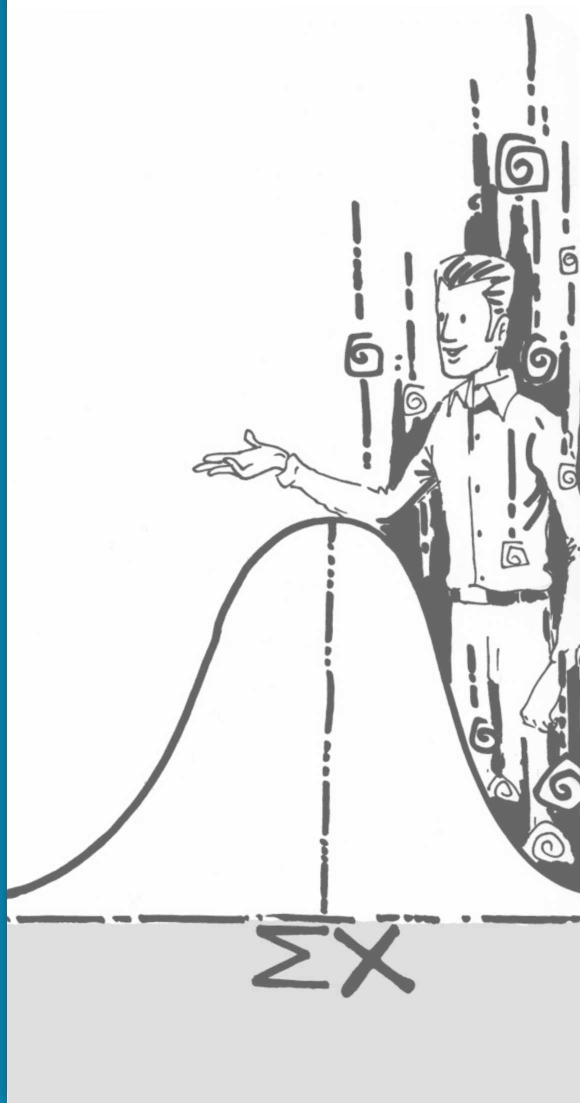
Module 7



Basic Statistics in Guidance and Counselling-I

Department of Educational Psychology and Foundations of Education

National Council of Educational Research and Training

While the individual man is an insoluble puzzle, in the aggregate he becomes a mathematical certainty. You can, for example, never foretell what any one man will be up to, but you can say with precision what an average number will be up to. Individuals vary, but percentages remain constant. So says the statistician.

—ARTHUR CONAN DOYLE

Basic Statistics in Guidance and Counselling I

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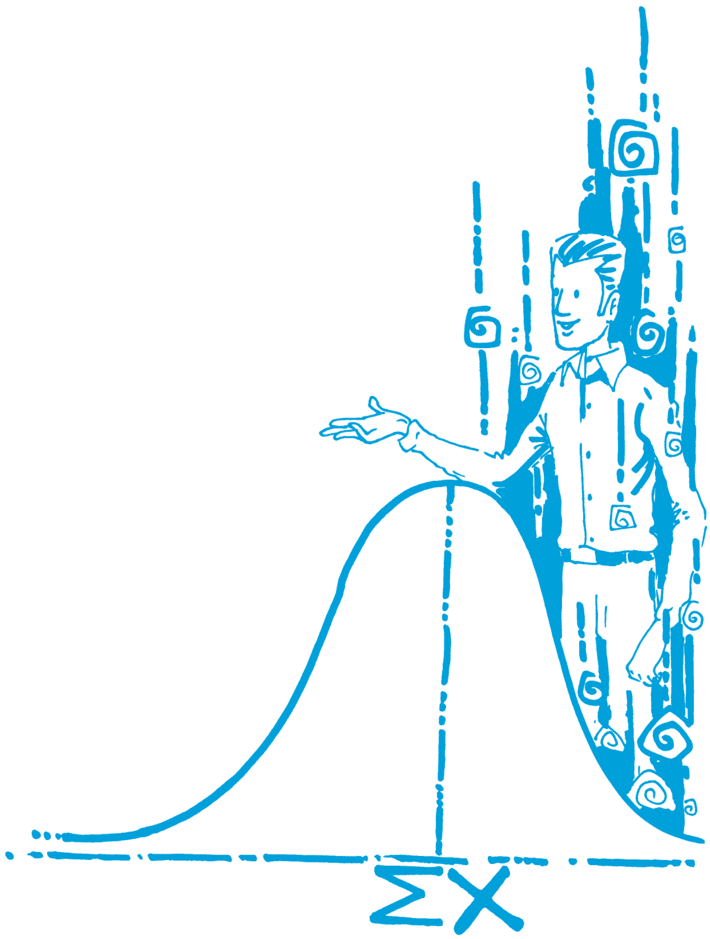

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About the Module

This is the seventh module of the course in Guidance and Counselling. It aims at developing basic statistical literacy especially in the context of guidance and counselling. The units covered in this module will help you to understand the statistical data given in the test manuals, and to analyse and interpret test scores.

The first unit deals with the importance of statistics in guidance and counselling. It also covers the three commonly used measures of central tendency, i.e., the mode, median and mean. These statistics are powerful because they can reduce huge arrays of data to a single, easily understood number. The second unit focuses on measures of variability which summarise information about the heterogeneity or variety in a distribution of scores. It may be noted that while measures of central tendency, locate the central points of the distribution, measure of variability indicate the amount of diversity in the distribution. The third unit of this module examines the concept of normal probability curve which is of great importance in statistics. It, in combination with the mean and standard deviation, can be used to construct precise descriptive statements about distributions that are normally distributed.

There are Self-check exercises and activities in every unit which will help you evaluate your progress through the module. Summary given at the end provides an overview of the unit, and references and suggested readings give additional sources of information.

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Importance of Statistics in Guidance and Counselling, and Measures of Central Tendency

Importance of Statistics in Guidance and Counselling, and Measures of Central Tendency 1

1.0 Introduction

When you use a large number of student scores or marks to derive meaning out of them you convert the scores into class interval form. Each class interval has a frequency that represents the number of scores falling in them. This type of arrangement of scores is also known as a frequency distribution table. After the scores have been tabulated into a frequency distribution, you require statistical techniques to identify the typical or average score for a distribution. Usually, you want to choose a value in the middle of the distribution because central scores are often the most representative. For example, all parents want to know at what age their baby will walk. When the long awaited event occurs, parents wonder whether their infant's accomplishment is typical or advanced. The information necessary to answer such questions requires a large sampling of infants and recording the age at which each took their first unassisted step. Then one would make a frequency distribution of such ages and graph it.

Next, what parents want to know is the "typical age" at which infants take their first step. This tendency of a group about a distribution is named the central tendency and the typical score lying between the extremes and shared by most of the students is referred to as a measure of central tendency. This unit will describe the importance of statistics in guidance and counselling, and several numerical indices of central tendency that can be used to characterise distributions and to compare one distribution to another.

1.1 Objectives

After going through this unit, you will be able to

- ***understand*** the importance of statistics in guidance and counselling.
- ***describe*** what is meant by 'measures of central tendency.
- ***calculate*** the mode, median and mean for a given distribution.

- ***identify*** the unique properties of the mode, median and mean.
- ***explain*** where each measure of central tendency is located in symmetrical and asymmetrical distributions.

1.2 Importance of Statistics in Guidance and Counselling

Human behaviour has never been and perhaps will never be understood in terms of a set of mathematical equations. However, mathematical equations can surely help to understand human behaviour. Statistics is the science of data which is used to understand human behaviour. Statistics can be defined as the scientific technique of collection, classification, tabulation, presentation, analysis and interpretation of the numerical data. The word statistics is the plural of statistic (singular) which refers to the result of applying a statistical algorithm to a set of data, as in economic statistics, crime statistics, teaching statistics, educational statistics etc. Similarly statistics is also applied to get meaning out of data acquired of psychological aspects of human behaviour.

During the process of guidance and counselling, measurement and evaluation of the individual in all the multiphasic aspects of his/her behaviour and personality make-up is done. Guidance movement began at a time when psychology had established itself fully as a science of human behaviour and had evolved under the impact of physical and biological science techniques that gave emphasis to the measurement of more intangible aspects of human behaviour. The period from 1900 to 1915 was characterised by pioneering efforts of Binet and his followers in America to bring psychological and educational measurement on scientific lines. The tools of psychological measurement from 1915 onwards pinned its faith in the use of psychological tests as a device for diagnosis and prediction of human behaviour.

Gradually, guidance and counselling which had based its psychological foundations on individual differences was accepting measurement techniques which could provide a near scientific and objective statement of these differences. And as tests did it much better than any other technique could possibly do at that time, guidance workers accepted it as a ready solution to all their problems of diagnosis and prediction of individual's psychological characteristics, attributes such as intelligence, aptitude, interests and personality qualities.

The numerical data yielded by psychological tests was considered easier to handle and interpret. It also gave an objective colouring to the entire guidance and counselling procedure thus making it appear more scientific.

For effective guidance and counselling work, the information about the individual is collected for understanding and guiding the individual(s). The information gathered has to be systematic, comprehensive and continuous and the evaluation procedure should take cognizance of the different aspects of human behaviour to give a holistic view and also the ever-changing and ever-developing pattern of his/her personality make-up.

We will now discuss the situations in which statistics is used to derive meaning out of data acquired. Data about the individual or many individual groups are collected from various sources using variety of tools and techniques. Statistics helps a counsellor to get meaning out of data. We shall discuss the various methods that are used for data to be recorded, organised, presented, interpreted and analysed. Teacher/counsellor must understand the implications of this data and its usefulness in detecting the potential of the pupil.

Organisation and Presentation of Data

After the data collected through tests or descriptive or qualitative measures, statistics provides you the methods to organise and process the data by presenting it in a tabulated form in a frequency distribution. This data can then further be presented graphically in the form of diagrams e.g. of Histogram, Frequency Polygon, Bar Charts, Pie Diagram etc.

The graphical representation is the pictorial presentation of data, it is a vivid representation of given facts and it also helps to dilute the abstractness of ideas.

