The mighty zero

Rintu Nath
Scientist – E
Vigyan Prasar, Department of Science and Technology, Govt. of India
A – 50, Sector – 62, NOIDA – 201 309
rnath@vigyanprasar.gov.in
rnath07@gmail.com

Abstract

Zero is a strange number and one of the great paradoxes of human thought. It means both nothing and everything. The concept of zero was first formulated by Indian Mathematician Brahmagupta during 628 AD. Developing the concept of zero is the most important discovery in the history of mathematics. Brahmagupta wrote Brahmasphutasiddhanta (The Opening of the Universe), and attempted to give the rules for arithmetic involving zero and negative numbers. Other Indian mathematician Mahavira and Bhaskara also developed the concept of zero and stated different mathematical operations involving zero. During 12th century, the Islamic and Arabic mathematicians took the ideas of the Indian mathematicians to further west. During the same time Chinese mathematicians started using the concept of zero and used symbol 'O' to represent zero. In the year 1202, Italian mathematician Leonardo Fibonacci wrote a book titled "Liber Abaci" and introduced 'modus Indorum' (method of the Indians). In this book Fibonacci advocated numeration with the digits 0–9 and place value, today known as Indo- Arabic numerals. However, the concept of zero took some time for acceptance. It is only around 1600 that zero began to come into widespread use. That is how shunyam given by our forefathers was recognised in the world and made its place permanently as zero.

Zero is neither positive nor negative and appears in the middle of a number line. It is neither a prime number nor a composite number. It cannot be prime because it has an infinite number of factors and cannot be composite because it cannot be expressed by multiplying prime numbers (0 must always be one of the factors).

Division by zero is a tricky one. The uniqueness of division breaks down when we attempt to divide any number by zero since we cannot recover the original number by the inverting the process of multiplication. Division by zero is undefined. This is the reason that in all computer programs or mathematical calculations, one should take care of this vital operation and there should have appropriate strategy to deal with this situation.

Zero is tiny number, but we should never ignore its might. Imagine the world without zero. Not only mathematics, but all branches of sciences would have struggled for more clear definitions in their individual contexts, had zero not exist in our number system. Thanks to the ingenuity of our forefathers.

Keywords: *Zero, Brahmagupta, Indo-Arabic numerals, number line, indeterminate, undefined*

Discovery of zero

Initially, the zero as a number was not available. There was the idea of empty space, which may be thought conceptually similar to zero. Babylonians around 700 BC uses three hooks to denote an empty place in the positional notation.

Almost during the same time, Greek mathematicians made some unique contributions to mathematics. Euclid wrote a book on number theory named *Elements*, but that was completely based on geometry and no concept of zero was mentioned.

Around 650 AD, the use of zero as a number came into Indian mathematics. The Indian used a place-value system and zero was used to denote an empty place. In fact there is evidence of an empty placeholder in positional numbers from as early as 200AD in India. Around 500AD Aryabhata devised a number system, which had no zero, as a positional system, but used to denote empty space. There is evidence that a dot had been used in earlier Indian manuscripts to denote an empty place in positional notation. For example, to represent '100' it would be two dots after 1.

In 628 AD, Brahmagupta wrote *Brahmasphutasiddhanta* (The Opening of the Universe), and attempted to give the rules for arithmetic involving zero and negative numbers. He explained that given a number then if you subtract it from itself you obtain zero. He gave the following rules for addition, which involve zero: *The sum of zero and a negative number is negative, the sum of a positive number and zero is positive; the sum of zero and zero is zero.* Similarly, he gave the correct rules for subtraction also.

Brahmagupta then said that any number when multiplied by zero is zero but when it comes to division by zero, he gave some rules that were not correct. However, it was an excellent attempt to visualise number system in the light of negative numbers, zero and positive numbers.

In 830, another Indian mathematician Mahavira wrote *Ganita Sara Samgraha* (Collections of Mathematics Briefings), which was designed as an update of Brahmagupta's book. He correctly stated the multiplication rules for zero but again gave incorrect rule for division by zero.

