

MATHEMATICS

Textbook for Class XI



राष्ट्रीय शैक्षिक अनुसंधान और प्रशिक्षण परिषद्
NATIONAL COUNCIL OF EDUCATIONAL RESEARCH AND TRAINING

Foreword

The National Curriculum Framework (NCF), 2005, recommends that children's life at school must be linked to their life outside the school. This principle marks a departure from the legacy of bookish learning which continues to shape our system and causes a gap between the school, home and community. The syllabi and textbooks developed on the basis of NCF signify an attempt to implement this basic idea. They also attempt to discourage rote learning and the maintenance of sharp boundaries between different subject areas. We hope these measures will take us significantly further in the direction of a child-centred system of education outlined in the National Policy on Education (1986).

The success of this effort depends on the steps that school principals and teachers will take to encourage children to reflect on their own learning and to pursue imaginative activities and questions. We must recognise that given space, time and freedom, children generate new knowledge by engaging with the information passed on to them by adults. Treating the prescribed textbook as the sole basis of examination is one of the key reasons why other resources and sites of learning are ignored. Inculcating creativity and initiative is possible if we perceive and treat children as participants in learning, not as receivers of a fixed body of knowledge.

These aims imply considerable change in school routines and mode of functioning. Flexibility in the daily time-table is as necessary as rigour in implementing the annual calendar so that the required number of teaching days are actually devoted to teaching. The methods used for teaching and evaluation will also determine how effective this textbook proves for making children's life at school a happy experience, rather than a source of stress or boredom. Syllabus designers have tried to address the problem of curricular burden by restructuring and reorienting knowledge at different stages with greater consideration for child psychology and the time available for teaching. The textbook attempts to enhance this endeavour by giving higher priority and space to opportunities for contemplation and wondering, discussion in small groups, and activities requiring hands-on experience.

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New Delhi
20 December 2005

Director
National Council of Educational
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Contents

<i>Foreword</i>	<i>iii</i>
1. Sets	1
1.1 Introduction	1
1.2 Sets and their Representations	1
1.3 The Empty Set	5
1.4 Finite and Infinite Sets	6
1.5 Equal Sets	7
1.6 Subsets	9
1.7 Power Set	12
1.8 Universal Set	12
1.9 Venn Diagrams	13
1.10 Operations on Sets	14
1.11 Complement of a Set	18
1.12 Practical Problems on Union and Intersection of Two Sets	21
2. Relations and Functions	30
2.1 Introduction	30
2.2 Cartesian Product of Sets	30
2.3 Relations	34
2.4 Functions	36
3. Trigonometric Functions	49
3.1 Introduction	49
3.2 Angles	49
3.3 Trigonometric Functions	55
3.4 Trigonometric Functions of Sum and Difference of Two Angles	63
3.5 Trigonometric Equations	74
4. Principle of Mathematical Induction	86
4.1 Introduction	86
4.2 Motivation	87
4.3 The Principle of Mathematical Induction	88

5. Complex Numbers and Quadratic Equations	97
5.1 Introduction	97
5.2 Complex Numbers	97
5.3 Algebra of Complex Numbers	98
5.4 The Modulus and the Conjugate of a Complex Number	102
5.5 Argand Plane and Polar Representation	104
5.6 Quadratic Equations	108
6. Linear Inequalities	116
6.1 Introduction	116
6.2 Inequalities	116
6.3 Algebraic Solutions of Linear Inequalities in One Variable and their Graphical Representation	118
6.4 Graphical Solution of Linear Inequalities in Two Variables	123
6.5 Solution of System of Linear Inequalities in Two Variables	127
7. Permutations and Combinations	134
7.1 Introduction	134
7.2 Fundamental Principle of Counting	134
7.3 Permutations	138
7.4 Combinations	148
8. Binomial Theorem	160
8.1 Introduction	160
8.2 Binomial Theorem for Positive Integral Indices	160
8.3 General and Middle Terms	167
9. Sequences and Series	177
9.1 Introduction	177
9.2 Sequences	177
9.3 Series	179
9.4 Arithmetic Progression (A.P.)	181
9.5 Geometric Progression (G.P.)	186
9.6 Relationship Between A.M. and G.M.	191
9.7 Sum to n terms of Special Series	194
10. Straight Lines	203
10.1 Introduction	203
10.2 Slope of a Line	204
10.3 Various Forms of the Equation of a Line	212
10.4 General Equation of a Line	220
10.5 Distance of a Point From a Line	225

11. Conic Sections	236
11.1 Introduction	236
11.2 Sections of a Cone	236
11.3 Circle	239
11.4 Parabola	242
11.5 Ellipse	247
11.6 Hyperbola	255
12. Introduction to Three Dimensional Geometry	268
12.1 Introduction	268
12.2 Coordinate Axes and Coordinate Planes in Three Dimensional Space	269
12.3 Coordinates of a Point in Space	269
12.4 Distance between Two Points	271
12.5 Section Formula	273
13. Limits and Derivatives	281
13.1 Introduction	281
13.2 Intuitive Idea of Derivatives	281
13.3 Limits	284
13.4 Limits of Trigonometric Functions	298
13.5 Derivatives	303
14. Mathematical Reasoning	321
14.1 Introduction	321
14.2 Statements	321
14.3 New Statements from Old	324
14.4 Special Words/Phrases	329
14.5 Implications	335
14.6 Validating Statements	339
15. Statistics	347
15.1 Introduction	347
15.2 Measures of Dispersion	349
15.3 Range	349
15.4 Mean Deviation	349
15.5 Variance and Standard Deviation	361
15.6 Analysis of Frequency Distributions	372

16. Probability	383
16.1 Introduction	383
16.2 Random Experiments	384
16.3 Event	387
16.4 Axiomatic Approach to Probability	394
Appendix 1: Infinite Series	412
A.1.1 Introduction	412
A.1.2 Binomial Theorem for any Index	412
A.1.3 Infinite Geometric Series	414
A.1.4 Exponential Series	416
A.1.5 Logarithmic Series	419
Appendix 2: Mathematical Modelling	421
A.2.1 Introduction	421
A.2.2 Preliminaries	421
A.2.3 What is Mathematical Modelling	425
Answers	433
Supplementary Material	466



BE A STUDENT OF STUDENTS

A teacher who establishes rapport with the taught, becomes one with them, learns more from them than he teaches them. He who learns nothing from his disciples is, in my opinion, worthless. Whenever I talk with someone I learn from him. I take from him more than I give him. In this way, a true teacher regards himself as a student of his students. If you will teach your pupils with this attitude, you will benefit much from them.

Talk to Khadi Vidyalaya Students, Sevagram
Harijan Seva, 15 February 1942 (CW 75, p. 269)

USE ALL RESOURCES TO BE CONSTRUCTIVE AND CREATIVE

What we need is educationists with originality, fired with true zeal, who will think out from day to day what they are going to teach their pupils. The teacher cannot get this knowledge through musty volumes. He has to use his own faculties of observation and thinking and impart his knowledge to the children through his lips, with the help of a craft. This means a revolution in the method of teaching, a revolution in the teachers' outlook. Up till now you have been guided by inspector's reports. You wanted to do what the inspector might like, so that you might get more money yet for your institutions or higher salaries for yourselves. But the new teacher will not care for all that. He will say, 'I have done my duty to my pupil if I have made him a better man and in doing so I have used all my resources. That is enough for me'.

Harijan, 18 February 1939 (CW 68, pp. 374-75)

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