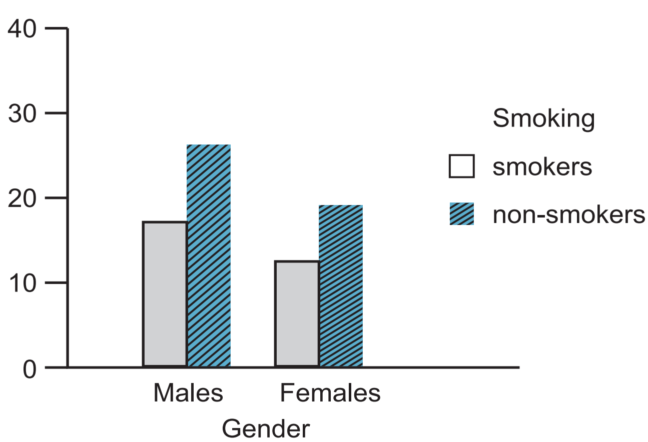
When data are presented in a histogram, the purpose is to show the frequency of occurrence of a phenomena or behaviour within certain classes or categories graphically. The frequencies within each interval of a histogram are represented by a rectangle, the base of which equals to size of the interval and the height of which equals the number of scores within that interval. The interesting property of the histogram is that the area of histogram corresponds to the total frequency (N) of the scored distribution and that the area of each rectangle in a histogram corresponds to the frequency within a given interval.

If instead of whole class interval, only midpoints are connected then resulting figure is called a frequency polygon. The midpoints of an interval is taken to represent the entire interval. In a histogram, we have assumed that the scores are spread uniformly over their intervals, whereas in a frequency polygon we assume that all scores in a given interval are at the midpoint of the interval.

The histogram is the easiest of all to understand and is usually the best if only one distribution is being represented, however if two or more distributions are to be compared, frequency polygons are usually better.

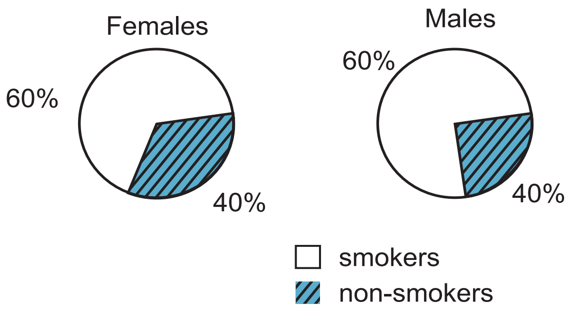
Bar Charts/Diagrams

The data are presented in a Bar/Chart when the levels of the variable are categorical (boys-girls, rural-urban, etc.) as in the example of male and female smokers



Pie Charts

Data presented in the form of Pie Chart differs from the graphs, in that it does not use axes but represents the subtotals as slices of a pie. Alternatively the areas could be expressed as percentages.



Analysis and Interpretation

Statistics helps in getting meaning out-of-data through the process of analysis and interpretation. After grouping of test scores into frequency tables and condensing them they can be analysed and interpreted. A single term which is representative of the entire distribution needs to be found. Since the scores presented in the entire distribution are usually found near the centre of the data, they are commonly called measures of central tendency.

There are five measures of central tendency : the arithmetic mean, the median, the mode, the geometric mean and harmonic mean. The mean, median and the mode are the most widely used measures.

When we want to have a general estimate of the performance of a person or number of persons on a task, mean helps to find out the average standing of the person or the group. It can also help to find out the typical behaviour or level of performance in the group.

The mean is indeed the word 'average' used by many persons. The mean scores can be obtained by adding all the scores and dividing the sum by the number of students $\left(\bar{X} = \frac{\sum X}{N}\right)$, where

$\bar{X}$ = Mean

$\sum X$ = Sum of the raw scores

N = Number of scores

There are various other methods to calculate the mean. (You will learn about it in subsequent units of this module). Mean is the most useful of all statistical measures. It is the base from which many other important measures are computed.

Example: On a test of achievement which was administered to 6 students, the following scores were obtained:

	X
	6 5 4 3 2 1 $\sum X = 21, N = 6$
Mean = 3.50	

Median is the point on scale which separates the top half of the group from the bottom half. It is a measure of position rather than of magnitude and is frequently found by inspection rather than by calculation. When there is an odd number of untied scores, the median is the middle score.

Example:

7, 6, 5 } 3 scores above

4 ⟶ Median

3, 2, 1 } 3 scores below

When there are even number of untied scores, the median is the mid-point between the two middle scores.

Example:

6, 5 } 2 scores above

4

3 ⟶

2, 1 } 2 scores below

When the distribution of scores is such that most scores are at one end and relatively few are at the other (skewed distribution) the median is preferable because it is not influenced by extreme scores at the either end of the distribution.

The mode is the item which occurs most frequently in a statistical series. In a simple ungrouped series of measures, mode is that single measure which occurs most frequently. Example in the series 6,8,9,10,11,11,11,13,14,14,15 the most often recurring measure is 11, so is the mode. The mode obtained by mere inspection is called the crude, apparent or inspectional mode.

The mode which is estimated, computed or refined can be determined with the formula,

Mode = 3 median – 2 mean

Because of the difference in the method of determining these measures, mean is often called the calculated average, median is known as counting average while the mode is called the inspected average.

Thus, these measures help in finding some meaning out of the data.

Measuring Group Performance

For understanding group behaviour or characteristics, a counsellor administers group tests. It generates a lot of data. The raw scores tell us nothing more than the number of answers marked correctly. A raw scores by itself means very little; as these are always understood in a context or against a norm.

The simplest technique used for this is indicating relative position by rank order. It simply consists of arranging the scores in order of size and then assigning a rank to each, the top scores receiving the rank and so on down the line.

The second method is of indicating a pupil's relative position in a group is by showing how far his/her raw scores is above or below the mean.

There are numerous methods of comparing standard scores like Z-scores, T-scores, stanine, etc. These are discussed in detail in unit 1 titled transformation of scores.

The third method of making scores comparable is by computing percentile ranks. It indicates a pupil's relative position in a group in terms of the percentage of pupils scoring below him. The points that divide the distribution into 100 equal divisions are called percentiles. The median is the 50th percentile, since 50 per cent of all the measures lie below this point. Similarly Q_1 is the 25th percentile and Q_3 the 75th percentile.

The percentile system enables us to compare the standing of a student on various tests. Percentile norms are suitable for any type of test such as an aptitude or an achievement test or a personality inventory.

Statistical methods also help us to compare the performance between two groups of individuals having different levels of performance or the effect of certain factors on the individual or group to see whether the difference between two sets of data is real or due to chance factors. The methods will also be discussed in second module on statistics.

Tool Development

Sometimes counsellor is also required to develop tools and tests suitable for his/her own need. In that case She/he should be well aware of the statistical methods to develop the tools which proves to be reliable and valid indicator of an individual's potential.

Some tools and tests prepared require to be standardised. The counsellor should be aware of the conditions of reliability and validity. Reliability is the degree of consistency that the instrument or procedure demonstrates. Whatever it is measuring, the tool has to do so consistently. Validity is that quality of a data-gathering instrument or procedure that enables it to measure what it is supposed to measure. Statistics helps you to establish these conditions to have a reliable and valid measure. More details on these measures have also been discussed in module on assessment and appraisal.

Research Reporting

Interpretations acquired in terms of scores have to be presented in a readable form based on research and analysis done with the help of the statistical methods. For example, scores transformed into percentile norms, stanine, t-score or z-scores etc. need to be interpreted judiciously and tentatively as the judgements given by a counsellor to a person seeking counselling are based on these statistical measures.

1.3 Meaning and Computation of Measures of Central Tendency

Measures of central tendency are the most frequently used statistical methods in psychological and educational studies. These measures provide a single figure that best summarises and describes the central location of a set of scores or observations. In this section you will study about their salient characteristics and also learn how to compute them for a set of scores or observations.

There are three situations in which a measure of central tendency is used:

- To compare the level of performance of a group with that of a standard reference group.
- To indicate a standard of performance not previously known.
- To compare the level of performance under two or more conditions or between two or more existing groups.

There are numerous measures of central tendency, but the most common ones are mode, median, and mean (see Figure 1.1). In the next section, we will define these three measures, discuss their computations and examine their properties.

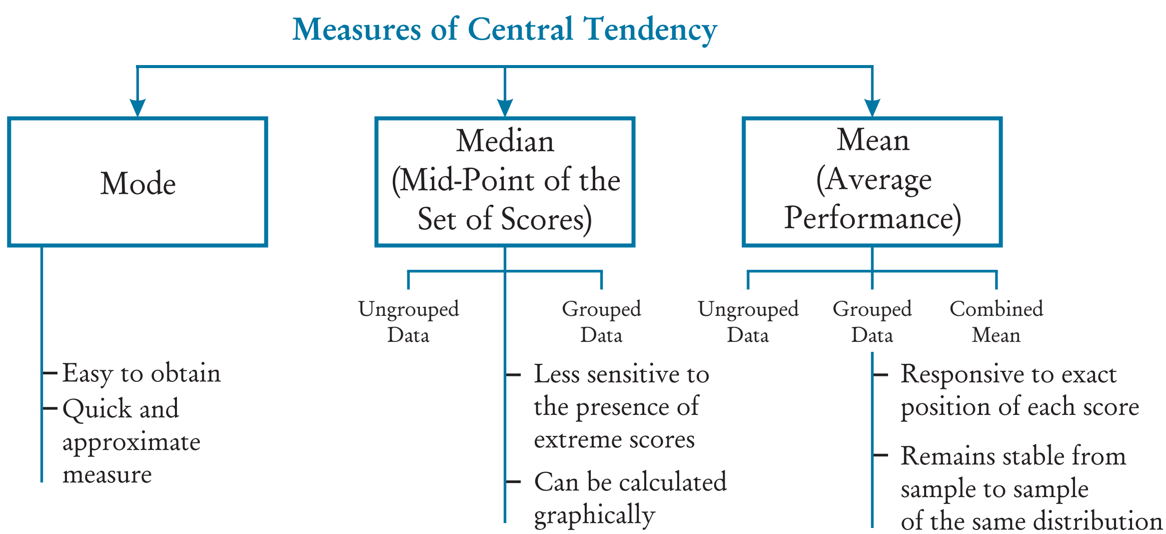


Fig. 1.1 : *Measures of Central Tendency*

1.3.1 The Mode

The common meaning of mode is 'fashionable' and its statistical meaning is the most common observation among a group of scores. Accordingly, in a frequency distribution the mode is the score that has the greatest frequency and its symbol is Mo.

Consider the following set of scores:

X: 3, 4, 4, 5, 5, 5, 6, 8

In the above distribution of scores, 4 has occurred twice, 5 has occurred three times, and other scores (3, 6, 8) have occurred only once. Since score of 5 has highest frequency, it is the mode value of the group.