After 500 years of Brahmagupta, mathematician Bhaskara tried to solve the problem of division by stating that any number divided by zero as infinity. Well, conceptually though it is still incorrect, however, Bhaskara did correctly state other properties of zero, such as square of zero is zero and square root of zero is also zero.

It is therefore clear that Indian mathematicians developed the concept of zero and stated different mathematical operations involved with zero. But how did the concept spread to all over the world?

From India to the world

The Islamic and Arabic mathematicians took the ideas of the Indian mathematicians to further west. Al-Khwarizmi described the Indian place-value system of numerals based on zero and other numerals. Ibn Ezra, in the 12^{th} century, wrote *The Book of the Number*, which spread the concepts of the Indian numeral symbols and decimal fractions to Europe.

In 1247 the Chinese mathematician Ch'in Chiu-Shao wrote *Mathematical treatise in nine sections* which uses the symbol 'O' for zero. In 1303, Chu Shih-Chieh wrote *Jade Mirror of the Four Elements*, which again used the symbol 'O' for zero.

In around 1200, Leonardo Fibonacci wrote *Liber Abaci* where he described the nine Indian symbols together with the sign '0'. However, the concept of *zero* took some time for acceptance. It is only around 1600 that *zero* began to come into widespread use after encountering a lot of supports and criticisms from mathematicians of the world.

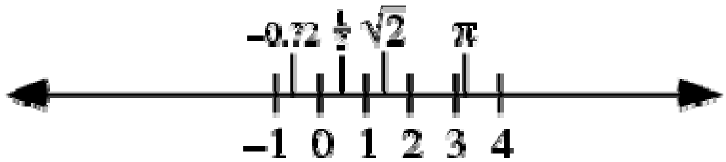
That is how *shunyam* given by our forefathers was recognised in the world and made its place permanently as *zero*.

Interestingly, the word *zero* probably came from Sanskrit word for *shunyam* or the Hindi equivalent of *shunya*. The word *shunyam* was translated to Arabic as *al-sifer*. Fibonacci mentioned it as *cifra* from which we have obtained our present *cipher*, meaning empty space. From this original Italian word or from alteration of Medieval Latin *zephirum*, the present word *zero* might have originated.

Unique number

The number 0 is neither positive nor negative and appears in the middle of a number line. It is neither a prime number nor a composite number. It cannot be prime because it has an infinite number of factors and cannot be composite because it cannot be expressed by multiplying prime numbers (0 must always be one of the factors).

Real numbers consist all rational (i.e. the numbers which can be express as *p/q,* like 2) and irrational numbers (which cannot be expressed as fraction, like $\sqrt{2}$). Now all these real numbers can be placed uniquely in a real line towards both positive and negative direction. Hence all positive, negative, even, odd, rational and irrational numbers correspond to only a single point on the line.



Among these real numbers, zero has the most important and unique position. It is in the intersection between positive and negative numbers. If one goes to the right side from zero, it is positive numbers and if one goes towards the left side of zero, it is all negative numbers. So essentially zero is neither positive nor negative number, it is the borderline for positive and negative numbers, or it is neutral in that sense. In fact this is the only number in the real number world, which is neither positive nor negative.

Zero as single entity has no power of its own. Even if one puts zero to the left side of any number still it is powerless. But if one starts adding it to the right side of a number, then zero starts showing its power and the number increases by ten times for each additional zero.

Division by zero

If zero is added with a positive or a negative number, then one will remain in the same number point in the real line scale, i.e. no change in the value. And if one multiplies any positive and negative real number with zero, result is zero.

However, division by zero is a tricky one. Brahmagupta himself could not describe the operation properly and later Bhaskara also mentioned it incorrectly.

Bhaskara said: if any number is divided by zero, it is infinity.

At first instance, assigning some positive number divided by zero as infinity or very high value seems logical. For example, if one continue to divide a real positive number by a smaller number, then result will go on increasing. Like:

10/10 = 1

10/1 = 10

10/0.01 = 1000

10/0.0001 = 10,0000

.

.

