Child may be asked to revisit the concept of ratio and then may be asked to compare two ratios.
- Practice may be given to identify the equivalent ratio.
- Practice may also be given to reduce the fractions to its lowest form.

Task 2

Indicator

Child can compare quantities using percentage.

Task

Compare the quantities of following components of air in atmosphere in terms of percentage.

.78 g Nitrogen
.21 g Oxygen
.01 g Other gases
} in 1 g of air

Sample Responses

(i) Amount of Nitrogen = .78 g

Amount of Oxygen = .21 g

Amount of other Gases = .01 g

Ratio of Nitrogen to Oxygen = $\frac{.78}{.21} = \frac{78}{21}$

Ratio of Oxygen to other Gases = $\frac{.21}{.01} = \frac{21}{1}$

(ii) Amount of Nitrogen = .78g So amount of Nitrogen = .78%

Amount of Oxygen = .21g So amount of Oxygen = .21%

Amount of other Gases = .01g So amount of other Gases = .01%

(iii) Amount of Nitrogen = .78g $= \frac{.78\times100}{100} = \frac{7800}{100} = 7800\%$

Amount of Oxygen = .21g $= \frac{.21\times100}{100} = \frac{2100}{100} = 2100\%$

Amount of other Gases = .01g $= \frac{.01\times100}{100} = \frac{1000}{100} = 100\%$

(iv) Amount of Nitrogen = .78g $= \frac{.78\times100}{100} = \frac{78}{100} = 78\%$

Amount of Oxygen = .21g $= \frac{.21\times100}{100} = \frac{21}{100} = 21\%$

Amount of other Gases = .01g $= \frac{.01\times100}{100} = 100\%$

(v) Content of Nitrogen $= \frac{.78}{1} = \frac{.78\times100}{1\times100} = \frac{78}{100} = 78\%$

Content of Oxygen $= \frac{.21}{1} = \frac{.21\times100}{1\times100} = \frac{21}{100} = 21\%$

Content of other Gases $= \frac{.01\times100}{1\times100} = \frac{.01}{1} = \frac{1}{100} = 1\%$

Reflections on Responses

- In response (i), child has not understood what is to be done. Child is on Stage I.
- In response (ii), child has little understanding of the concept. Child is on Stage II.
- In response (iii), child has understood the concept but does not know about percentage conversion. Child is on Stage III.
- In response (iv), child has followed the process correctly but makes computational error. Child is on Stage IV.
- In response (v), child has understood the problem and solved it correctly. Child is on Stage V.

Remedial Measures

Child may be asked to write few decimals and convert them into fraction, such as $0.76 = \frac{76}{100}$. More practice of questions like conversion of the fraction into percentage i.e.,

0.76 = $\frac{76}{100}$ = 76% or 0.08 = $\frac{8}{100}$ = 8% and 0.8 = $\frac{8}{10}$ = $\frac{80}{100}$ = 80% can be given.

Task 3

Indicator

Child can solve daily life problems by using ratio and proportion.

Task

Child proportion is asked to solve the following problem.

Shaina pays ₹ 7,500/- as rent for three months. How much she has to pay for a whole year, if the rent per month remains the same?

Sample Responses

(i) Rent of 3 months = ₹ 7,500

So rent of 1 year = 7,500 × 12

= ₹ 90,000

(ii) Ratio of the duration = 3 months : 1 year

Let the rent of the whole year = ₹ x

∴ Ratio of rents = ₹ 7500 : ₹ x

∴ 3 : 1 = 7500 : x

$\frac{3}{1} = \frac{7500}{x}$

$x = \frac{\cancel{7500}^{2500}}{\cancel{3}}$

= x = ₹ 2,500/-

(iii) 1 Year = 12 months

Ratio of the duration = 3 months : 12 months

= 3 : 12 = 1 : 4

Let the rent of whole year = ₹ x

∴ Ratio of the rents = ₹ x : ₹ 7,500

$\therefore x : 7,500 = 1 : 4$

or $\frac{x}{7,500} = \frac{1}{4}$

so, $x = \frac{\cancel{7,500}^{1875}}{4}$

$\therefore x = ₹\ 1,875/-$

(iv) 1 Year = 12 months

Ratio of the duration = 3 months : 12 months

$$= \frac{3}{12} = \frac{1}{4} = 1 : 4$$

Let the rent of whole year = ₹ x

Ratio of rents = ₹ 7,500 : ₹ x

$= 7,500 : x = 1 : 4$

$$\frac{7,500}{x} = \frac{1}{4}$$

$\therefore x = 7,500 = ₹\ 1875$

(v) 1 year = $\frac{4}{12}$ months

Ratio of the duration = 3 months : 12 months

$$= \frac{3}{12} = \frac{1}{4} = 1 : 4$$

Let the rent of whole year = x

Ratio of rents = ₹ 7,500 : x

$= 7,500 : x = 1 : 4$

so, $\frac{7,500}{x} = \frac{1}{4}$

therefore, $x = 7,500 \times 4 = ₹\ 40,000$

(vi) Ratio of the duration = 3 months : 12 months

$$= \frac{3}{13} = \frac{1}{4} = 1 : 4$$

Let the rent of whole year = ₹ x

Ratio of rents = Rs 7,500 : ₹ x

$= 7,500 : x = 1 : 4$

or $\frac{7,500}{x} = \frac{1}{4}$

so, $x = 7,500 \times 4$

$= ₹\ 30,000/-$

Reflections of Responses

- In response (i), child has no idea about the concept to be used. Child is on Stage I.
- In response (ii), child is not able to apply the understanding of ratio (units of two quantities which is to be compared should be same) and proportion correctly. Child is on Stage II.
- In response (iii), though child has the understanding of ratio and proportion. child is not able to apply it. Child is on Stage III.
- In responses (iv) and (v), child is able to apply the understanding of ratio and proportion but does some careless mistake. Child is on Stage IV.
- In response (vi), child is able to solve the problem correctly. Child is on Stage V.

Remedial Measure

Child may be asked to revisit the concept of ratios.

Task 4

Indicator

Child can solve daily life problems using unitary method.

Task

Teacher asked the students to solve the following problem.

Cost of 4 dozen bananas is ₹ 60. Calculate the number of bananas that can be purchased for ₹ 12.50/-

Sample Responses

(i) Cost of 4 dozens bananas = ₹ 60

Cost of 1 dozen banana = ₹ $\frac{60}{4}$ = ₹ 15

For ₹ 15 number of bananas = 1

For ₹ 1 number of bananas = $\frac{1}{15}$

For ₹ 12.50 number of bananas = 12.50×15

(ii) For ₹ 60 number of bananas = 4 dozens

For ₹ 1 number of bananas = $\frac{4}{60}$

For ₹ 12.50 number of bananas = $\frac{4}{60} \times 12.5$

(iii) Cost of 4 dozen bananas = ₹ 60

1 dozen = 12

4 dozen = $12 \times 4 = 48$

∴Cost of 48 bananas = ₹ 60

Cost of 1 banana = ₹ $\frac{60}{48}$

= ₹ $\frac{5}{4}$

No. of bananas for ₹ 12.50 = $12.50 \times \frac{5}{4}$

= 62.50

(iv) Cost of 4 dozen bananas = ₹ 60

Cost of 1 dozen bananas = ₹ $\frac{60}{4}$ = ₹ 15

Cost of 12 bananas = ₹ 15

Cost of 1 banana = ₹ $\frac{15}{12}$ = ₹ $\frac{5}{4}$

For ₹ $\frac{5}{4}$ number of banana = 1

For ₹ 1 number of bananas = $\frac{5}{4}$

For ₹ 12.5 number of bananas = $\frac{5}{4} \times 12.5$

= 15.625

(v) Cost of 4 dozen bananas = ₹ 60

Cost of 1 dozen banana = ₹ $\frac{60}{4}$ = ₹ 15

Cost of 12 bananas = ₹ 15

Cost of 1 banana = ₹ $\frac{15}{12}$ = ₹ $\frac{5}{4}$

For ₹ $\frac{5}{4}$ number of bananas = 1

For ₹ 1 number of bananas = 1 $\frac{5}{4}$

$= \frac{4}{5}$

For ₹ 12.50, number of bananas = $\frac{4}{5} \times 12.5$

$= 10$

Reflections on Responses

- In response (i), child is not able to find the cost of one banana correctly and also not able to find the number of bananas for ₹ 12.50. Child has not understood the concept. Child is on Stage I.
- In response (ii), child has little understanding of unitary method. Child is on Stage II.
- In response (iii), child has understanding of unitary method but is not able to apply it properly. Child is on Stage III.
- In response (iv), child is able to apply unitary method to solve the problem but does a careless mistake. Child is on Stage IV.
- In response (v), child is able to solve the problem correctly. Child is on Stage V.

Remedial Measure

Child may be asked to find the value of one unit when value of more than one unit is given. Practice of identifying the quantity whose unit value is to be found in the given problem should be given. For example, if a car covers a distance of 150km in 10 litres, find how much distance the car will cover in 1 litre and also find the consumption of petrol 1km distance covered.

Task 5

Indicator

Child can find percentage of a given quantity.

Task

Child is asked to find the percentage decrease if the population of a city decreased from 25,000 to 24,500.

Sample Responses

(i) Original population = 25,000

New population = 24,500

Decrease in population = 25,000 – 24,500 = 500

Therefore, % = 500%

(ii) Original population = 25,000

New population = 24,500

$$\% \text{ decrease } = \frac{24,500}{25,000} \times 100\%$$

$$= 98\%$$

(iii) Original population = 25,000

New population = 24,500

Difference= 25,000 – 24,500 = 500

$$\% \text{ decrease } = \frac{500}{24,500} \times 100\%$$

(iv) Original population = 25,000

New population = 24,500

Decrease = 500

Let decrease be = x %

x % of 24,500 = 500

$$\frac{x}{100} \times 24,500 = 500$$

$$x = \frac{500 \times 100}{24,500}$$

$$= \frac{22}{49} \%$$

(v) Original population = 25,000

New population = 24,500

Decrease = 24,500 – 25,000 = 500

$$\% \text{ decrease } = \frac{500}{25,000} \times 100$$

= 20%

(vi) Original population = 25,000

New population = 24,500

Decrease = 25,000 – 24,500 = 500

$$\% \text{ decrease} = \frac{500}{25{,}000} \times 100$$

= 2 %

Reflections on Responses

- In response (i), child is not able to find the decrease percentage. Child is on Stage I.
- In response (ii), child is able to understand the concept of percentage but is not able to find percentage of which quantity is to be found. Childe is on Stage II.
- In response (iii), child is able to understand the concept of percentage but is not able to apply it completely. Child is on Stage III.
- In responses (iv) and (v), child is able to understand the concept of percentage but does careless mistake. Child is on Stage IV.
- In response (vi), child is able to solve the problem. So child is on Stage V.

Remedial Measure

Initially child may be asked to find increase/decrease of a given quantity. Then child may be explained how to find its percentage.

Task 6

Indicator

Child can find profit and loss using percentage.

Task

If a shopkeeper buys 80 articles for ₹ 2,400/- and sells them for a profit of 16%. Child is asked to find the selling price of an article?