Sometimes it is possible for a distribution to have more than one mode. Consider the following distribution:

X: 3, 4, 4, 4, 5, 6, 6, 7, 7, 7, 8

In this case, the mode values are 4 and 7 and such a distribution is called bimodal. A distribution that contains more than two modes is called multimodal.

Self-check Exercise 1

Find the mode for the following set of scores:

X: 5, 4, 3, 2, 1, 2, 6, 4, 4, 1

1.3.2 The Median

The median is another measure of central tendency. It is the point along the scale of possible scores below which 50% of the scores fall. Thus, the median is the value that divides the distribution into two equal halves. Exactly one-half of the scores are less than or equal to the median. Because, exactly 50% of the scores fall at or below the median, this value is equivalent to the 50 percentile. The symbol of median is Mdn. In the following two sections, the methods of calculating median from different groups of data (i.e., ungrouped and grouped data) are shown.

1.3.2.1 Ungrouped Data

To find the median from ungrouped data, we first arrange the scores in rank order from lowest to highest.

(i) When there is an odd number of cases in the distribution, the median can be calculated by taking the score value corresponding to the middle case, which will be the $(n + 1)/2$nd case from the bottom of the distribution. If the distribution has five scores: 3, 4, 6, 7, 10, the median will be $(5 + 1)/2$nd case, i.e. 3rd case from the bottom. The score value of 3rd case is 6, so the median is 6.

(ii) When there is an even number of scores in the distribution, the median is halfway between the score values of the middle cases, which will be the average of the score values of the two middle cases. Thus, median will be average of cases $n/2$ and $(n/2) + 1$ from the bottom of the distribution. If the distribution has six scores:

3, 5, 6, 7, 10, 14, the middle two cases will be n/2 = 6/2 = 3rd case and (n/2) + 1 = 3 + 1 = 4th case. The median will be the average of the score values of these two middle cases, i.e., (6 + 7)/2 = 13/2 = 6.5

1.3.2.2 Grouped Data

The procedure for calculating median for grouped data is same as for calculating P_{50}. The computational formula for the median is:
(Computational Formula for the Median)

$$Mdn = P_{50} = LL + (i)\,\frac{(n/2 - \text{cum f below})}{f}$$

where, LL = Lower real limit of class interval containing P_{50}

i = width of the class interval

$n/2$ = half the cases (i.e., number of scores lying below the median)

cum f below = number of scores lying below LL

f = frequency of scores in the interval containing the median.

Consider the following distribution for calculation of median for the grouped data:

Example 1: Calculation of Median for Grouped Data

Score	f	cum f
195-199	1	50
190-194	2	49
185-189	4	47
180-184	5	43
175-179	8	38
170-174	10	30
165-169	6	20
160-164	4	14
155-159	4	10
150-154	2	6
145-149	3	4
140-144	1	1
	N = 50	

The computation of median is done in the following steps:

1. Find out $\frac{n}{2}\left[\frac{50}{2} = 25\right]$
2. Find out the class interval in which the median (i.e., 25) lies. In this example, it is located in the class interval 170-174.
3. Find out LL (lower limit of the class interval below median). The value is 169.5.
4. Find out cumulative frequency below the class interval in which median lies (in this case it is 20).
5. Find out f (the frequency of class interval in which the median lies), which is 10.
6. Find out i (the length of the class interval), which is 5 in this case.
7. Substitute all these values in the formula.

$$Mdn = 169.5 + (5)\frac{0.5 \times 50 - 20}{10}$$

$$= 169.5 + \frac{5}{10} \times 5$$

$$= 172.00$$

Self-check Exercise 2

Find the median for the following set of scores:

X: 3, 10, 8, 4, 10, 7, 6

1.3.3 The Mean

The mean is the most widely used method among all the measures of central tendency. It is the sum of all the scores in a distribution divided by the total number of scores. It is also called arithmetic mean. A distinction is made between the mean of a sample and mean of a population. However, this distinction is not crucial at a purely descriptive level and it is important only at inferential statistics level. The mean of a sample is represented by $\overline{X}$, read as X bar, and mean of a population by μ the Greek letter "mew." Notice that the scores do not have to be ordered to find a mean.

1.3.3.1 Ungrouped Data

The defining raw score formulas for mean

$$\mu_x = \frac{\Sigma X}{N} \text{ (mean of a population)}$$

$$\overline{X} = \frac{\Sigma X}{n} \text{ (mean of a sample)}$$

Here,

X = refers to raw score

S = refers to sum of all

N = size of the population

n = size of the sample

Let us calculate mean for the following sample scores:

X
8
3
4
10
7
1
$\Sigma X = 33$

$$\overline{X} = \frac{\Sigma X}{n} = \frac{33}{6} = 5.5$$

1.3.3.2 Grouped Data

When scores are grouped in the form of class intervals, we assume that the midpoint of the interval is the mean of the scores in that interval and we use the mean to represent the scores in the interval. To calculate the sum of scores, we multiply each midpoint by the number of cases in the corresponding interval and then sum these products over all class intervals. Let us consider the following example:

Example 2 : Calculation of Mean for Grouped Data

Scores	X (midpoint)	f	f X
90-94	92.0	2	184
85-89	87.0	4	348
80-84	82.0	9	738
75-79	72.0	7	504
70-74	72.0	10	720
65-69	67.0	12	804
60-64	62.0	9	558
55-59	57.0	5	285
50-54	52.0	3	156
45-49	47.0	3	141
40-44	42.0	2	84
		n = 66	Σf X = 4522

Note : The frequency is given to you in this example.

$$\bar{X} = \frac{\Sigma fX}{n}$$

$$= \frac{4522}{66} = 68.51$$

1.3.3.3 Mean of Combined Sub-groups

Sometimes means of several sub-groups are known and it is desired to find the mean of all of the scores when the sub-groups are pooled. Under such circumstances, the following formula can be used:

$$\bar{X}_c = \frac{n_x\bar{X} + n_y\bar{Y} + \cdots}{n_x + n_y + \cdots} \text{ (mean of combined sub-groups)}$$

where,

$\overline{X}_c$ = Mean of the combined sub-groups

n_x = Number of scores in group X

$\overline{X}$ = Mean of group X

n_y = Number of scores in group Y

$\overline{Y}$ = Mean of group Y

Suppose two sub-groups have been given the same test and the mean of each group is known. Let us calculate the combined mean.

Given : $\overline{X} = 40 \quad \overline{Y} = 30$

$n_x = 50 \quad n_y = 25$

$$\overline{X}_c = \frac{n_x\overline{X} + n_y\overline{Y}}{n_x + n_y}$$

$$= \frac{(50)(40) + (25)(30)}{50 + 25} = \frac{2000 + 750}{75} = \frac{2750}{75} = 36.66$$

Self-check Exercise 3

Compute mean for the following set of scores:
X : 24, 18, 19, 12, 23, 20, 21, 22

1.4 Properties of Measures of Central Tendency

By now it must be clear that each measure has certain distinguishing characteristics which can make the measure more useful depending upon the situation. For a better understanding let us put them together.

1.4.1 Properties of the Mode

- The mode is easy to obtain.
- Mode can be used when a quick and approximate measure of central tendency is desired.
- The Mode does not provided a lot of detailed information.

1.4.2 Properties of the Median

- The median shows how many scores lie below or above it. The median does not show the relative distance of a score in a given distribution. Thus, the median is less sensitive than the mean to the presence of a few extreme scores in a given distribution. Therefore, in the distributions that are strongly asymmetrical (i.e., skewed) or have a few very deviant scores, the median may be the better choice if you wish to represent the bulk of the scores and not give undue weight to the relatively few deviant ones.

- Sometimes we encounter a distribution that is open-ended, where the upper limit of the top class interval is not specified and consequently the mid point of the interval is unknown. In such open-ended distributions, the mean cannot be calculated, but calculation of the median remains possible.
- The median of the distribution can be calculated graphically, but a mean cannot be.
- The median is not as useful a measure of central tendency as the mean for purposes beyond a descriptive level.

1.4.3 Properties of the Mean

- The Mean is responsive to the exact position of each score in the distribution. Any increase or decrease in the value of any score changes $\sum X$ and thus, also changes the value of the mean.
- The mean is the balancing point of a distribution. If we express the scores in terms of the amount by which they deviate from their mean, taking into account the negative and positive deviations, their sum is zero. Thus, the sum of the negative deviations from the mean exactly equals the sum of the positive deviations i.e., $\sum X = 21, N = 6$.
- The mean is more sensitive to the presence or absence of scores at the extremes of the distribution than are the median or the mode.
- When a measure of central tendency is required that reflects the total of the scores, the mean is the best choice because it is the only measure based on this quantity.
- One of the most important characteristics of the mean is its stability from sample to sample. If a distribution is bell-shaped, the means of the successive samples will vary least among themselves. Thus, the mean is most resistant to sampling fluctuations.
- When linear transformations are applied, the value of the mean is affected by all four processes (addition, subtraction, multiplication, and division). Linear transformations preserve a proportional or straight-line relationship between the original scores and their transformations. Accordingly, addition or subtraction of a constant value increases or decreases the mean by the same amount. Similarly, if we multiply or divide each score by a constant, the mean will be changed in the same way.
- When we need to do further computations, the mean is likely to be the most useful of the measures of central tendency.

You have thus seen that each measure has certain limitations and advantages. The selection of a particular measure, therefore, needs to be done with care.

1.5 Measures of Central Tendency in Symmetrical and Asymmetrical Distributions

In normal distributions (you will study about normal distributions in a subsequent unit) that are perfectly symmetrical, (i.e., those in which the left half is a mirror image of the right half) the mean, median, and mode all have the same value. If the mean and median of a distribution have different values, the distribution cannot be symmetrical. The more skewed or lopsided a distribution is, the greater the discrepancy between these two measures. Figure 1.2 shows three pictures of (a) a negatively skewed, (b) a positively skewed, and (c) a normal distribution. The mode (Mo), Mean (x) and Median (Mdn) are shown for each distribution.