$10/10^{-99}$ = 10^{100}

and so on

Therefore, as one keep dividing by a smaller number and go towards zero, the result increases. But still the smaller number is not equal to zero. Therefore, one is not actually doing any division by zero, rather predicting a trend, which might be possible if divisor reaches a value, closer to zero or very small numbers. But whatever the smallest number one can think, another number smaller than that exists. Moreover, it is to be understood that infinity is a concept, an abstract thing, not a number as defined in our number system and all rules of mathematics are invalid while one consider operation with infinity. For example, if infinity is added with infinity result is not twice the value of infinity. It is still infinity!

It is therefore wrong to say that a number divided by zero is infinity. In fact, in the very first place it is wrong to attempt to divide a number by zero.

Let us understand the actual explanation for this situation:

A division is essentially the inverse of multiplication rule. That means if you divide 10 by 2, then you will get 5. And if you multiply 5 with 2, then you will get your original value back again. Using algebra, we can put it like this:

$$\text{If } (a / b) = c, \text{ then } a = (b * c)$$

Let's see what will happen if we follow the infinity theory. Assume that $a = 10$ and $b = 0$. Now, if you attempt to do (a / b) and assume $c =$ infinity, then according to rule of multiplication, we get 10 = (0 * infinity). But the rule of multiplication for zero says that anything multiplied with zero is zero. That means, applying the multiplication rule in the right hand side gives us finally 10 = 0, which is not possible. So we cannot get back 10 by multiplying the elements in the right hand side. We will get some absurd result as above while attempting and evaluating something divided by zero.

The uniqueness of division breaks down when we attempt to divide any number by zero since we cannot recover the original number by the inverting the process of multiplication. And zero is the only number with this property and so division by zero is *undefined* for real numbers. So we should *never* attempt to do a division with zero. In fact, it is meaningless to attempt to do this operation.

This is the reason that in all computer programs or mathematical calculations, one should take care of this vital operation and there should have appropriate strategy to deal with this situation. Imagine, a remotely controlled rocket is going towards a distant star and the computer installed in it, is doing millions of vital calculation every second. But the scientists who programmed the computer just inadvertently forgot to tell the computer what it should do if something like division by zero occurs. And unfortunately if it occurs, the computer will stop working and it will wonder what to do with this undefined operation. So all the efforts of the scientists will be a waste! Zero is so powerful.

Indeterminate

Another interesting case is zero divided by zero. Mathematically speaking, an expression like zero divided by zero is called *indeterminate*. To put it simply, this is a sort of expression, which cannot be determined accurately. If we see the expression properly, you can't assign any value to it. That means (0 / 0) can be equal to 10, 100 or anything else and interestingly the rule of multiplication also holds true here since 10 or 100 multiplied by zero will give the product as zero. So the basic problem is that we cannot determine the exact or precise value for this expression. That's why mathematically (0 / 0) is said to be *indeterminate*.

Zero to the power zero is also *indeterminate.* Mathematically, this situation is similar to zero divided by zero. Using limit theorem, it can be found that as *x and a tend to zero*, the function a^x takes values between 0 and 1, inclusive. So zero to the power zero is also termed as *indeterminate*. But modern day mathematicians are giving many new theories and insights regarding proper explanation of zero to the power zero. Some mathematicians say that accepting $0^0 = 1$ allows some formulas to be expressed simply while some others point out that $0^0 = 0$ makes the life easier. So this expression is not as naïve as it looks like!

Factorial of zero

The factorial of zero is equal to one. This is because the number of permutations one can do with zero elements is only one. This also can be proved mathematically. Remember here that the factorial of one is also one.

The mighty zero

Zero is tiny number, but we should never ignore its might. Imagine the world without zero. Not only mathematics, but all branches of sciences would have struggled for more clear definitions in their individual contexts, had zero not exist in our number system. Numbers from 2 to 9 are absent in binary system, and so are 8 and 9 in octal system. However, zero is everywhere and it is one of the most significant discoveries of mankind. Thanks to the ingenuity of our forefathers.

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