Sample Responses

(i) C.P. of 80 articles = ₹ 2,400/-

$$\text{Profit \%} = \frac{16}{100}$$

S.P. = C.P. + P

$= ₹\ 2{,}400 + \frac{16}{100} = \frac{2{,}416}{100}$

(ii) C.P. of 80 articles = ₹ 2,400/-; Profit % = 16%

P% of C.P.

$= \frac{16}{100} \times 2{,}400$

= ₹ 384/-

S.P. of 80 articles = C.P. – P

= 2400 – 384

= ₹ 2,016/-

(iii) C.P. of articles = ₹ 2,400/-

P = P% of C.P.

$= \frac{16}{100} \times 2{,}400$

= ₹ 384/-

S.P. = C.P. + P

= ₹ (2,400 + 384)

= ₹ 2,784/-

(iv) C.P. of 80 articles = ₹ 2,400/-

Profit % = 16%

C.P. of an article = $₹\ \frac{2{,}400}{80}$ = ₹ 300/-

Profit = P% of C.P.

$= \frac{16}{100} \times 300$ = ₹ 48/-

S.P. = C.P. + P

= ₹ (300 + 48) = ₹ 300/-

∴ Selling Price of an article = ₹ 300/-

(v) C.P. of 80 articles = ₹ 2,400/-

Profit % = 16%

C.P. of an article = ₹ $\frac{2,400}{80}$ = ₹ 300/-

Profit = P% of C.P.

= $\frac{16}{100}$ × 300 = ₹ 48/-

S.P. = C.P. + P

= ₹ (300 + 48) = ₹ 348/-

∴ Selling Price of an article = ₹ 348/-

Reflections on Responses

- In response (i), child is not able to find the percentage of cost price and hence is not able to find correct selling price. Child is on Stage I.
- In response (ii), child is able to find percentage of cost price, also knows what is to be done further but does not know the relation between C.P. and S.P. Child is on Stage II.
- In response (iii), child is able to find selling price of 80 articles but is not finding the S.P. of an article. Child is on Stage III.
- In response (iv), child can solve the problem but does some careless mistake. Child is on Stage IV.
- In response (v), child can solve the problem correctly. Child is on Stage V.

Remedial Measure

Child may be asked to revisit the concept of percentage. Practice of more such questions can be given.

3.6 Data Handling

Data handling has emerged as an important tool and application of mathematics now-a-days. In day-to-day life, child comes across various situations where organisation and representation of data is required. The collection, recording and presentation of data helps to organise child's experiences and draw results from them. Representation of data in the form of pictures like pictograph, bar graph, histogram, pie chart etc., is useful in deriving results at a glance. Also there may be situations where a child has to compare different collections of data. Such comparison of data needs finding a representative value of the data for which child can use mean, mode and median. A child should be made aware that data handling plays an important role everywhere — be it school level in preparing time table or marksheet or attendance register, in market, showing market price of commodities or comparison of rates of different items, in industries, comparison of demand-supply, in business, in share market and in games.

At Upper Primary Stage, child's experiences and thinking takes wider shape than the primary level. Through newspapers, magazines and media or even through textbooks, a child faces a flow of information in the form of tables and graphs. It enhances her level of understanding and opens a door towards the practical field of mathematics.

At this stage, emphasis is mainly on collection, interpretation and representation of data and a child should be encouraged to conduct small surveys in her class itself or in her vicinity so that she can herself collect, represent and analyse data.

Indicators of Learning

Child can

1. collect and record data.
2. organise the given data in the form of a table using tally marks.
3. interpret data arranged in a tabular form.
4. read and interpret a pictograph.
5. draw a pictograph to represent data given in a tabular form.
6. read and interpret a bar graph.
7. draw a bar graph/double bar graph of the given data.
8. arrange the given raw data in the form of grouped frequency distribution (continuous).
9. find range of the given raw data.
10. read and interpret a histogram.
11. draw a histogram for the given data.
12. read and interpret a pie chart.
13. draw a pie chart for the given data.
14. calculate mean of given raw data.
15. calculate median of a given raw data.
16. calculate mode of a given raw data.
17. identify events which are more likely, less likely or equally likely to happen.
18. enlist the outcomes of the given random experiment and calculate probability of events.

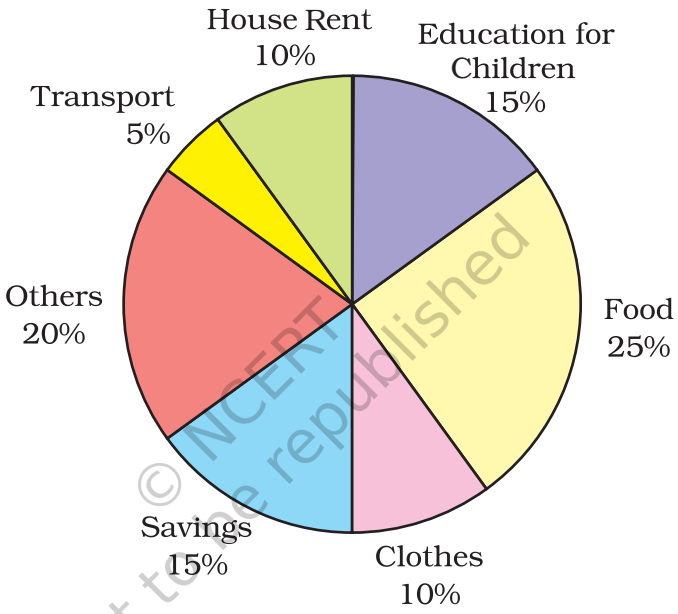
Task 1

Indicator

Child can interpret a pie chart.

Task

Teacher gives the following pie chart which shows the expenditure (in percentage) on various items and savings of a family during a particular month.



Teacher asks the children to answer the following, based on the above pie chart:

(a) On which item, the expenditure was maximum and how much is it?

(b) Expenditure on which item is equal to the total savings of the family?

(c) If the monthly savings of the family is ₹ 3000, what is the monthly expenditure on clothes?

Sample Responses

	(a)	(b)	(c)
(i)	food	House Rent	10%
(ii)	food, 25%	Transport + Clothes	$3000 \times \frac{15}{10} = ₹450$

(iii) food, 25% Education

15% of total of expenditure = 3000
therefore, total expenditure

$$\frac{3000 \times 100}{100} = ₹20,000$$

So, expenditure on clothes

$$\frac{20,000 \times 100}{100} = ₹10,000$$

(iv) food, 25% Education

Ratio of monthly savings to expenditure of clothes

$= 15 : 10 = 3000 : x$

and $\frac{15}{10} = \frac{3000}{x}$

So, $x = 2000 \times \frac{10}{15} = 1333.33$

so, expenditure on clothes

= ₹1333.35

(v) food, 25% Education

15% of total expenditure (x)

= savings

so, 15% of x = 3000

$$x = \frac{3000 \times 100}{15}$$

= 20,000

Total expenditure = ₹ 20,000

expenditure on clothes

= 10% of x

$$\frac{10}{100} \times 20,000$$

= ₹ 2,000

Reflections on Responses

- In response (i), child is not able to read and interpret the pie chart correctly. Child is on Stage I.
- In response (ii), child is able to read the pie chart correctly for (a) only. Child is on Stage II.

- In response (iii), child is able to interpret the pie chart correctly but could not find answer of part (c). Child is on Stage III.
- In response (iv), child is able to interpret the pie chart correctly but makes careless mistake in answering the part (c). Child is on Stage IV.
- In response (v), child is able to interpret the pie chart correctly. Child is on Stage V.

Remedial Measures

- Child needs to revisit the concept of pie chart.
- The child may revisit the concept of percentage, ratio and proportion, including unitary method.
- To improve and practice computational competencies, some questions can be given such as

  (i) find 15% of ₹ 1000

  (ii) find x, if x% of ₹ 500 is 300 etc.

Task 2

Indicator

Child can draw a histogram of the given data.

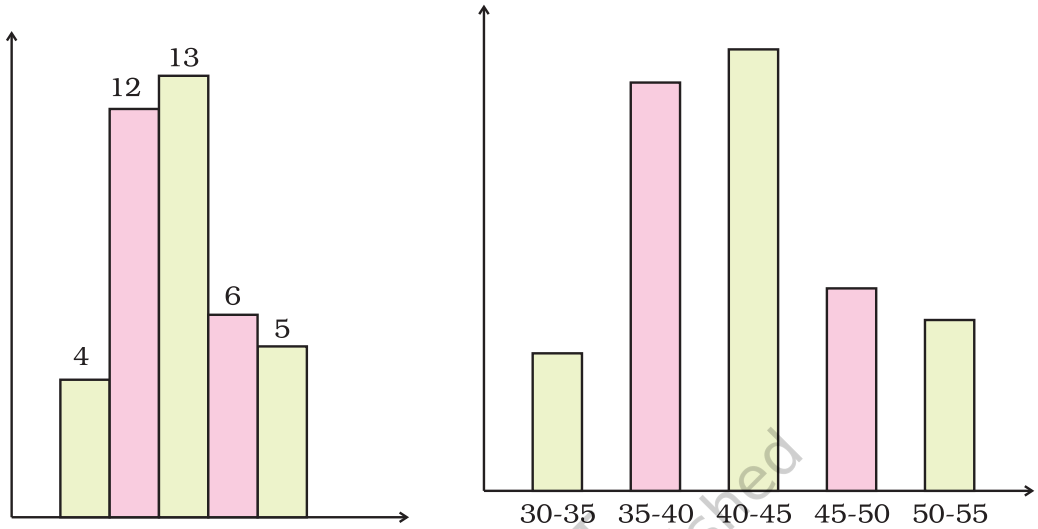
Task

Child is asked to draw a histogram for the following data of distribution of weight (in kg) of 40 students.