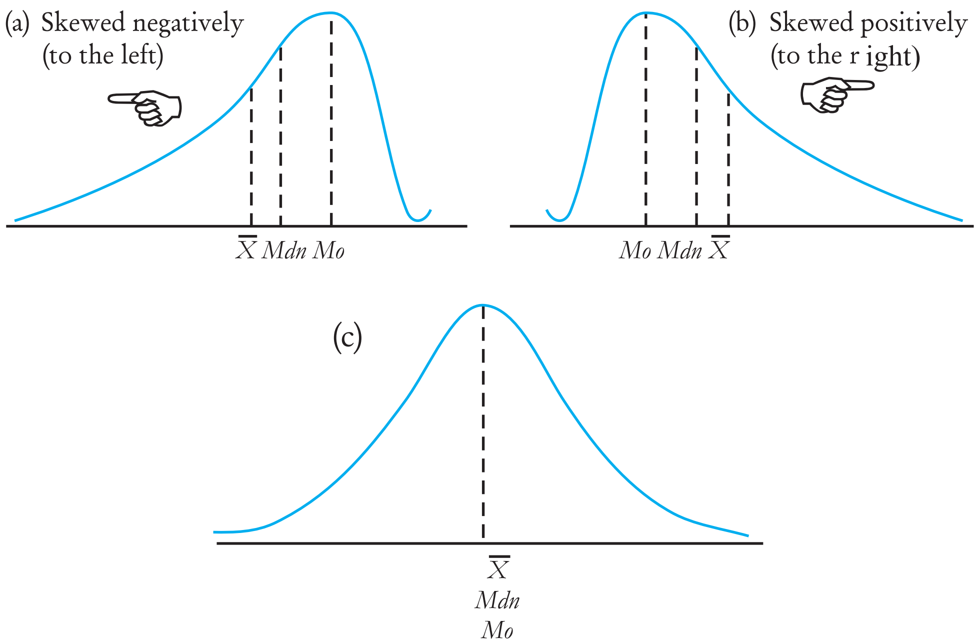


Fig. 1.2 : $\overline{X}$ *Mdn, and Mo in Skewed Distributions and in the Normal Distribution*

As can be seen in a negatively skewed distribution (a), the mode has the highest score value, and the median falls at a point about two-thirds of the distance between that and the mean. The mean has the lowest value. In a positively skewed distribution (b), just the opposite situation occurs. As a consequence, the relative position of the median and the mean may be used to determine the direction of skewness.

1.6 Summary

A measure of central tendency provides a single summary figure that describes the central location of a distribution of scores. The three measures in common use are: mode, median and mean.

The mode is the score with greatest frequency. It is the only measure that can be used for quantitative data, but it is subject to substantial sampling variation and is of little use in inferential statistics.

The median shows the point along the scale of possible scores that divides the lower half of scores from the upper half. It is of great use in open-ended distributions and is less affected by extreme scores than is the mean. The mean is the sum of all the scores divided by the total number of scores. It is the balancing point of a distribution and is responsive to the position of each score in the distribution. It is of greatest significance at the inferential level.

In a skewed distribution the mean is always pulled towards the skewed end of the distribution and the values of mean and median are different. With an increase in the difference in values of mean and median, the skewness increases.

Finally, all four processes of addition, subtraction, multiplication and division of score transformation affect the mean and other measures of central tendency.

Self-evaluation Exercises

1. How can statistics be useful in guidance and counselling? Explain.
2. What do you understand by the term "measures of central tendency"? List the most common measures of central tendency.
3. In each of the situations described below, what measures of central tendency would you most likely compute?
   a) The average achievement of a group.
   b) The most popular fashion of the day.
   c) Determining the mid-point of the scores of a group in an entrance examination.
4. Find the mean I.Q. for the eight scores given below:
   80, 100, 105, 90, 112, 115, 110, 120
5. Compute median for the following data:
   8, 3, 10, 5, 2, 11, 14, 12
6. The mean of a set of scores is 20. What will the mean become if 15 points are subtracted from each score?

Answer Key to Self-evaluation Exercises

1. Refer Section 1.2
2. Refer Section 1.3
3. (a) Mean (b) Mode (c) Median
4. Mean = 104
5. Median = 9
6. The mean will be 5.

Answer Key to Self-check Exercises

Self-check Exercise 1

The mode is 4, as it has occurred three times.

Self-check Exercise 2

The median is 7.

Self-check Exercise 3

The mean is 19.9.

Suggested Readings

Garrett, H.E. and Woodworth, R.S. 1981. *Statistics in Psychology and Education*. Vakils, Feffer and Simons Ltd., Bombay.

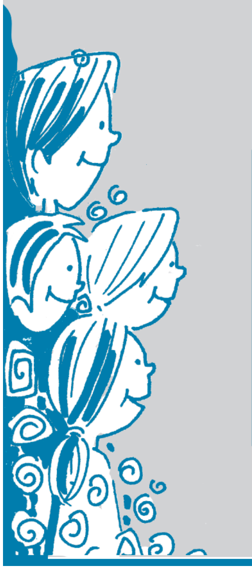
Minium, E.W. and Clarke, R.B. 1982. *Elements of Statistical Reasoning*. John Wiley and Sons, Inc., New York.

Minium, E.W., King, B.M. and Bear, G. 1993. *Statistical Reasoning in Psychology and Education*. John Wiley and Sons, Inc., New York.

Pagano, R.R. 1998. *Understanding Statistics in the Behavioural Sciences*. Brooks/Cole Publishing Company, Minnesota, USA.

2

Measures of Variability



Measures of Variability 2

2.0 Introduction

In the previous unit, you learned about three important measures of central tendency, namely mode, median and mean. However, to provide more meaningful guidance to the students, you will need to have some additional information as well. Whereas a measure of central tendency provides a statement about the central location of a distribution of observations, a measure of variability is a summary of the spread of observations around a central point. Variability information is often as important as information on central tendency.

This unit will describe several numerical indicators of measures of variability that can be used to characterise distributions.

Several examples can be taken to understand the usefulness of measures of variability. Suppose a test of achievement administered to a group of 50 boys and 50 girls shows that both the groups have a mean performance of 35. So far as the means go, there is no difference in performance of the two groups. An inspection of the data, however, shows that boys' scores range from 15 to 51 and the girls' scores range from 19 to 45. This difference in range shows that in a general sense boys' scores are more variable than girls' scores. Difference in variability is often of great importance for purposes of education and vocational guidance. If a group is homogeneous, most of its scores will fall around the same point on the scale, the range will be relatively short and the variability will be small. But, if the group contains individuals of widely differing capacities, scores will be strung out from high to low, the range will be relatively wide and the variability will be large. This, of course, has implications for providing guidance to the students. This unit will describe several numerical indicators of measures of variability that can be used to characterise distributions.

2.1 Objectives

After going through this unit, you will be able to

- ***explain*** what is meant by measures of variability.
- ***calculate*** range, semi-interquartile range, deviation scores, variance, and standard deviation.
- ***describe*** the unique properties of range, semi-interquartile range, and standard deviation.

2.2 Meaning and Computation of Measures of Variability

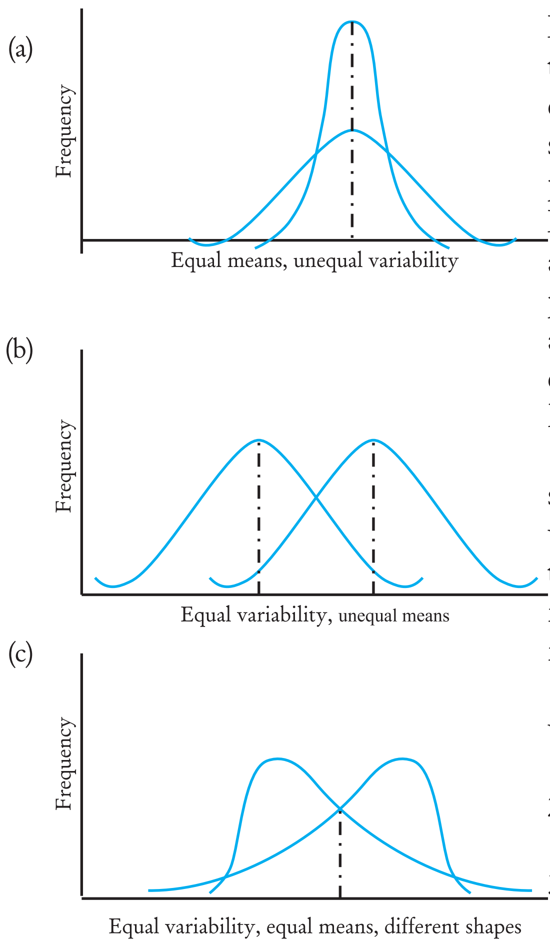


Fig. 2.1 : *Differences in Central Tendency, Variability, and Shape of Frequency Distributions*

Measures of variability express quantitatively the extent to which the scores in a distribution scatter about or cluster together. They describe the spread of an entire set of scores, however they do not specify how far a particular score diverges from the centre of a group. Measures of variability do not provide any information about the shape of the distribution or the level of performance of a group. To describe any distribution adequately, therefore, you usually need a measure of central tendency, a measure of variability, and also knowledge about the shape of the distribution.

Figure 1 shows three different situations: 2.1 (a) shows that it is possible to have two distributions with equal means but unequal variability; 2.1 (b) shows two distributions with equal variability but unequal means; and 2.1 (c) shows two distributions having equal means, equal variability but differing in shape.

There are three situations in which measures of variability are used:

1. Comparison with a known standard,
2. Establishment of a standard previously unknown, and
3. Comparison of two or more sets of scores obtained under different conditions.

In addition to above uses, a good measure of variability also serves two other valuable purposes:

(a) Variability gives an indication of how accurately the mean describes the distribution. If the variability is small, then the scores are all close together, and each individual score is closer to the mean. In this situation, the mean is a good representative of all the scores in the distribution. On the other hand, when variability is large, the scores are all spread out, and they are not necessarily close to the mean. In this case, the mean may be less representative of the whole distribution.

(b) Variability also gives an indication of how well an individual score represents the entire distribution. This is particularly important in the area of inferential statistics where relatively small samples are used to answer general questions about large populations. If the scores in a distribution are all clustered together, then any individual score will be reasonably accurate representative of the entire distribution. But if the scores are all spread out, then a single value selected from the distribution often will not be representative of the rest of the group.

Thus, from the above account you learn that the measures of central tendency and variability are closely related. Whenever one appears, the other usually is close at hand.

In the following account five measures of variability: the range, the quartile deviation, the deviation scores, the variance and the standard deviations will be considered.