Weight (in kg)	Number of Students
30 - 35	4
35 - 40	12
40 - 45	13
45 - 50	6
50 - 55	5

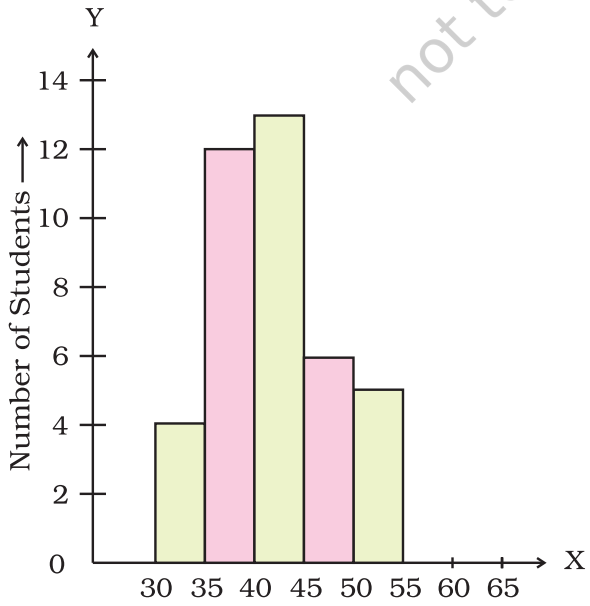
Sample Responses

(i)

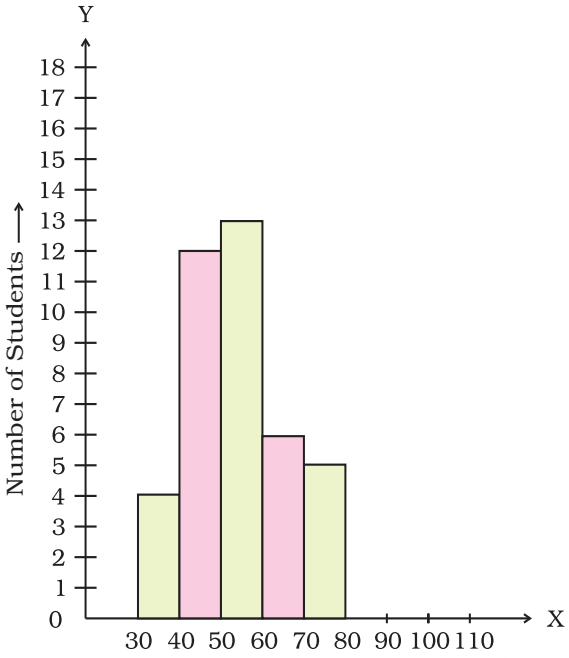


(a) (b)

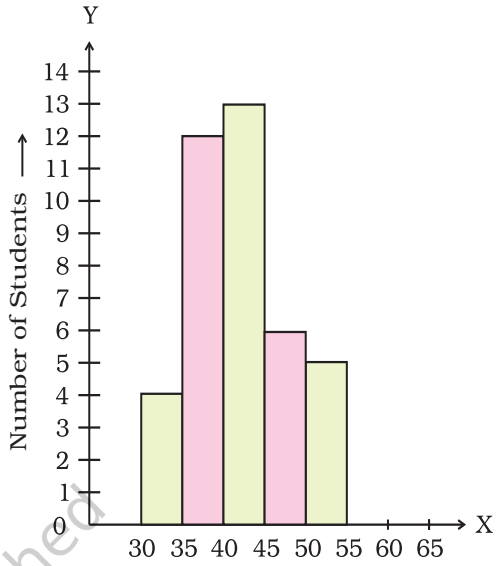
(ii)



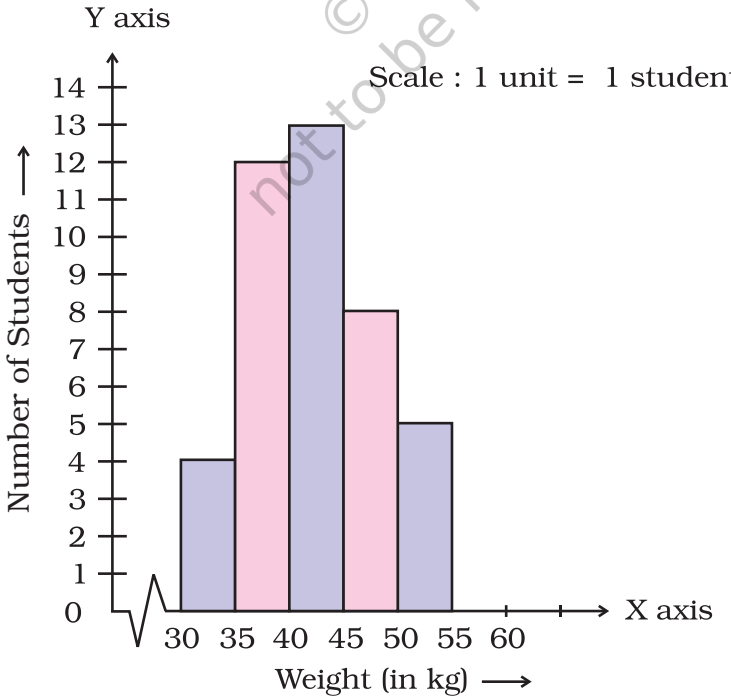
(iii)



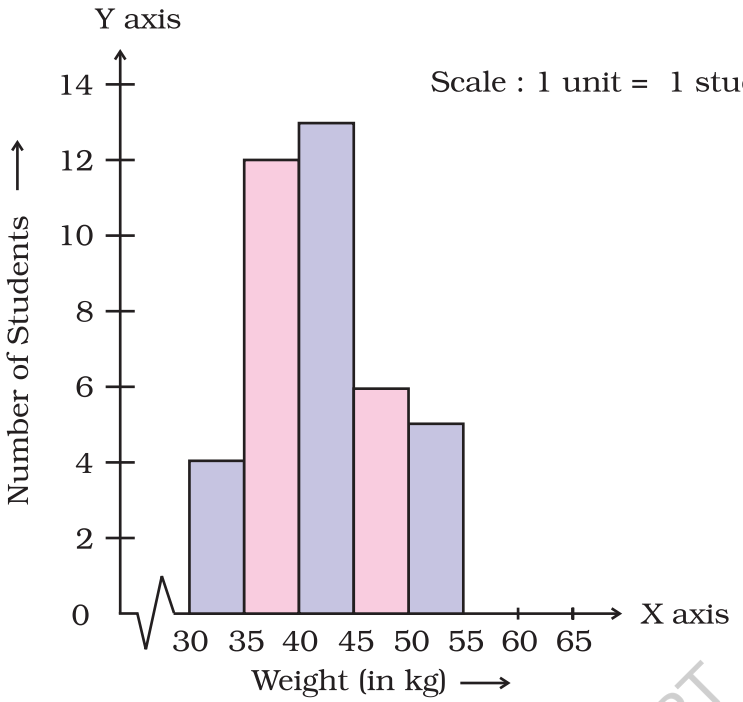
(iv)



(v)



(vi)



Reflections on Responses

- In response (i), child has not made equal division along the axes and also has not made bars according to any scale. So child is on Stage I.
- In response (ii), the child has made equal divisions along the axes but has not taken correct intervals for rectangles did not take proper scale, has not shown kink etc. So, child is on Stage II.
- In responses (iii) and (iv), child has not shown kink on x-axes, did not mention the scale choosen [in (iv)]. So, child is on Stage III.
- In response (v), child is able to construct histogram correctly but does a careless mistake in drawing rectangle corresponding to interval 45-50. So, child is on Stage IV.
- In response (vi), child has constructed histogram correctly. So, child is on Stage V.

Remedial Measures

- Child may be asked to observe various histograms so that child can understand that it is necessary to make equal divisions on axes, label the axis properly, choose a suitable scale.
- Child may be asked to draw more histograms for practice.

Task 3

Indicator

Child can calculate mean of a given raw data.

Task

Teacher asks the children to calculate mean of the data :

10, 12, 8, 0, 9, 5, 7, 9

Sample Responses

(i) 10, 12, 8, 0, 9, 5, 7, 9

Mean = 10 + 12 + 8 + 0 + 9 + 5 + 7 + 9 = 60

(ii) $\text{Mean} = \frac{10+12+8+0+9+5+7+9}{7} = \frac{60}{7} = 8.57$

(iii) $\text{Mean} = \frac{10+12+8+0+9+5+7+9}{8} = \frac{56}{8} = 7$

(iv) $\text{Mean} = \frac{10+12+8+8+9+5+7+9}{8} = \frac{68}{8} = 8.5$

(v) $\text{Mean} = \frac{10+12+8+0+9+5+7+9}{8} = \frac{60}{8} = 7.5$

Reflections on Responses

- In response (i), the child does not know the concept of mean. Child is on Stage I.
- In response (ii), the child knows the formula for calculation of mean but ignores observation 0 from the number of observations. Child is on Stage II.
- In response (iii), the child knows meaning of mean and how to calculate mean, but makes computational error in finding sum of observations. Child is on Stage III.
- In response (iv), the child knows meaning of mean and how to calculate mean, but does careless mistake by writing '8' in place of '0' while copying. Child is on Stage IV.
- In response (v), the child knows the concept of mean and calculates mean correctly. Child is on Stage V.

Remedial Measures

- For Stage I, student needs revisit of the concept of mean.
- For Stage II, more questions involving 0 as one of the observation may be given.
- For Stage III and IV, more questions for calculation of mean have to be provided.

Task 4 Project-Based

Indicator

Child can interpret a bar graph.

Task

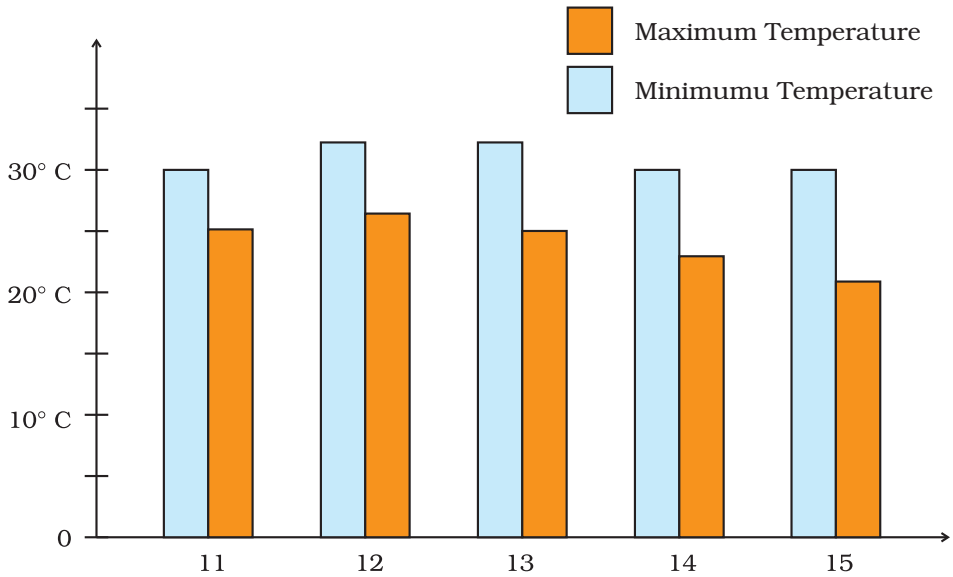
Teacher asked students to collect different bar graphs appearing in newspapers and magazines and told them to interpret it.

Sample Responses

(i) Few children brought cuttings of graphs but actually they were not bar graphs.

(ii) Some children brought cuttings showing bar graphs but were not able to tell what the graph is about.

(iii) Some children could tell what the graph is about but could not read accurately. For example, in the following graph the child is not able to find the difference correctly of maximum and minimum temperature of a particular date.



(iv) Some children can interpret the graph accurately for each day but make careless mistake in interpreting temperatures for only one day.

(v) Some children are able to read and interpret the bar graph correctly. They have conceptual understanding of bar graphs.

Reflections on Responses

- The child in response (i), is on Stage I.
- In response (ii), the child has understood the concept of a bar graph but is not able to interpret it. So, child is on Stage II.
- In response (iii), the child is on Stage III.
- In response (iv), the child is on Stage IV.
- In response (v), the child is on Stage V.

Remedial Measure

Teacher may divide class in the groups each of four and five students and place one student out of them who is on Stage V, in each group. Group discussion may help the children to develop conceptual understanding of bar graph.

3.7 Introduction to Graphs

Indicators of Learning

Child can

1. read and interpret a line graph.
2. draw a line graph for the given data.
3. understand the concept of co-ordinates of a point.
4. locate a point in a plane.
5. plot a point in a plane.
6. read and interpret a linear graph.
7. draw a linear graph of the given data.

Task 1

Indicator

(i) Child can draw a linear graph of the given data.

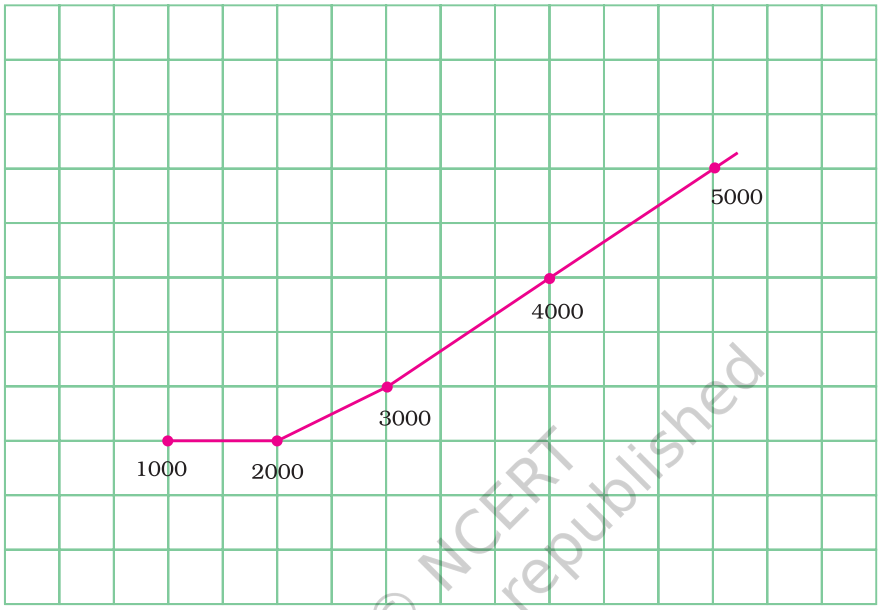
(ii) Child can interpret a linear graph.

Task

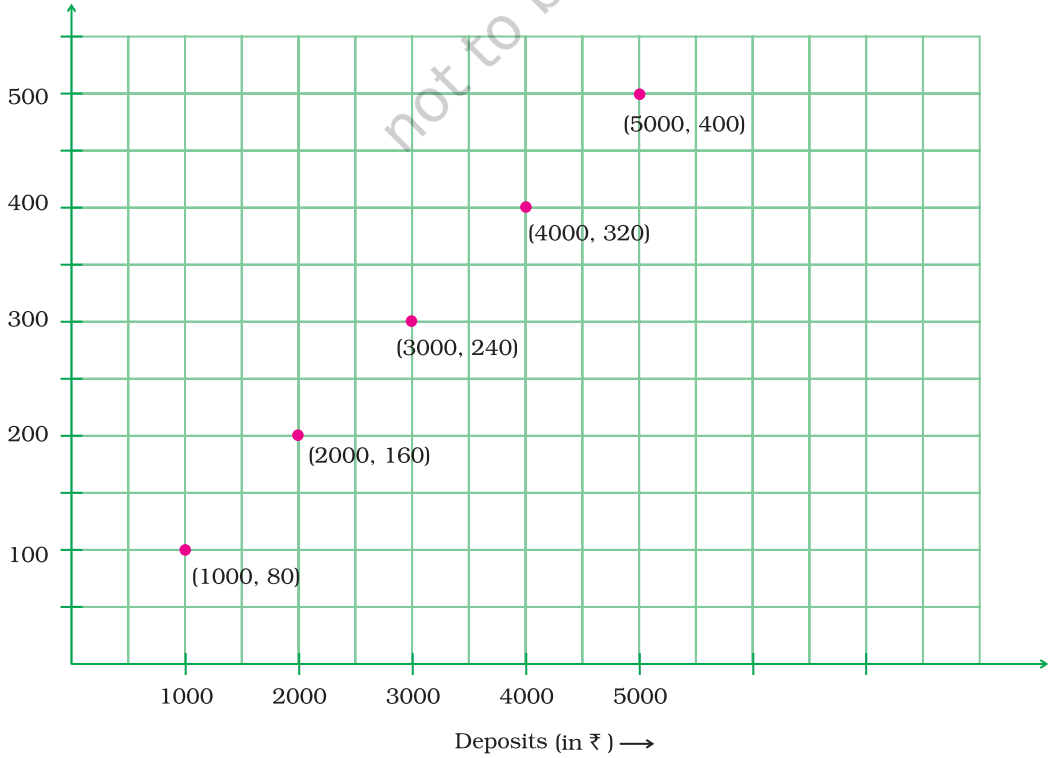
Teacher asks the children to draw a graph for the following table and also to find simple interest on ₹ 2,500/- using the graph

Deposit (in ₹)	1,000	2,000	3,000	4,000	5,000
Simple Interest (in ₹)	80	160	240	320	400

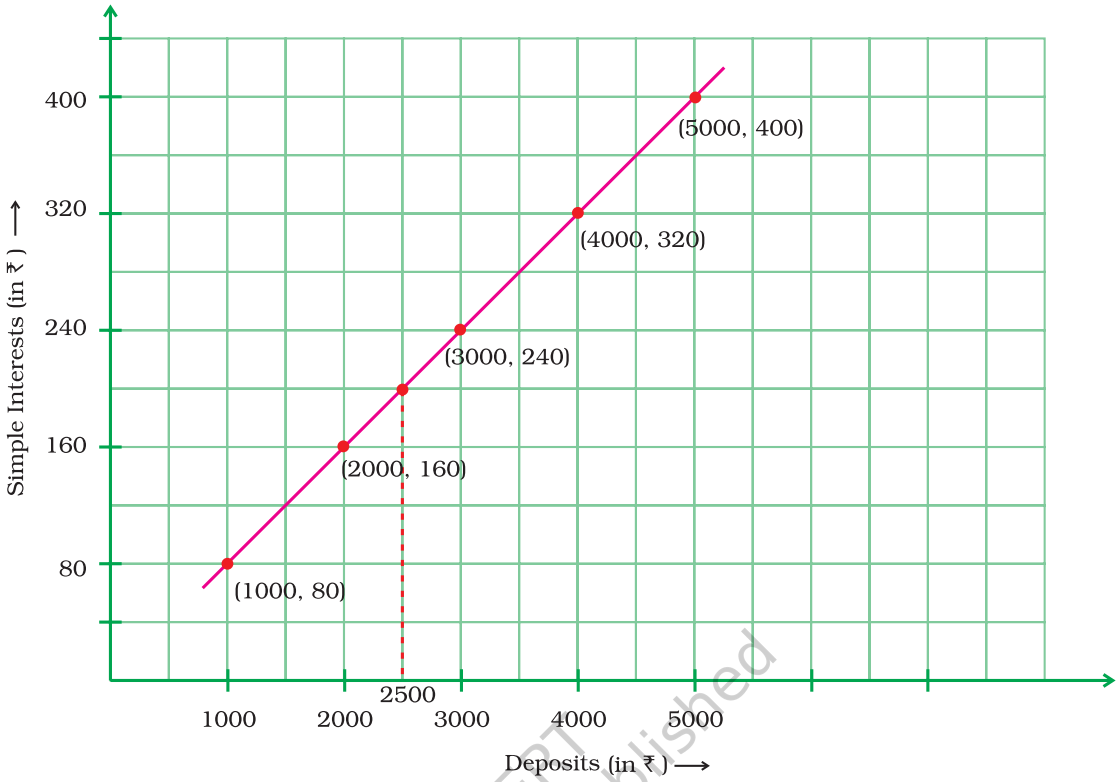
(i)



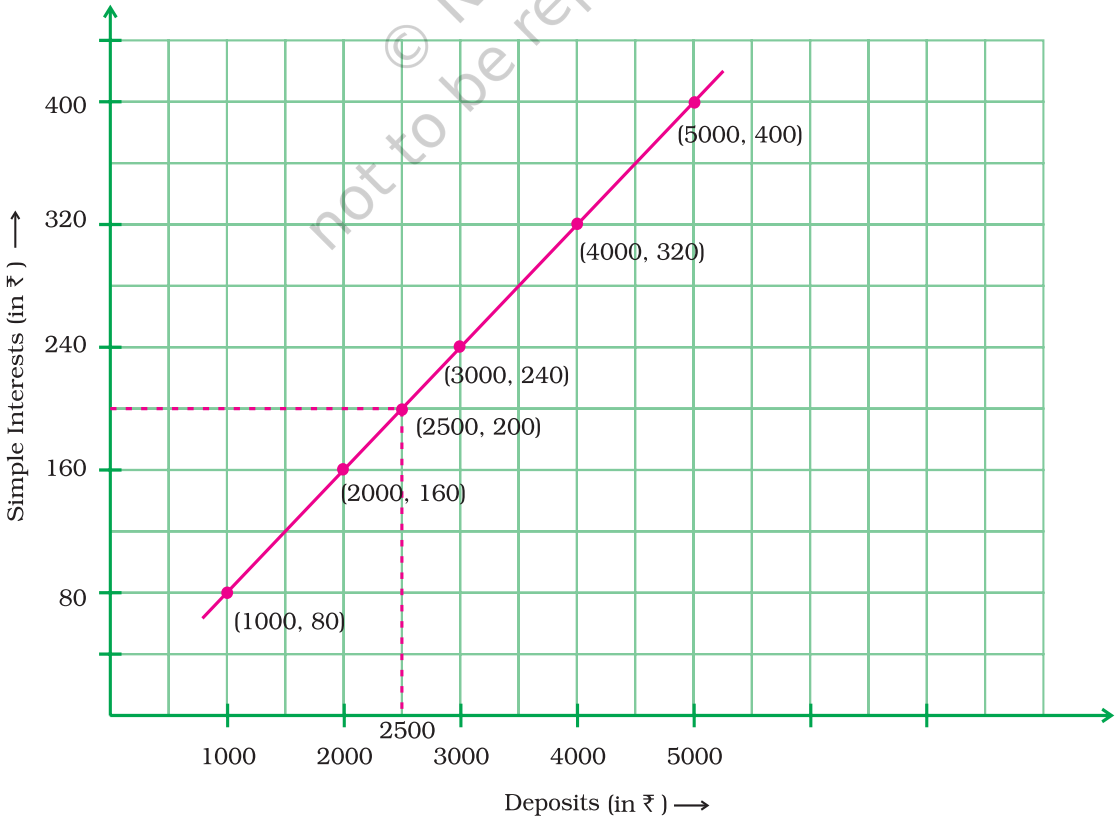
(ii)