2.2.1 The Range

You will note that the simplest measure of variability is range. It is the difference between the highest and lowest score in the distribution. Like other measures of variability, the range is a distance and not, like measures of central tendency, a location. Consider the following examples:

X_1 = 3, 5, 5, 8, 13, 14, 18, 23	X_2 =37, 42, 48, 53, 57
Highest Score = 23	Highest Score = 57
Lowest Score = 3	Lowest Score = 37
Range = Highest Score – Lowest Score	Range = Highest Score – Lowest Score
= 23 – 3	= 57 – 37
= 20	= 20

In both these examples, the range is 20 in spite of the difference in the number of scores and their magnitudes. The range is easy to calculate but gives us a relatively crude measure of variability, because it really measures the spread of only the extreme scores and not the spread of any of the scores within the distribution.

Self-check Exercise 1

A. Calculate the range for the following hypothetical scores of two groups of students on an objective type examination:

(i) 2, 3, 5, 8, 10

(ii) 18, 12, 28, 15, 20

2.2.2 The Semi-interquartile Range

The semi-interquartile range, symbolised by the letter Q, is a more sophisticated measure that depends only on the relatively stable central portion of a distribution — specifically, on the middle 50% of the scores. It is defined as one-half the distance between the first and third quartile points. Accordingly, its formula is as follows:

$$\text{Semi-Interquartile Range} = Q = \frac{Q_3 - Q_1}{2} = \frac{P_{75} - P_{25}}{2}$$

The **quartile points** are the three score points that divide the distribution into four parts, each containing an equal number of cases. These score points are symbolised as Q_1, Q_2 and Q_3 and are equal to P_{25}, P_{50} and P_{75} respectively, the calculations of which have already been discussed in the previous unit on percentiles.

2.2.3 The Deviation Scores

So far, we have been mainly dealing with raw scores. In order to know about variance and standard deviation, we need to know about expressing a score in another form, i.e., the deviation score form. A deviation score expresses the location of a score by indicating how many score points lies above or below the mean of the distribution. In deviation form, a deviation score is defined as:

$X - \bar{X}$: deviation score for sample data

$X - m_x$: deviation score for population data

As an illustration consider the following data:

Table 2.1: Calculation of Deviation Score

X	$X - \quad = x$	
2	$2 - 6 = -4$	
4	$4 - 6 = -2$	Calculation of $\bar{X}$
6	$6 - 6 = 0$	$\bar{X} = \frac{\Sigma x}{N} = \frac{30}{5} = 6$
8	$8 - 6 = +2$	
10 N=5	$10 - 6 = +4$	$\bar{X} = AM + \frac{\Sigma x}{N}$
$\Sigma X = 30$	$\Sigma x(X - \bar{X}) = 0$	$N = 5$

When we subtract the mean $(\bar{X} = 6)$ from each of the raw scores, the resulting deviation scores state the position of the scores relative to the mean. A plus sign (+) indicates that the original raw score is greater than the mean, where as a minus sign (–) indicates that it is less than the mean. For example, the raw score of 8 becomes $(8-6) = +2$, which says that this score is 2 points above the mean of 6. Similarly, the raw score of 4 becomes $(4 - 6) = -2$, indicating that its location is 2 points below the mean. In general, if the scores cluster tightly together, then the deviation from the mean will be small. If the scores scatter widely, then the deviations from the mean will be large.

Because deviation scores indicate the distances of the raw scores from the mean, it appears to be an attractive measure of variability. However, it has proved to have a mathematical inattractability, which severely limits its use. Being a measure of distance between each score and the mean, the resulting deviations are either positive or negative. The algebraic sum of all the deviations taken from means is always zero, i.e., $\sum(X-\overline{X})=0$ and thus, no measure of variability results. This necessitates disregarding the positive and negative signs attached to deviation scores and yields a measure with such awkward mathematical properties that it is useless in inferential statistics. Another way to get rid of the signs is to square the deviation scores and this leads to a very useful measure of deviation called the variance.

2.2.4 The Variance

You read about the problem in the deviation scores arising out of ignoring of the algebraic signs of the deviations taken from the mean. This can be resolved by squaring the deviations. The variance is defined as the mean of the squares of the deviation scores. The symbol for the variance of a population is σ^2 (the Greek letter sigma) and that for a sample is S^2. The defining formula for the variance is:

Variance of a population : $\sigma_x^2 = \dfrac{\Sigma(X - \mu_x)^2}{N}$

Variance of a sample : $S_x^2 = \dfrac{\Sigma(X - \overline{X})^2}{n}$

The numerator of the variance, $\sum(X-\overline{X})^2$ is used very frequently in other statistical formulas and its abbreviated name is the **sum of squares**, which is defined as the sum of the squared deviations from the mean. It is often symbolised by **SS**. The steps for calculating the variance are as follows:

Step 1: Record each score

Step 2: Calculate $\overline{X}$

Step 3: Obtain deviation scores by subtracting $\overline{X}$ from each value of X. Check that $\sum(X-\overline{X})$ is equal to zero

Step 4: Square each deviation score

Step 5: Sum the values of the squared deviation scores to get the sum of squares (SS)

Step 6: Divide the sum of squares by n using the formula $S_x^2 = \dfrac{\Sigma(X-\overline{X})^2}{n}$

Consider the data in Table 2 for the calculation of the variance:

Table 2.2 : Calculation of Variance by Deviation Score Method

X	$X - \overline{X}$		$(X - \overline{X})^2$	
1	1 – 6 =	– 5	25	SX = 30
9	9 – 6 =	+ 3	9	n = 5
5	5 – 6 =	– 1	1	$\overline{X}$ = SX/n
8	8 – 6 =	+ 2	4	= 6
7	7 – 6 =	+ 1	1	
SX = 30	$S(X - \overline{X})$ =	0	$S(X - \overline{X})^2$ = 40	
			= 8	

The variance is the most important measure that finds its greatest use in advanced statistical procedures, especially in inferential statistics. For use as a basic description of results, however, it has a fatal flaw as its calculated value is expressed in squared units of measurement. Consequently, it is of little use in descriptive statistics.

2.2.5 The Standard Deviation

In a variance, the values in the numerator are expressed in squared units of measurement. This defect, however, can easily be remedied by taking the square root of the variance. By doing this we return to the original units of measurement and obtain a widely used index of variability called the **standard deviation**. In computing standard deviation the squared units of numerator values are always taken from mean, never from the median or mode. The standard deviation is the most stable index of variability and is customarily employed in experimental work and in research studies.

Like variance, standard deviation can also be calculated by deviation-score method. It can also be computed by another method called the "raw-score method," which is relatively simpler and avoids problems pertaining to decimals. You will look at both methods of calculating the standard deviation.

2.2.5.1 Deviation-score Method

The steps of calculating standard deviation by deviation-score method are basically same as the ones summarised above for variance except an additional last step of taking the square root of the variance value. The final defining formula become as follows:

Standard Deviation of a population : $\sigma_x = \sqrt{\dfrac{\Sigma(X - \mu_x)^2}{N}}$ or $\sqrt{\dfrac{\sigma_x}{N}}$

Standard Deviation of a sample : $S_x = \sqrt{\dfrac{\Sigma(X - \bar{X})^2}{n}}$ or $\sqrt{\dfrac{SS_x}{n}}$

If we obtain standard deviation of the sample of scores given in the section on variance in Table 2, we simply take the square root of the variance:

$$S_x = \sqrt{\frac{\Sigma(X - \bar{X})^2}{n}} = \sqrt{\frac{40}{5}} = \sqrt{8} = 2.83$$

2.2.5.2 Raw-score Method

Standard deviation can also be calculated by using the raw-score method, which is easier to use and avoids potential difficulties concerned with a decimal remainder. For most instances, particularly when there are decimals or a large number of scores, the raw-score method is a better practical choice for computation. By this method SS can be calculated by raw scores without the necessity of calculating deviation scores. Algebraically it can be shown that:

$$SSx = \Sigma(X - \bar{X})^2 = \Sigma X^2 - \frac{(\Sigma X)^2}{n}$$ (Raw-score Equivalent of SS)

The steps in calculating the standard deviation by the raw-score method are as follows:

Step 1: Record each score

Step 2: Record the square of each score

Step 3: Calculate ΣX and ΣX^2

Step 4: Calculate SS_x in accordance with the following

formula: $SS_x = \Sigma X^2 - \frac{(\Sigma X)^2}{n}$

Step 5: Substitute the numerical values for SS and n in the formula for S_x :

$$S_x = \sqrt{\frac{SS_x}{n}}$$

Step 6: Complete the calculate of S_x

Let us calculate standard deviation for the following data using raw-score method.

Table 2.3 : Calculation of Standard Deviation by Raw-score Method

X	X^2	
		$SS_x = \Sigma X^2 - \frac{(\Sigma X)^2}{n}$
10	100	
12	144	$= 2471 - \frac{(135)^2}{8}$
13	169	
15	225	$= 2471 - 2278.13$
18	324	$= 192.88$
20	400	$S_x = \sqrt{\frac{SS_x}{n}}$
22	484	
25	625	$= \sqrt{\frac{192.88}{8}} = \sqrt{24.11}$
SX = 135	SX² = 2471	
n = 8		$= 4.91$

Standard deviation is a very important measure in a variety of contexts, especially the normal curve. Though details about these relationships will be discussed further in the unit on normal curves, it is necessary to notice here that in an ideal normal distribution, if you start at the mean which normally lies at the middle and move towards both end points you can then express the standard deviations as:

$\mu \pm 1\sigma$ contains about 68% of the scores

$\mu \pm 2\sigma$ contains about 95% of the scores

$\mu \pm 3\sigma$ contains about 99.7% of the scores

Self-check Exercise 2

A. Calculate variance from the following data:
X : 6, 2, 8, 5, 4, 4, 7

B. Calculate standard deviation for the following data using the raw-score method:
X : 52, 50, 56, 68, 65, 62, 57, 70

2.3 Properties of Measures of Variability

Let us recapitulate the properties of various measures of variability based on our earlier discussion.

2.3.1 Properties of Range

- You have seen that the range is easier to compute than the other measures of variability and its meaning is direct. Therefore, the range is ideal for preliminary work or in other circumstances where precision is not an important requirement.
- The range has some major shortcomings as a measure of variability. Only the two outermost scores of a distribution affect its value, the remainder could lie anywhere between them. The range, thus, is not sensitive to the total condition of the distribution.
- The range is of little use beyond the descriptive level. In most situations, it varies more with sampling fluctuations than other measures do.
- In many types of distributions, including the normal distribution, the range is dependent on sample size, being greater when sample size is greater.