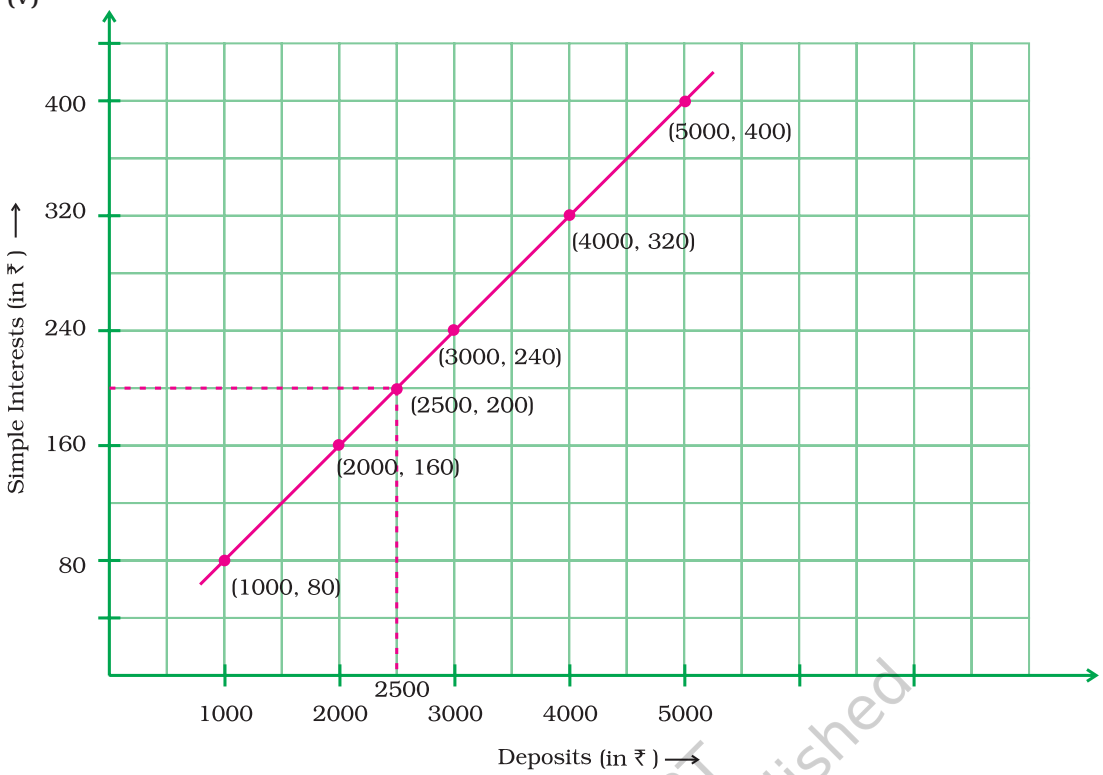
(iii)



(iv)



(v)



Reflections on Responses

- In response (i), child has no understanding of labeling the axes, scale or plotting the corresponding points and is not able to interpret the graph. So child is on Stage I.
- In response (ii), child has divided both the axes into equal parts. Though child has written the coordiantes of the points correctly, child is not able to plot them correctly. So child is on Stage II.
- In response (iii), child is able to draw the graph correctly and also located the point on the graph corresponding to ₹ 2,500/- but could not write simple interest corresponding to this deposit. So child is on Stage III.
- In response (iv), child is able to draw a correct line graph by choosing suitable scale but does a careless mistake in finding the simple interest on ₹ 2,500/-. So child is on Stage IV.
- In response (v), child is able to draw a correct line graph and is also able to find the simple interest of ₹ 2,500/- (i.e., ₹ 200/-). So child is on Stage V.

Remedial Measures

- Responses (i) and (ii) show lack of conceptual understanding of a linear graph. So the child needs to revisit the concept. Teacher may show different line graphs to the children.
- For responses (iii) and (iv), sufficient practice may be given to the children for reading/locating points on the graphs, taking different values of the points on x-axis.

CHAPTER 4

RECORDING AND REPORTING

4.1 INTRODUCTION

We have discussed earlier that assessment is essentially a process of gathering information related to mathematical learning abilities of the students. The main purpose of assessment is to provide positive feedback to the students and develop more appropriate need based teaching strategies by the teacher. This is crucial so as to enable students to develop their own confidence and to nurture emerging mathematical abilities. In Chapter 3, we have suggested some sample tasks to receive feedback from the students continually during the day-to day teaching-learning. However, one of the main objectives of assessment is also to provide commentary on the child's mathematical understanding in consolidated form to the individual child/parents/guardians. This consolidation would be carried out periodically. For this purpose, a continuous recording and reporting is necessary.

Recording and reporting of student's performance in both scholastic and coscholastic areas are very crucial components of any assessment procedure. On one hand, the process of recording refers to systematic documentation of evidences of student's growth and development through various tools and techniques of assessment. On the other hand, reporting refers to categorisation/ organisation of students into appropriate learning stages as mentioned in Chapter 3, commensurate with the documented evidences of student's growth. This is done for identifying and helping the students for further learning, removing misconceptions, promotion, selection, certification etc. Recording and reporting should also be a basis for better learning strategy and future curriculum designing.

The task of reporting student's progress cannot be separated from the procedures used in assessing student's learning and development. If indicators have been clearly defined in terms of performance and relevant tasks and techniques and if other assessment procedures have been properly used, recording and reporting becomes a matter of summarising the results and presenting them in an understandable form. The task is still a perplexing one, since the evidence of learning and development has to be presented on a very brief report which is understandable to a variety of users (i.e., students, parents, teachers, counsellors and administrators). As with other facts of the learning programmes, **the main focus of the recording and reporting system should be the improvement of the student's learning and development. No Assessment for the sake of assessment itself.**

Reporting student's progress becomes difficult especially when the vast array of assessment data is summarised as a single letter grade (e.g., A, B, C, D, E) as grades do not tell us much about what is being learnt and how it is learnt. Should the assigned grade represent achievement only or should effort and work habits be also included? How should the various aspects of achievement (e.g. test, project reports, group work, home assignments and lab work) be weighed and combined? Should the achievement be judged in relation to other pupils, some absolute standard or the individual's learning potential? What distribution of grades (i.e., A, B, C, D and E) should be used and how should this be determined? There are no simple answers to these questions. Although some schools provide guidelines for assigning grades, practices vary from school to school and frequently from teacher to teacher within the same school. Many schools have circumvented the problem of using a single letter grade by supplementing it with a more elaborate reporting system.

Even the student's statement 'I don't know' provides us valuable information, and we must not see it as total lack of understanding on the part of the child, and give zero mark to the child. In addition to telling us that the child does not know the answer, it does tell us that the child is clear, confident and comfortable enough to say that she/he does not know that concept. Moreover, the general tendency on the part of a teacher is to ignore the misconception which exists in the minds of the students related to a particular mathematical concept. The need of the hour is to help the student to convert a misconception into a right concept.

Thus, in the light of the above discussion, there is a need to record the assessment data of each student in a stage-wise manner for better understanding of the student's learning of the subject, particularly the five dimensions of mathematical learning such as

- Mathematical concepts as per the syllabus (elaborated into various dimensions and sub-sections as examples).
- Mathematical reasoning (able to follow an argument or able to provide an argument, justification, etc., able to explain how and why she/he did, what she/he did and reflect on the procedures followed by others).
- Attitude towards mathematics (persists at a task, confident about ability to do etc.).
- Using mathematical knowledge and techniques to solve problems (can solve problems in more than one way, if possible), applying and connecting to day-to-day situations.
- Language and communication (initiates questions, shares and explains to peers and translates verbal form into mathematical form and vice-versa etc.).

Recording and reporting of all the above mentioned dimensions will also provide teachers with insights into the nature of individual attention to be given

during and after the teaching-learning process. It will also generate a sense of accomplishment among students.

4.2 How to do Reporting and Recording?

Methods of recording student's performance in scholastic areas are different from the methods which are used for recording it in coscholastic areas. This difference is mainly because of the different tools and techniques used for collecting evidences of student's growth and development in these areas. While assessing student's performance in scholastic areas the techniques used are oral, written and practical project work while the achievement test/paper-pencil test is used as a tool. The achievement test may be used at the end of the unit or at the end of a term or at the end of the course. Accordingly, these tests are known as unit tests, term end tests or term tests and summative tests. On the other hand, student's performance in coscholastic areas is assessed using observational, peer appraisal and self-appraisal techniques. Tools mainly used for this are observation schedule, checklist, rating scale, anecdotal record, peer appraisal sheet, personal interview forms etc.

For recording and reporting student's performance in subject-based curricular areas, following points may be kept in mind:

1. Techniques used may be oral, written, practical or project work. More weightage may be given to best three/four items including only one written test chosen by the students in their portfolio at the end of the term.
2. The tests/tasks may be used as a tool based on competencies or capabilities (as per indicators in terms of stages). They may form the basis for student's development in mathematical learning.
3. Periodicity of tests may be maintained to ensure the continuity in evaluation. Please note that continuity in assessment does not mean to put a student for all the time in a stressful situation in the name of tests.
4. To keep a close watch on the learning progress of students, strong areas may be identified by analysing the performance of individual student or students in groups. Diagnostic tests may be developed on the difficult concepts. After diagnosing the learning difficulty, remedial instructions may be provided to students with learning difficulties.
5. While observing a student through a group activity, a teacher may maintain the formal observation notes in the form of mental notes. She/he may take cognizance of various aspects of each student's participation as well as individual work characteristics. Some of the aspects to be noted would include;
    - Does the child ask questions? (Some difficult and challenging questions).
    - Is she/he able to follow arguments and make her own?

- What does she/he do when confronted with a new type of problem? etc.

When assessment is done across the whole class, the teacher could also gain insight into her/his pedagogic practices and identify content areas where she/he needs to focus more or alter current practices. It would also give him/her a sense of appropriateness of the overall level of the tasks.

Informing parents (or guardians) of their child's school progress is a basic function of grading and reporting system. These reports will help parents understand the broader learning objectives and how well their children are achieving the intended indicators of that particular mathematical programme. This information is important from several viewpoints. First, by knowing what the school is attempting to do, parents are better able to cooperate with the school in promoting their children's learning and development. Second, information concerning their successes, failures and special problems enables parents to give them the emotional support and encouragement needed. Third, knowing their children's strengths and weaknesses in learning provides a basis for helping them make more sound educational and vocational plans, in general, and mathematical learning, in particular. To serve these purposes adequately, the reports should contain as much information and details as parents can easily comprehend and use.

Some important points that the teacher should keep in mind while she/he did the recording of the student's progress are as follows:

1. Recording should be done in terms of stages of learning ability of the student based on the indicators related to the mathematical learning.
2. The general practice is that students are informed in terms of grades without giving the meaning to it. Thus, it will be appropriate to record the assessment by highlighting the particular stage with emphasis on qualitative description e.g.,

		I Term	II Term	III Term
Existing Practice	Rani	A	B	B
Suggestive	Rani	Stage 1	Stage 2	Stage 3

How to Report

Student's parents will be reported by providing the stages of mathematical abilities learnt in terms of stages alongwith qualitative descriptions.

Stage 1: Needs revisiting the concept.

Stage 2: Little understanding of the concept.

Stage 3: Understands the concepts but is unable to apply.

Stage 4: Makes careless mistakes.
Stage 5: Gives correct expected response. Can move on to next indicator.

The central point that we must keep in our mind is that assessment must not be viewed in terms of end product only, rather it is merely one of the different aspects of the entire process of learning.

Recording and reporting is beneficial for all the stakeholders viz., teachers, students, parents, curriculum planners etc. Therefore, it should be done in such a manner which is understandable to each one of them.