2.3.2 Properties of Semi-interquartile Range

- The semi-interquartile range is closely related to median because both are defined in terms of percentile points of the distribution.
- Semi-interquartile range is less sensitive to the presence of a few very extreme scores than is the standard deviation.
- If the distribution is open-ended, it is not possible to calculate the standard deviation or range. With open-ended distributions, the semi-interquartile range may be the only measures of variability that is reasonable to compute.
- Unlike mean, if we add a constant to each score in the distribution or subtract a constant from each score, it does not affect standard deviation or any other measure of variability. However, when scores are multiplied or divided by a constant; the resultant measure of variability is also multiplied or divided by that same constant.

2.3.3 Properties of Standard Deviation

- The standard deviation, like the mean, is responsive to the exact position of every score in the distribution. If a score is shifted to a position more deviant from the mean, the standard deviation will increase. If the shift is to a position closer to the mean, the standard deviation decreases.

- The standard deviation is more sensitive than the semi-interquartile range to the presence or absence of scores that lie at the extremes of the distribution. Therefore, standard deviation may not be the best choice when the distribution contains a few very extreme scores or when the distribution is badly skewed.
- When we calculate deviations from the means, the sum of squares of these values is smaller than if they had been taken from median or mode, i.e., $\sum(X - \overline{X})^2$ is minimum.
- One of the most important points favouring use of standard deviation is that it shows very good resistance to sampling fluctuation.
- Standard deviation is an indispensable measure of variability in many procedures of both descriptive statistics and inferential statistics.

Thus, in many ways, the properties of the standard deviation are related to those of the mean.

2.4 Summary

To describe a distribution adequately, we require information about a measures of variability, central tendency and shape of the distribution. Measures of variability are summary figures that express quantitatively the extent to which the scores in a distribution scatter about or cluster together. The four important measures of variability are: range, semi-interquartile range, variance and standard deviations.

The range is the distance between the highest and lowest score in the distribution. Though it is easy to obtain, it has the shortcoming of being unresponsive to the location of intermediate scores.

The semi-interquartile range is one-half the distance between the first (Q_1) and the third (Q_3) quartile points. It is responsive to the number of scores lying above or below the outer quartile points, but not to their exact location. For this reason it is particularly useful with open-ended distributions. However, it shows poor resistance to sampling fluctuations and is of little use in inferential statistics.

The variance is the mean of the squared deviations taken from the mean. Since it is a quantity expressed in squared units, it is of little use at descriptive level. However, it has great importance in inferential statistics because of its resistance to sampling variation.

The standard deviation is the square root of the variance and since it is expressed in original score units, it is the widely used measure both at descriptive and inferential level.

The measures of variability are not affected by addition or subtraction processes of score transformation, however, they are affected by multiplication and division.

Self-evaluation Exercises

1. What are the different measures of variability? Discuss them in brief?
2. A distribution consists of five scores. For four of the scores the deviations from the mean are +5, +2, +1, and –8. What is the deviation of the fifth score?
3. Which measure of variability is best for open-ended distributions? Why?
4. Why is standard deviation considered to be the best measure of variability? Outline important characteristics of standard deviation.
5. Compute standard deviation for the following data:
   X: 30, 35, 36, 39, 42, 44, 46, 38, 34, 35
6. Compute SS for the following five scores:
   X: 1, 6, 4, 3, 8, 7, 6
7. Give two important properties of semi-interquartile range.
8. Give the symbol/formula for each of the following:
   (a) A raw score
   (b) Variance of a sample
   (c) Variance of a population
   (d) Standard deviation of a sample
   (e) Standard deviation of a population.

Answer Key to Self-evaluation Exercises

1. See the section titled “Meaning and Computation of Measures of Variability ”
2. Zero
3. Semi-interquartile range
4. See the section titled “Properties of Measures of Variability”
5. 4.66
6. 36
7. See the section title “Properties of Measures of Variability”
8. (a) X

   (b) $S_x^2 = \frac{\Sigma(X-\bar{X})^2}{n}$

   (c) $\sigma_x^2 = \frac{\Sigma(X-\mu_x)^2}{N}$

   (d) $S_x = \sqrt{\frac{\Sigma(X-\bar{X})^2}{n}}$

   (e) $\sigma_x = \sqrt{\frac{\Sigma(X-\mu_x)^2}{N}}$

Answer Key to Self-check Exercises

Self-check Exercise 1

A. (i) The range is 8; (ii) The range is 16.

Self-check Exercise 2

A. The variance is 3.55.

B. The standard deviation is 6.91.

Suggested Readings

Garrett, H.E. and Woodworth, R.S. 1969. *Statistics in Psychology and Education*. Vakils, Feffer and Simons Ltd., Bombay.

Minium, E.W. and Clarke, R.B. 1982. *Elements of Statistical Reasoning*. John Wiley and Sons, Inc., New York.

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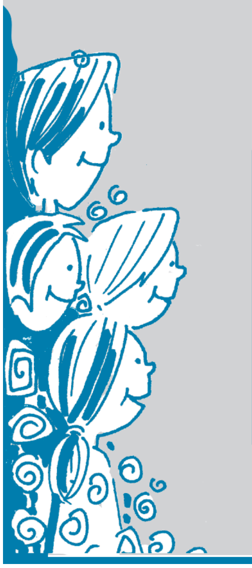
Pagano, R.R. 1998. *Understanding Statistics in the Behavioural Sciences*. Brooks/ Cole Publishing Company, Minnesota, USA.

3

Normal Probability Curve



Normal Probability Curve 3

3.0 Introduction

In eighteenth century, Abraham De Moivre, a French Mathematician discovered that mathematical relationships explained the probabilities associated with various games of chance. He developed the equation and the graphic pattern that describes it. During the 19th Century French Astronomer, La Place and a German Mathematician, Gauss, independently arrived that the same principle and applied it to areas of measurement in biology, psychology, sociology and other sciences. The theory describes the fluctuation of chance, errors of observation and measurement. It is necessary to understand the theory of probability and the nature of curve of normal distribution in order to comprehensive many important statistical concepts, particularly in the area of standard scores, the theory of sampling and inferential statistics.

3.1 Objectives

After going through this unit, you will be able to

- ***distinguish*** between a normal distribution and a non-normal distribution.
- ***understand*** the properties and significance of the Normal Probability Curve (NPC).
- ***verify*** the characteristics the NPC.
- ***find*** the norms for a particular group.

3.2 Normal Distribution

In order to make appropriate decisions, the variability or fluctuation of the all relevant data is require to be carefully and thoroughly analysed. The examination of variability assists in determining how different a student's test score from his or her peers.

One of the first indicators of educational problems is the teacher's observation that the student is lagging behind his or her classmates in academic performance. Generally one or two days of poor performance is not regarded as indicative of serious instructional problems. However, if the problem persists for sometime, serious attention may be warranted. The data obtained from many formal tests allowed the test scores of an individual student to be compared to those of students of a similar age or grade. The observations of differences may be made informally over time. These observations lead to the question of exactly how different is the student's performance from his/her peers.

Normal probability curve provides the answer to the question 'How much do the data fluctuate and deviate?'

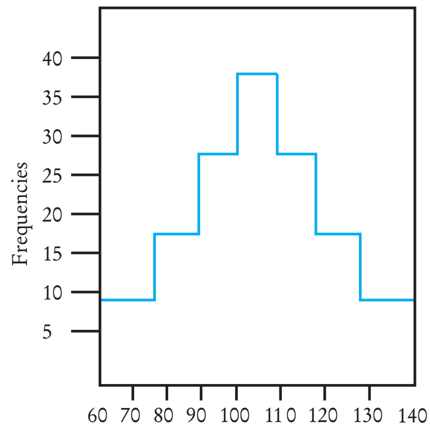


Fig. 3.1 : *Distribution of I.Q.s*

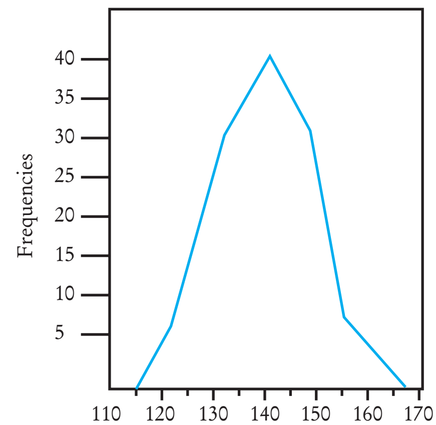


Fig. 3.2 : *Heights in cms*

The term normal distribution means any frequency distribution whose form corresponds to that of the normal curve.

Figure 3.1 is a Histogram representing the frequency distribution of data from psychology. Figure 3.2 represents data from physiology.

It is apparent, even upon superficial examination that both the groups have the same general form. The measures are concentrated closely around the centre and taper off from this central high point or crest to the left and right. There are relatively few measures at the 'low-score' and 'high-score' and a maximum number of scores are at the middle position. All the measures moreover, are symmetrically distributed that is to say there are as many measures on one side of the distribution as on the other side.

One idealised form of this kind of distribution is called the *normal probability curve.* It is typical of many biological, psychological and physical measurements. This smoothened curve, familiar to mathematicians and biologists is also called the Gaussian Curve after the great German Mathematician Gauss, who investigated its properties and wrote the equation for it.

As an example : put 10 coins in a jar, shake them up, toss them on the table and count the number of heads from 0 to 10 that appear on each toss. The measurement in this case is the number of heads that turn up. If you repeat this time after time, for say, 100 tosses, and each time tabulate the number of heads, the distribution you get approaches the normal probability curve as shown in Figure 3.3.

Many human characteristics such as height, weight, strength of grip, learning ability, mechanical ability, cooperativeness, social dominance etc. yield normally distributed

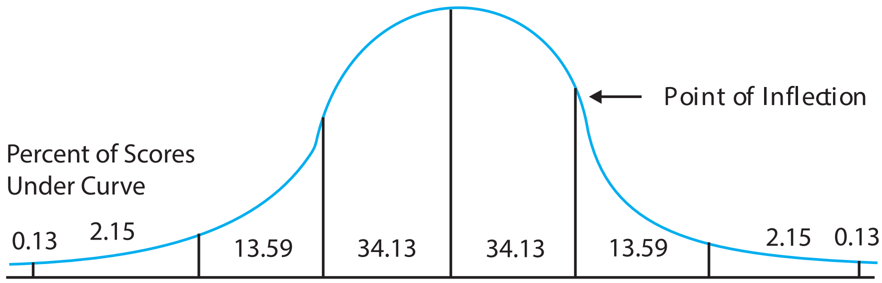


Fig. 3.3 : *Normal Probability Curve*

measurements like these. These distributions form a bell-shaped curve which is known as the normal curve. However, some traits like colour of human skin or colour of eye is not distributed normally because people fall into fairly distinct groups on the basis of these traits.

The normality of data is a concept of great usefulness in statistical theory and practice and a counsellor should have some understanding of the normal curve. It is the foundation of statistical theory that the greater a deviation from the mean the less frequently it occurs.

3.3 Characteristics of Normal Probability Curve

A normal probability curve has all of the following characteristics:

- The curve is symmetrical around its vertical axis.
- The terms cluster around the centre (the median).
- The mean, median and the mode of the distribution have the same values.
- The curve has no boundaries in either direction for the curve never touches the base line, no matter how far it is extended. The curve is a curve of probability, not of certainty.

Each of these characteristics has implications for interpreting the data from a normal probability curve. The significance of normal probability curve is discussed below.

3.4 Significance of Normal Probability Curve

In a normal distribution, the following characteristics hold true:

- The normal curve is symmetrical about the mean.

  The number of cases below the mean in normal distribution is equal to the number of cases above the mean. Hence, the mean and the median coincide in a normal curve.
- The height of the curve is at its maximum at the mean.

  Hence, the mean and the mode of the normal distribution coincide.
- There is one maximum point of the normal curve, which occurs at the mean. The height of the curve declines as we go in either direction from the mean. This dropping off is slow at first, then rapid and then slow again.

  Theoretically, the curve never touches the base line. Its tail approaches but never reach the base line.

Hence, the range is unlimited.

- Distributions of many physical traits exhibit the bell-shaped form; hence, it seems probable that the same would also be true for many mental traits.

The normal curve provides the relationship of the amount of area under the curve lying between certain limits on the base line.

The space included between the mean and $+1.00\ \sigma$ is .3413 of the total area under the curve.

The percentage of cases that fall between the mean and $+1.00\sigma$ is .3413.

The probability of events occurring (observation) between the mean and $+1.00\sigma$ is .3413.

Because one half of the curve is above the mean and the .3413 of the area is between mean and $+1.00\sigma$, the area of the curve between $\pm 2\sigma$ to $\pm 3\sigma$ is .215.

For example:

If there were a normal distributions of 10,000 cases, 3413 of them would be expected between the mean and $\pm 1\sigma$, 1359 cases lie between $\pm 1\sigma$ to $\pm 2\sigma$ and 215 cases lie between $\pm 2\ \sigma$ to $\pm 3\sigma$.

The normal curve is important in education and psychology because the scores obtained on educational and psychological tests for almost any group of pupils, frequently present a bell-shaped form of distribution. Most of the tests are deliberately constructed to yield an approximately symmetrical distribution of scores. Nearly in all test construction the author tries to adjust the difficulty of items in such a way so as to get a normal distribution of scores.

Normality and non-normality must be inferred from the distribution of test scores themselves. The resulting symmetrical distribution of test scores should not be taken as a conclusive proof of normality of the trait being measured. We prefer the normal curve in test construction because this type of distribution generally fits the data better and such a curve, because of its properties, is more useful is interpreting the test results. However, it should be remembered that the theoretical justification and empirical use of the normal curve are two quite different matters.

In dealing with the complex variables of the social sciences traits like musical ability or mechanical ability, normal distribution is rarely possible. These traits may or may not exhibit a normal distribution because these abilities are too little known to justify.

Self-check Exercise 1

Example : Determine the percentage of cases in normal distribution, within given limits.

Given : Normal distribution of intelligence test scores with a mean = 100, and S. D. = 16

(a) What percentage of the cases fall between an I.Q. of 84 and 116?

(b) What percentage of the cases lie above an I.Q. of 132?

What percentage of the case lie below an I.Q. of 92?

3.5 Practical Applications of Normal Probability Curve

In the field of guidance and counselling, the normal curve could be applied–

1) To calculate the percentile rank of scores in a normal distribution.
2) To normalise the frequency distribution, an important process in standardising a psychological test or inventory.
3) To test the significance of observed measures in experiments, relating them to the chance of fluctuation of errors that are inherent in the process of sampling and generalising about populations from which the samples are drawn.

3.6 Skewness

All distributions, particularly of sample data, are not identical to or even close to a normal curve. There are two types of distributions that can occur : skewed and bimodal.

A distribution is said to be skewed when the mean and the median fall at different points in the distribution and the balance or centre of gravity is shifted to one side or the other. The word skewed means 'lacking symmetry' or 'distorted'.

Skewness depends upon the manner in which the scores in a series scatter about the average value. When the scatter is greater on one side of the point of central tendency than on the other, the distribution is skewed.

In a normal distribution, the mean equals the median exactly and the skewness is, of course, zero (0).

As the distribution approaches the normal form, the closer are the mean and median and skewness is less. Distribution is said to be negatively skewed or to the left when scores are massed at the high end of the scale (the right-end) and are spreadout more gradually towards the low end (or left).

Distributions are skewed positively when there is a piling up of scores at the low end and a long tail running up into high scores. The median lies in the middle while the mean and the mode remain on opposite sides of the median.

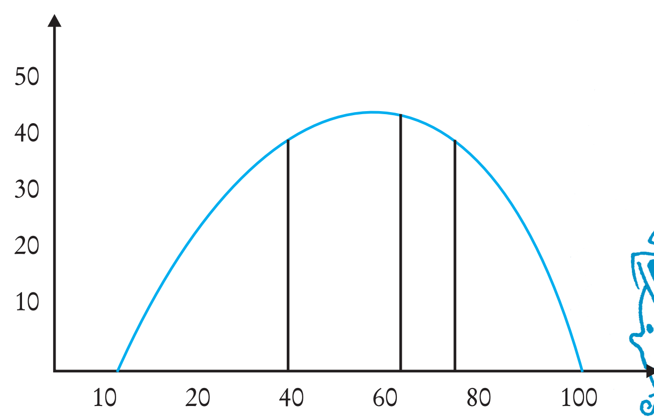


Fig. 3.4 : *Negative Skewness*

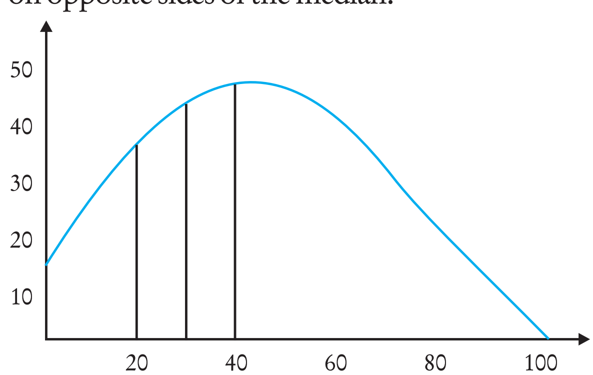


Fig. 3.5 : *Positive Skewness*

Note that the mean is pulled more toward the skewed end of the distribution than is the median. In fact, the greater the gap between the mean and the median, the greater the skewness will be.

When skewness is negative, the mean lies to the left of the median and when skewness is positive the mean lies to the right of median.

Measures of Skewness

Skewness can be measured by the formula

$$SK = \frac{3\,(Mean - Median)}{\sigma}$$

Skewness will be zero when the mean and median coincide.

Kurtosis

Kurtosis is a 'peakedness' or 'flatness' of a frequency distribution as compared with the normal.

When there is a high concentration of scores in the neighbourhood of the point of central tendency, the distribution is relatively narrow across the shoulders. Relatively high and narrow distributions are described as leptokurtic.

When there is low concentration of scores in the neighbourhood of the central tendency the distribution is relatively broad across the shoulders. Such relatively flat-topped distributions are described as platykurtic.

A normal distribution is called mesokurtic (mesos means middle or medium).

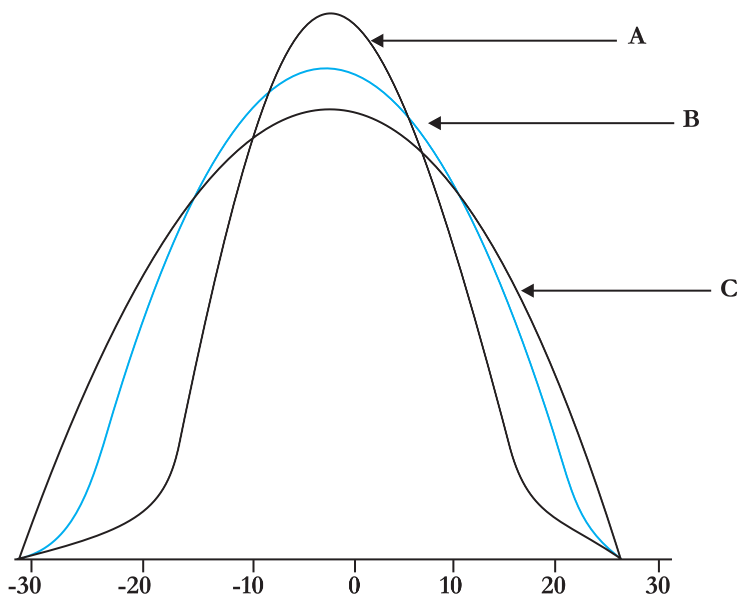


Fig. 3.6 : *Leptokurtic (A) Normal or Mesokurtic (B) Platykurtic (C) Curves*

3.6.1 Causes of Skewness and Kurtosis

There are reasons for each of the above distributions to occur. It is beneficial for a counsellor to know why distributions diverge from the normal form.

Following are few common causes of asymmetry :

(i) Selection

If the sample chosen is biased one, the distribution of the scores will not exhibit the bell-shaped form.

(ii) Unsuitable or Poorly Made Tests

If a test is too easy, scores will pile up at the high score end of the scale and will give negative skewness whereas if the test is too hard, scores will pile up at the low score end of the scale giving a positively skewed curve.

(iii) Non-Normal Distributions

Skewness or kurtosis or both will appear when there is a real lack of normality in the trait being measured. Non-normal curves often occur in medical statistics.

(iv) Errors in the Use of Tests

Errors in timing or in giving instructions, errors in scoring, differences in motivations, all of these factors cause some students to score higher and others to score lower than they normally would and distribution give skewness.

Activity 1

Take the scores of any test conducted by you, plot a graph, identify type of skewness, and find out the reasons for skewness.

3.6.2 Other Forms of Distribution

Bimodal distributions have two modes (see Fig.3.7) rather than the single mode of normal or skewed distribution.