If possible we can think of a position/situation where grading is not merely the translation of marking system. It must reflect the proficiency level of the student in terms of qualitative statements/ description. As far as possible, at the upper primary stage, we should try to do away with current summative form of assessment.

However, for the meaningful and understandable form of assessment, we are proposing a direct grading system.

Direct Grading System

In direct grading, the performance exhibited by an individual is assessed in qualitative terms and the impression so obtained by the examiner is directly expressed in terms of letter grades namely, A, B, C, D and E. This method may be profitably used for the assessment of both cognitive and non-cognitive areas of learning.

For qualitative assessment, according to direct grading, following scheme of grading may be used for summative form of assessment only at the end of each term, if necessary.

Stage	Grade
Stage 1	E
Stage 2	D
Stage 3	C
Stage 4	B
Stage 5	A

While reporting the summative assessment, teacher may take decision according to her/his own best of judgements. For example, if there are five types of assessment (written test, project, oral etc.) and the child achieves 'A' in first, 'B' in second and third and C in fourth and fifth. The teacher may award overall 'B' grade to the child.

Have a look at Table 4.1 below:

Table 4.1 **Suggestive Plan of Various Activities in Mathematics to be Assessed during an Academic Year**

Activities Planned	Term I			Term II			Term III		
Months	MI (1)	MI (2)	MI (3)	MII (1)	MII (2)	MII (3)	MIII (1)	MIII (2)	MIII (3)
Class Exercise (e), Home assignment (h)	h1, e1	h2, e2		h3, e3	h4, e4			h5, e5	h6, e6
Activity (a)			a1			a2			a3
Projects (P)					P1			P2	
Quiz (q)					q1				
Group Discussion (gd)								gd	
Puzzle (PE)							PE		
Field Trip (f)					f1				
Surveys (s)						s			
Presentations (PT)				PT					
Written test (w)				w1			w2		w3
Any other									

Table 4.1 shows the suggestive plan of various tasks to be assessed during an academic year while teaching mathematics. It can be seen from the table that there can be a variety of assessment tools available with the teacher like class exercise, activity, projects, quiz, puzzle, field surveys, presentation, interviews, written test and any other task.

Table 4.2 shows an example of a plan of student's record in a term.

Table 4.2 **Sample Plan of Student's Record of Portfolio: Term I**

Name of the Student	Activity	Project	Class Exercise	Surveys	Interview	Field Trip	Quiz/Puzzle	Written Test	Any Other	Learning Difficulties	Remedial
Shreyas		P1	e1		i1			w1			
Sameena	a1	P1	e1					w1			
....											
....											

Teacher may not assess all the students on the same task. For example as in Table 4.2 above, teacher assesses Sameena on activity while Shreyas on interview through two different tasks during Term I, as teaching, learning and assessing students is a continuous activity.

"Any other" in Table 4.1 can include arranging an exhibition after students have done their projects. Such type of activities can be based on what students have already done.

In Table 4.1 in the symbols MI(1), MI(2), MI(3) etc., M indicates month, I, II, III indicate term numbers and number in the brackets indicates month of that term and so on.

Some Suggestions

- The written task should be done during one or two regular periods only.
- As far as possible, projects should be carried out during school hours. If the nature of project is such that it has to be done outside school hours, teachers should encourage students to do the project on their own with minimal help from parents.
- Assessment tools available with the teacher are class exercise, activity, projects, quiz, puzzle, field trip, surveys, interviews, written test and any other task.
- The written task will help teacher to gauge the development of children through some form of written work such as short test, a test based on problem solving, expressing their ideas or any other.

Table 4.3 Childs' Assessment Portfolio

Mathematics

✓ Indicates student's participation

Name: _____

Class/Section: _____

Roll No.: _____

	Chapters	Oral Test	Problem Solving Work Sheet	H.W.	U.T. Pen Paper Test	Quiz	Paper Presentation	Lab Activity	Project Work	Identification of Weak Areas	Proficiency Level Teacher's Remarks Remedial Measure Taken
Ist TERM	1. Knowing our Numbers										S-4 Understands the concepts, has mathematical reasoning but commits careless errors in haste. Separate copy is maintained for regular practice.
	2. Whole numbers							✓			
	3. Playing with numbers	✓	×		✓	✓					
	4. B. Geometry Concept			✓				✓		Geometry	
	5. U. Elementary Shapes		✓		✓		✓		✓		
IInd TERM	6. Integers	✓	✓					✓		Integers	S-5 Improved his computation skill. Is able to apply his mathematical knowledge in real life situations, is inquisitive in nature.
	7. Fractions		✓					✓			
	8. Decimals	✓						✓			
	9. Data Handling		✓				✓	✓			
	10. Mensuration	✓	✓	✓	✓	✓					
IIIrd TERM	11. Algebra	✓	✓		✓			✓	✓	Algebra	S-4 Has developed interest in Maths and most of the concepts are clear but has to develop skill in geometry.
	12. Ratio and Proportion	✓	✓			✓				Ratio and Proportion	
	13. Symmetry			✓				✓			
	14. Practical Geometry		✓		✓						

A suggested termwise portfolio of a student is given in Table 4.3.

Table 4.4 Qualitative Profile in Mathematics

Traits/Aspects	Feedback and Observation
Concept	Excellent ability to understand and apply.
Mathematical Reasoning	Good at mathematical reasoning, has computational skill and does logical thinking.
Written Work	Very neat and systematic, good ability of drawing graphs and doing constructions using instruments.
Application and Project Work	Excellent in approach towards project work, has mathematical skills of application, computation presentation etc.

S1 : Needs revisiting the concept.

S2 : Needs some more understanding of the concept.

S3 : Understands the concept but unable to apply.

S4 : Makes slippery mistakes/avoidable mistakes.

S5 : Gives correct/expected response and can move on to the next higher indicator.

Table 4.5

Roll No.	Subject	Tasks	Content	Description in Qualitative Statements	Stage	Grade	Overall State-ment
Mahesh R. No. 15	Mathe-matics	1. Written Test	• Knowing our Numbers • Integers • ⋮	1. ..... 2. ..... 3. ..... ⋮	I II II ⋮	A B B... ⋮	B
		2. Presenta-tions		.....	.....	.....	A
		3.	.....	.....	.....	.....	B
		4.	.....	.....	.....	.....	A
					Overall Grade– B		

Note: Records to be maintained by the teacher in each child's portfolio.

Here, qualitative statements column and stages are for the teacher and students and Grade column is for the parents.

In the light of the above discussions, we propose a suggestive format for the Progress Report of child's performance in Mathematics scholastically and non-scholastically.

Table 4.6

Subjects	Tasks	Term I Grade	Term II Grade	Term III Grade	* Overall Remarks
Mathematics	Activity Project H. Assignment Survey Presentation Field Visit Quiz,Group Discussion Puzzle Written Test Any Other				
Science					
Social Science					
Attendance Presentation Engagement					
Teacher's Comments					
Principal's/ Headmaster's Signature					
Teacher's Signature					
Parent's Signature					

***Note:** Remarks of the teacher would be based on the indicators of learning.

CHAPTER 5

STRAIGHT FROM THE CLASSROOMS

During the development of this Source Book, the teachers involved in this work used some of the methods of Assessment discussed earlier in their classrooms. These are presented here with minimal changes.

5.1 CASE STUDY

In Class VI, students learn about large numbers. To familiarise with large numbers and to make them aware of its use in our daily life, following task was given to them by the teacher. Through this task students were also able to read and compare large numbers.

Each student was asked to bring at least five advertisements of various products valued more than ₹ 1,000 along with their prices mentioned and the photographs given in newspapers.

- Class was divided into groups of 3 to 4 students and all the members of the group were supposed to mix all the cuttings brought by them.
- Each group was suggested to segregate the products into two categories; with values less than ₹ 10,000 and with values ₹ 10,000 or more.
- Initially, to understand the level of understanding of numbers at Class VI level, teacher restricted the activity upto 4 digit numbers only.
- Each group was suggested to paste the cuttings on a drawing sheet.
- Every student was asked to note down the names of the products and its

value (both in numerals and in words) in increasing order in their note books.

Observations and additional input given by teacher

- Few students brought the products valued less than ₹ 1,000. Teacher realised that these students were not able to understand the given task. Some of them were not able to make out that the numbers with four digits are more than one thousand. They were guided accordingly.

- Few students who were not able to segregate the values into two categories, were helped by other students in reading and in segregation correctly. For example, one student was placing values ₹ 14,009 and ₹ 1,499 in one category, but his groupmates helped him to understand the difference by counting the number of digits in the given values.

- Teacher checked with every group whether they have done the proper segregation. Few students needed guidance (that the four digit numbers are always less than ten thousand).

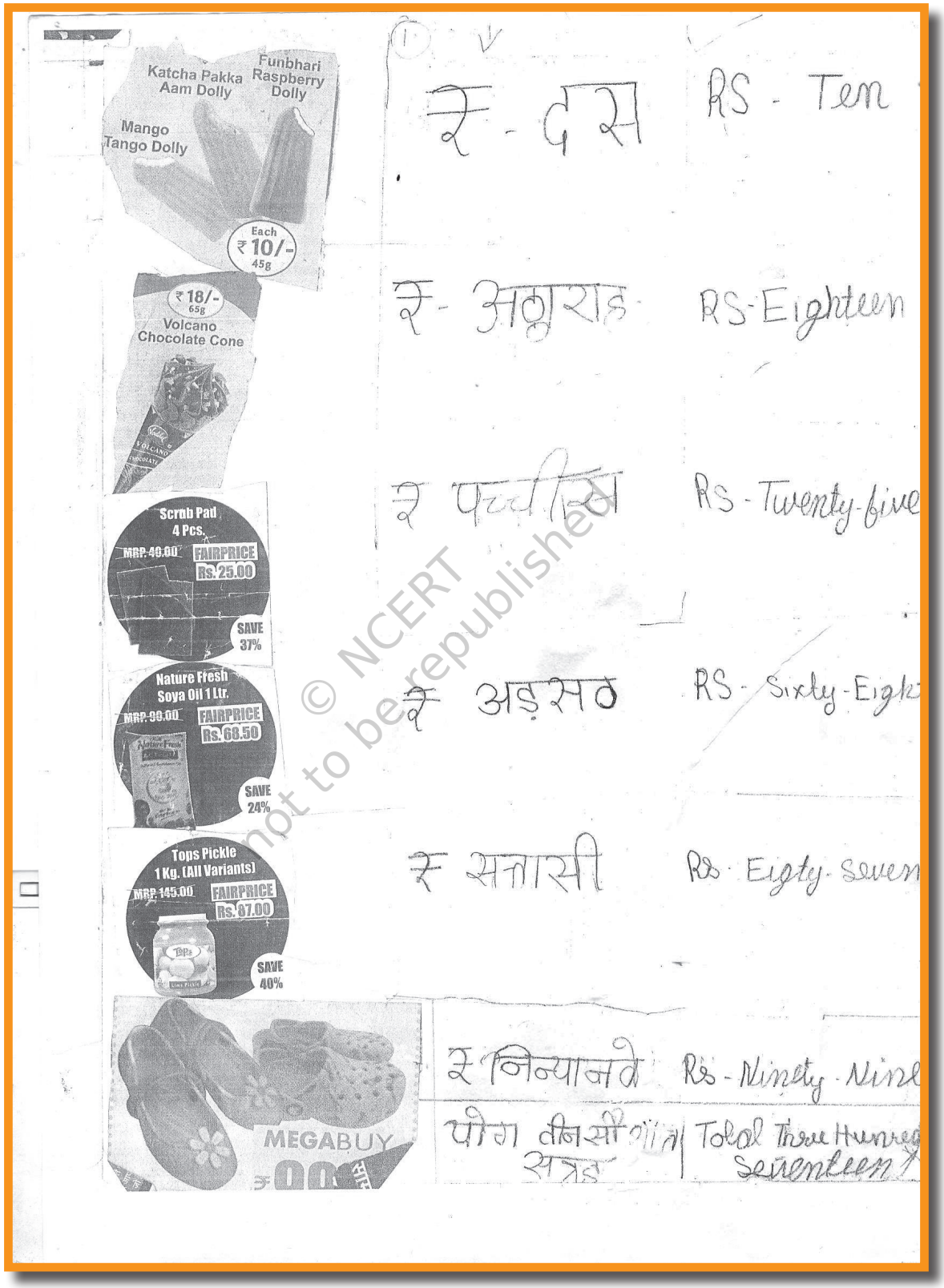
- It was observed that few students were not able to write the number names correctly. Teacher asked these students to read the name of first three digit number separately and then to read the name along with fourth digit which is at thousands place. For example, one student was reading 7,099 as seven hundred ninety nine. He was helped by the teacher in reading it correctly.