Bimodal curve is obtained when there are two distinct peaks in a distribution e.g.,: (1) Score of students of a class in which two distinct groups: such as bright students and not so bright students, or (2) a distribution of heights of a group of college students which includes half Japanese and half Americans.

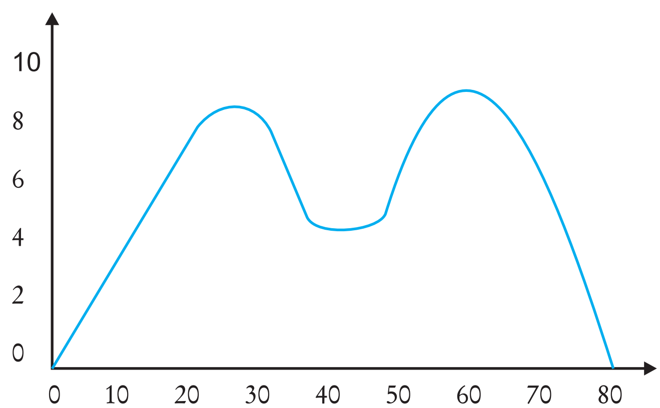


Fig. 3.7 : *Bimodal Curve*

If a distribution has more than two peaks, you get a multimodal curve.

J-Type Curve

If a curve is drawn showing the income of Indian people, it will exhibit J-type curve. There are many people with small and moderate income while only a few with very large income. Such a distribution of income will be positively skewed.

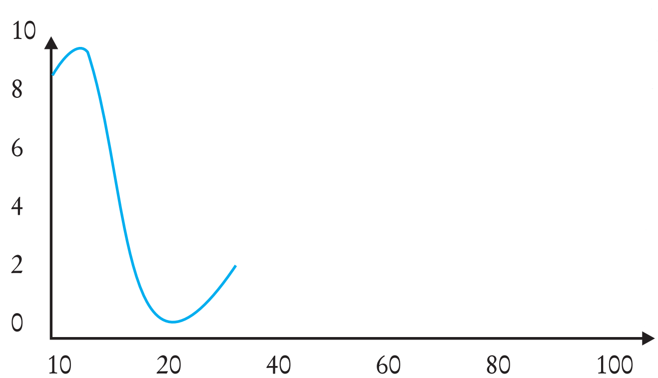


Fig. 3.8 : *J-Type Curve*

U-Type Curve

U-shaped curves are rarely encountered in mental and physical measurements. Such a distribution is possible when there are two distinct group in a class e.g., if a neurotic inventory is given to a group composed about equally of normal and of mentally-ill people, the normal people will tend to have low scores while the abnormal people will tend to have high scores. This type of curve makes for a dip in the centre of the distribution.

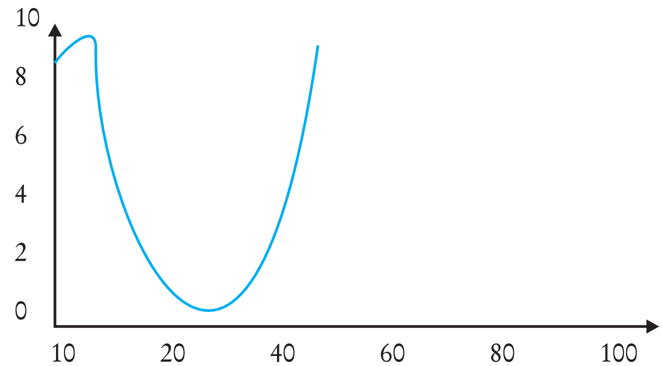


Fig. 3.9 : *U-Type Curve*

3.7 Summary

The term normal distribution means any frequency distribution whose form corresponds to that of the normal curve.

When all the measures moreover are symmetrically distributed the mean, the median and the mode are all the same. This kind of distribution is called the normal probability curve.

A distribution is said to be skewed when the mean and the median fall at different points in the distribution and the balance or centre of gravity is shifted to one side or the other. In a normal distribution, the skewness is zero. Distributions are skewed positively when there is a piling up of scores at the low end of the scale. Distributions are skewed negatively when scores are massed at the high end of the scale.

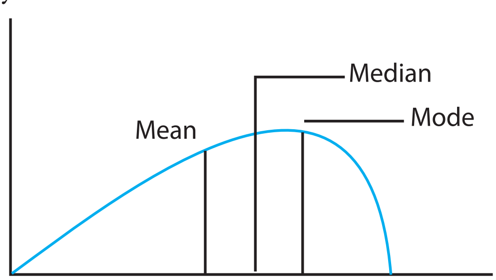
In the field of guidance and counselling, the normal curve could be applied (i) to calculate the percentile rank of scores in a normal distribution, (ii) to normalise the frequency distribution, an important process in standardising a psychological test or inventory, and (iii) to test the significance of observed measures in experiments, relating them to the chance of fluctuations of errors that are inherent in the process of sampling and generalising about populations from which the samples are drawn.

Self-evaluation Exercises

1. The mean and standard deviation of normal distribution of 500 scores are 75 and 15 respectively :
   a. How many per cent of scores lie between 45 and 90?
   b. How many per cent of scores lie below 60?
2. List the characteristics of normal probability curve.
3. Mention five traits that give a normal distribution and two traits that do not give a normal distribution.
4. A given test contains items of low difficulty. What type of distribution of scores is likely to occur, draw a curve, and show position of mean, median and mode.
5. Can you verify properties of normal probability curve in an achievement test? Illustrate with the help of an example.

Answer Key to Self-evaluation Exercises

1. a. 81.85 b. 15.87
2. Characteristics of normal probability curve –
   a) The curve is symmetrical around its vertical axis.
   b) The terms cluster around the centre (the median)
   c) The mean, median and the mode of the distribution have the same values.
   d) The curve has no boundaries in either direction for the curve never touches the base line, no matter how far it is extended. The curve is a curve of probability, not of certainty.
3. The traits that give a normal distribution are –
   e) Height f) Weight g) Strength of grip
   h) Learning ability i) Cooperativeness
   The traits that do not give a normal distribution are –
   j) Colour of human skin k) Colour of eyes
4. Negatively skewed curve

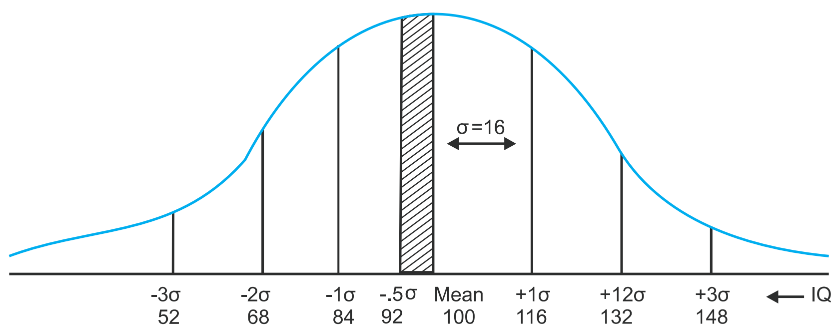


5. Collect the scores of any achievement test conducted by you and verify.

Answer Key to Self-check Exercise

Self-check Exercise 1

a. An I.Q. of 116 is 16 points above the mean and an I.Q. of 84 is 16 points below the mean.
There are 68.26% cases between $+1\sigma$ to -1σ
Hence 68.26% of the scores in distribution fall between IQ of 84 and 116.



b. An I.Q. of 132 is 32 score units above the mean i.e., $\frac{32}{16} = 2\alpha$ units above the mean.
We know that 47.72% of the cases lie between the mean and this point, that is between 100 and 132.
Since distribution is symmetrical, 50% of the cases will lie above the mean.
Hence 50-47.72 = 2.28% of the groups will have their I.Q. above 132.
If a student A has an I.Q. of 132, only 2.28% of the students will have I.Q. greater than that of student A.

c. An I.Q. of 92 is below the mean
The score of 92 is -0.5 α from the mean
Between mean and -0.5 α are 17.072% of cases. Since distribution is symmetrical 50% of the cases will lie below the mean.
Hence 50 – 17.07 = 32.93% of the cases lie below an I.Q. of 92.
If a student B has an I.Q. of 92, there are 32.93% students who have I.Q. below 92.

Suggested Readings

Anastasi, A. and Susana, U. 1997. *Psychological Testing.* Prentice-Hall International Inc., New Jersey.

Dandekar, W. N. 1996. *Evaluation in Schools.* Shrividya Prakashan, Pune.

Best, J. W. and Kahn, J. V. 1995. *Research in Education* (7th ed.) Prentice-Hall of India Private Limited, New Delhi.

List of Course Material

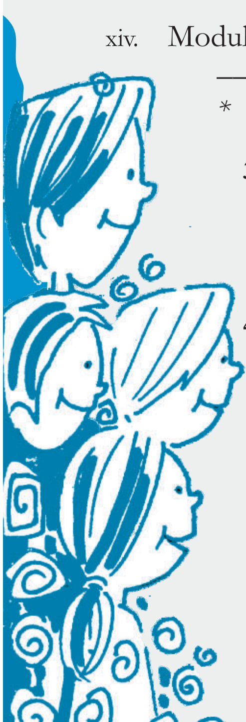
1. **Course Guide**

   Major inputs include objectives, scope, rules, syllabi as well as procedures for admission, transaction and evaluation for all the three phases of the course.

2. **Course Modules***

   i. Module- I : Introduction to Guidance
   ii. Module-II : Counselling Process and Strategies
   iii. Module-III : Guidance for Human Development and Adjustment
   iv. Module-IV : Career Development-I
   v. Module V : Career Information in Guidance and Counselling-I
   vi. Module VI : Assessment and Appraisal in Guidance and Counselling-I
   vii. Module VII : Basic Statistics in Guidance and Counselling-I
   viii. Module VIII : Guidance in Action
   ix. Module IX : Special Concern in Counselling
   x. Module X : Developing Mental Health and Coping Skills
   xi. Module-XI : Career Development-II
   xii. Module XII : Career Information in Guidance and Counselling-II
   xiii. Module XIII : Assessment and Appraisal in Guidance and Counselling-II
   xiv. Module XIV : Basic Statistics in Guidance and Counselling-II

* *Each module consists of number of self-learning units.*



3. **Practical Handbook**

   Provides areas and strategies for conducting and undergoing practicum, field experience and internship.

4. **Tutor Guide**

   Lists guidelines for tutors, supervisors for course transaction and evaluation during all the three phases of the course.

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