- Some of them were not able to arrange the values in correct order. But after understanding the place value of fourth digit (thousands place) correctly and reading the number accurately, they were able to arrange the values in correct order.

- When the students were able to perform the given task satisfactorily, they were asked to proceed with five digit numbers in a similar way.

Following are some of the responses of the students:

Response 1

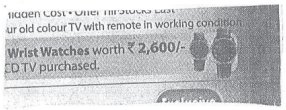


Response 2

दो हजार एक सौ –
Two Thousand one hundred

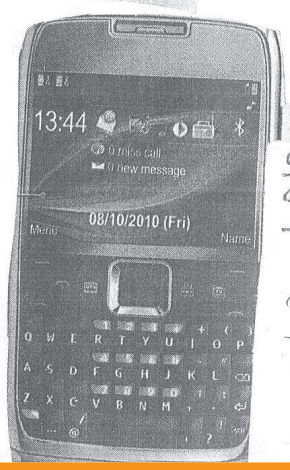
दो हजार एक सौ पिचानवे –
Two Thousand one hundred ninty Five

दो हजार चार सौ नब्बे –
Two Thousand four hundred ninty



दो हजार छः सौ –
Two Thousand Six hundred

दो हजार छः सौ नब्बे –
Two Thousand Six hundred ninty



~~₹ 12000~~ ₹ 2999

दो हजार नौ सौ निनानवे –
Two Thousand nine hundred ninty nine

331
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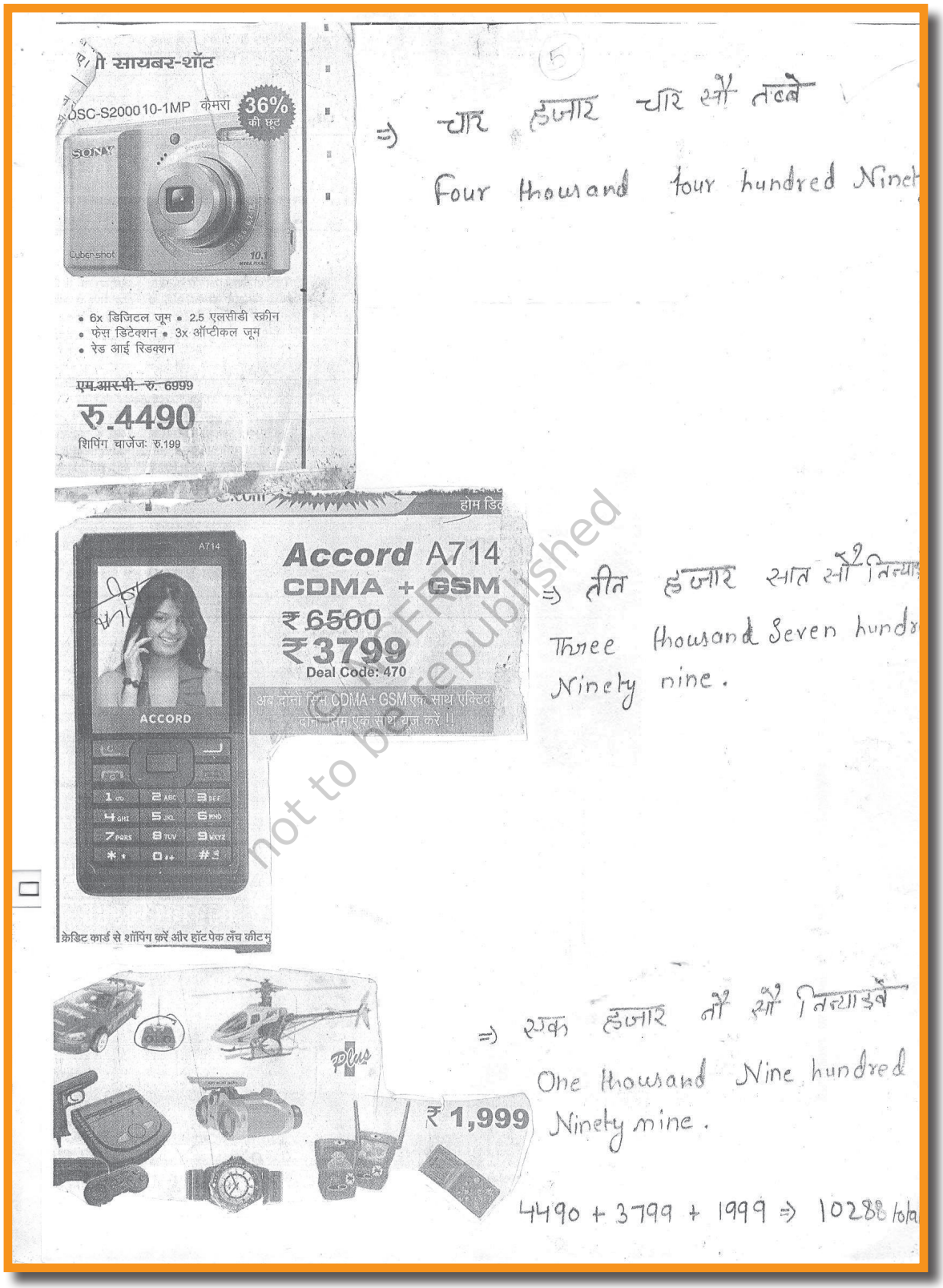
2490
2600
2690
+2999
15974 Total

Response 3



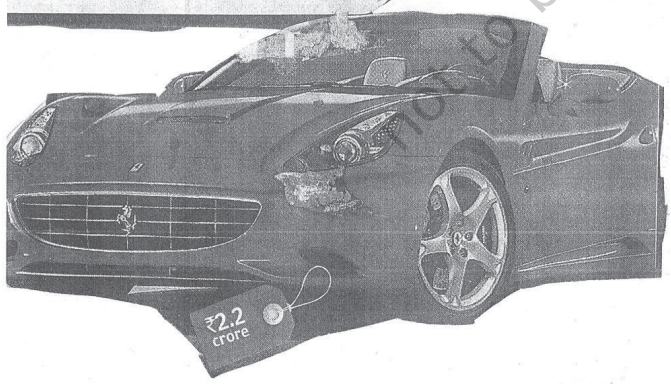
Response 4

=) उन्नीस हजार सात सौ नब्बे

=) Nineteen Thousand Seven hundred Ninety.

=) सत्ताईस हजार नौ सौ नब्बे

=) Twenty Seven Thousand Nine hundre Ninety.



=) दो करोड़

=) Two crores

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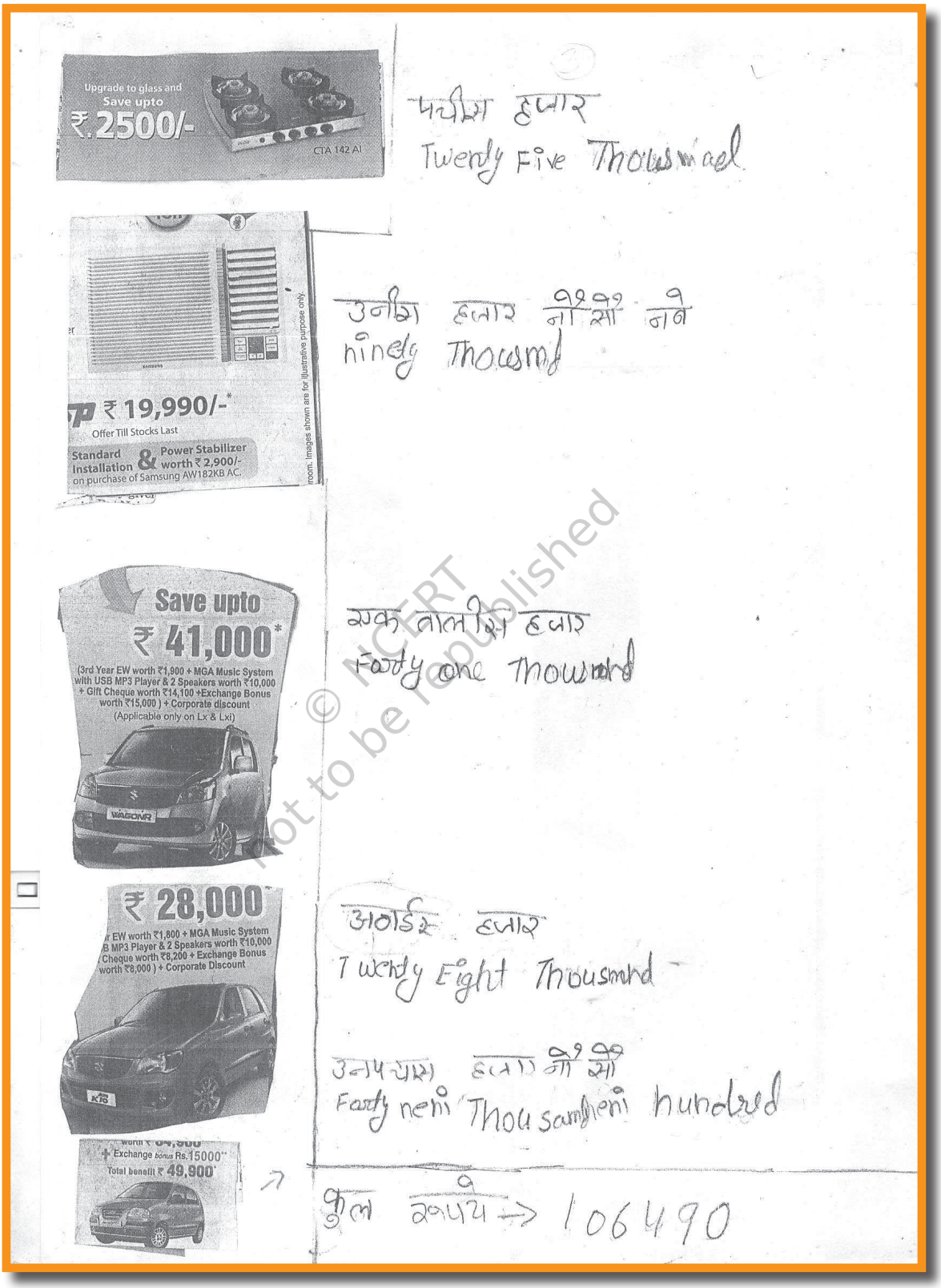
Response 5



5.2 Case Study

This is a case of Class VIII that happened while teaching the chapter, Practical Geometry. The children were taught to construct a quadrilateral when some vertex angles and sides of the quadrilateral are given. While the class was being taken, students could follow the steps with the teacher and could draw the quadrilaterals in their class work copies.

The next day, first 5 minutes were spent by the teacher to check how much of the concept could the students grasp after teaching it in the previous class. This was done by asking the students to write their names on a piece of paper and construct another quadrilateral with the same kind of parameters which were used for construction in the last class. After spending few minutes of the period on this, the papers were collected by the teacher and the teaching continued. Later, the teacher evaluated each paper and labelled them as I, II, III, IV or V according to the five stages defined in the proposed system. There was an amazing observation. It was found that in the class strength of 40, there were about 32 students in Stage I. This was alarming!

The next day, after evaluating the papers, the teacher discussed with the students and tried to find the reason for this performance by them. During the discussion, the fact which came out, would have never got noticed with the current or the past system. It was found during the discussion that, when these children were in Class VII, the mathematics teacher left the school in mid-session. It took a long time to get another teacher to continue the mathematics classes. In the process, the chapter on construction of basic angles was not covered and as a result of this, students did not know how to construct the basic angles and also the angles that were multiples of 15. Such a diagnosis was quick, convenient and a boon for the teaching-learning process. The teacher could find out the exact measure that was required to be taken in order to continue with the lesson and the actual teaching-learning happened in the right form with appropriate measures, which was not possible otherwise.

This also brings to light the fact that **teaching, learning and assessment need to go hand in hand.**

5.3 Case Study

This activity was integrated with the teaching-learning process by giving importance to each student's way of responding and learning. I conducted this activity on volumes and surface areas in Class VIII with 40 students while teaching Mensuration chapter in a regular period of 40 minutes. This was diagnostic assessment and was conducted with more of an emphasis on peer learning. This assessment of learning, generated opportunities for self- assessment and peer assessment. This activity became a source of interactive feedback which allowed students to adjust, rethink and relearn. Thus, it helped in successful and concrete learning.

Initially, the students were asked to bring empty containers which were cuboidal in shape from their homes, for example, match box, containers of tooth paste, empty packs of *Agarbatti*, soap case, cold cream etc. Students took interest in picking up cuboidal shapes of various types from their homes.

I asked students to measure the length, breadth and height of their cuboids with ruler and note down on a paper. They were instructed to calculate (i) volume, and (ii) total surface areas of their cuboids. I also asked them to draw a rough sketch of their cuboids. The dimensions they noted down involved decimals also. The multiplication of dimensions in decimals assessed their previous knowledge of decimal multiplication while calculating volumes. I could assess their concept of mentioning correct units for volume and surface areas. After assessing each student's activity, I graded them in 5 different stages, namely:

In **Stage I**, I listed those students who had not understood the concept and a few of them who had brought only cubes.

In **Stage II**, I grouped the students who had brought cuboidal containers but as they had no knowledge of correct formulae, they had calculated volumes and surface areas wrongly.

In **Stage III**, I grouped the students who had brought correct cuboidal shapes, they had used correct formulae for computations but had committed mistakes in mentioning correct units for volumes and surface areas.

In **Stage IV**, I listed the students who lacked the skill of drawing three dimensional figures of cuboids.

In **Stage V**, I selected the students who could accurately noted down the dimensions of length, breadth and height after measuring them. They had used correct formulae for calculations of volume and total surface area. They had understood the concept thoroughly as they had drawn correct figures too.

This activity gave me an interactive feedback and I used this assessment for my remediation. While taking remedial measures, the students of Stage V were made group leaders of 8 groups, each group consisting of four students. Here, I provided cuboids of same shape and size to each group. Now they were asked to write down the dimensions with proper units. The emphasis was on measurement of three dimensions which cleared their concept of expressing volumes in cubic units.

Similarly, after calculating total surface area why the answer has to be mentioned along with square units was clearly explained by group leaders and me. Peer assessment confirmed common answers. This activity gave a scope for thorough revision of decimal multiplication too. They were re-explained to put the decimal at proper place after multiplication. Thus through this activity, a concrete learning of the concepts took place and the students recognised which all containers around them had cuboidal shape. They practically learned by doing this activity how the volume and surface area can be calculated.

During remediation, all the stages were taken care of step by step thus helping successful and concrete learning. I found this activity very interesting and convenient to assess the students. I found that students had developed clear concepts of volume and surface area, in comparison to my previous method of just solving questions from exercise on the blackboard and students noting them down in their notebooks without understanding the practical application of the concept and meaning of the terms volume and surface area etc.

Ms Sharma
Mathematics Teacher
Uttar Pradesh

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"Winning doesn' t always mean being first----------winning means you're doing better than you' ve done before."

— Bonnie Blair

APPENDIX

Following are the suggestive measures that should be taken into consideration while recording and reporting:

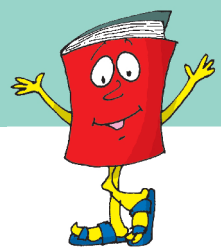
1. Project should be subject-specific and should ensure learning of the particular subject-concept. It can be done during any term, as teachers can decide, which chapter can be best explained through the project.
2. Certain activities such as survey, research, discovery etc., can be scheduled to be done during vacation. Lab activities can be done hand-in-hand while covering the corresponding chapters.
3. Field trip can be done during any term, as per the convenience of the school.
4. Pen and paper tests to give the achievement report at the end of every chapter should be designed to incorporate questions to check all the five stages of the child for that chapter. This report will show the maximum potential of the child in that chapter. Hence, before conducting this test, a series of diagnostic and remedial measures should be taken during the teaching periods of that chapter. All the achievement tests should be taken in stamped school sheets and should be retained by the teacher to be produced at the time of need.
5. Lastly, a report card which consists of five such recording and reporting sheets for five subjects will be filled by the respective subject teachers.

(All subject teachers will posses a photocopy of the above reporting scheme to record observations continuously and these observations and recordings can be copied down to the report card at the Term-end). Schools can divide terms in terms of months as per their convenience.

Note: It is recommended that enough flexibility in terms of time be given for submission of project. Students should be free to utilise as much time they need to completely explore the project and do them on their own.

Notes

Notes



A Time to Reflect

Why Should Children be Assessed?

Since we are all concerned about children's learning, assessment needs to be undertaken for a number of reasons:

- Support and improve every child's learning and development.
- Observe what changes and progress takes place over time.
- Identify individual and special needs and requirements.
- Plan teaching-learning situations in a more suitable way.
- Enhance the child's self-understanding and personal development.
- Achieve curriculum aims/syllabi objectives.
- Improve on-going teaching-learning in classroom.
- Provide evidence of children's progress to communicate to parents and others.

What Should be Assessed?

- Children's learning/performance in different subject areas.
- Achievement of skills – academic, inter personal, etc.
- Interests, attitudes and motivation amongst other aspects.
- Change and progress over time.
- Children's response to educational inputs, situations and/or opportunities.

When Should Assessment be Made?

- Continuously throughout the year.
- Periodic reflection by the teacher 3 or 4 times in a year.

How Should the Assessment Process be Undertaken?

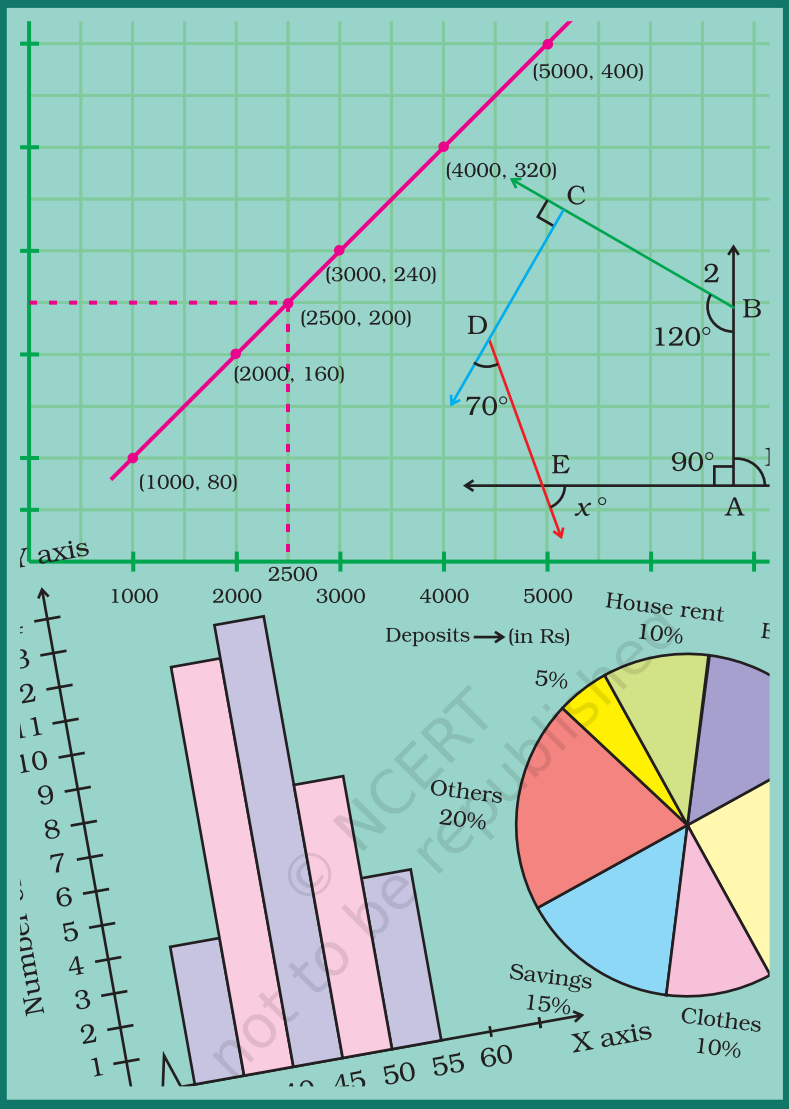
Steps that can be followed are:

- Collecting information/evidence.
  - A variety of sources
  - Different ways/methods
- Recording of information/evidence.
- Making sense of collected information/evidence.
- Sharing and communicating feedback on assessment.

How can Assessment Information be Used?

It can be used to :

- Improve children's learning and performance.
- Provide the right kind of learning opportunities, materials, aids, equipment etc.
- Bring out the best in children.
- Improve teaching-learning processes.
- Cater to differences in children and their special needs.
- Move children from one level of learning to a higher level.



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विद्यया ऽमृतमश्नुते

एन सी ई आर टी
NCERT

राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

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