MATHEMATICS

MATHEMATICS

(CLASSES IX–XII)

This syllabus continues the approach along which the syllabi of Classes I to VIII have been developed. It has been designed in a manner that maintains continuity of a concept and its applications from Classes IX to XII.

The salient features of the syllabus are the following:

(i) The development and flow is from Class I upwards, not from college level down.

(ii) It is created keeping in mind that the time for transacting it is approximately 180 hours, a realistic figure based on feedback from the field.

(iii) The time given for developing a concept/series of concepts is allowing for the learner to explore them in several ways to develop and elaborate her understanding of them and the inter-relationships between them. While transacting the syllabus, we expect that the learner would be allowed a variety of opportunities for exploring mathematical concepts and processes, to help her construct her understanding of these.

(iv) The focus is on developing the processes involved in mathematical reasoning. Accordingly, the learner requires plenty of opportunity and enough time to develop the processes of dealing with greater abstraction, moving from particular to general to particular, moving with facility from one representation to another of a concept or process, solving and posing problems, etc.

(v) Linkages with the learner's life and experiences, and across the curriculum, need to be focused upon while transacting the curriculum. The idea is to allow the learner to realize how and why mathematics is all around us.

(vi) We note that it is at the secondary stage, the child enters into more formal mathematics. She needs to see the connections with what she has studied so far, consolidate it and begin to try and understand the formal thought process involved. With this in view two areas, related to mathematical proofs/reasoning and mathematical modelling, have been introduced from Class IX to XII, in a graded manner. Since these areas are thought of for the first time at these stages and the required awareness is lacking, it was decided to have these topics as appendices in the textbooks. This will give an opportunity to teachers and students to get exposure to these concepts. It is proposed that these topics may be considered for inclusion in the main syllabi in due course of time.

SECONDARY STAGE

General Guidelines

1. All concepts/identities must be illustrated by situational examples.
2. The language of 'word problems' must be clear, simple, and unambiguous.
3. All proofs to be produced in a non-didactic manner, allowing the learner to see flow of reason. Wherever possible give more than one proof.
4. Motivate most results. Prove explicitly those where a short and clear argument reinforces mathematical thinking and reasoning. There must be emphasis on correct way of expressing their arguments.
5. The reason for doing ruler and compass construction is to motivate and illustrate logical argument and reasoning. All constructions must include an analysis of the construction, and proof for the steps taken to do the required construction must be given.

CLASS IX

Units

I. Number Systems
II. Algebra
III. Coordinate Geometry
IV. Geometry
V. Mensuration
VI. Statistics and Probability

Appendix: 1. Proofs in Mathematics,
2. Introduction to Mathematical Modelling.

Unit I: Number Systems

Real Numbers **(Periods 20)**

Review of representation of natural numbers, integers, rational numbers on the number line. Representation of terminating/non-terminating recurring decimals, on the number line through successive magnification. Rational numbers as recurring/terminating decimals.

Examples of nonrecurring/non terminating decimals such as $\sqrt{2}, \sqrt{3}, \sqrt{5}$ etc. Existence of non-rational numbers (irrational numbers) such as $\sqrt{2}, \sqrt{3}$ and their representation on the number line. Explaining that every real number is represented by a unique point on the number line, and conversely, every point on the number line represents a unique real number.

Existence of $\sqrt{x}$ for a given positive real number x (visual proof to be emphasized). Definition of nth root of a real number.

Recall of laws of exponents with integral powers. Rational exponents with positive real bases (to be done by particular cases, allowing learner to arrive at the general laws).

Rationalisation (with precise meaning) of real numbers of the type (and their combinations) $\frac{1}{a+b\sqrt{x}}$ and $\frac{1}{\sqrt{x}+\sqrt{y}}$ where x and y are natural numbers and a, b are integers.

Unit II: Algebra

Polynomials (Periods 25)

Definition of a polynomial in one variable, its coefficients, with examples and counter examples, its terms, zero polynomial. Degree of a polynomial. Constant, linear, quadratic, cubic polynomials; monomials, binomials, trinomials. Factors and multiples. Zeros/roots of a polynomial/equation. State and motivate the Remainder Theorem with examples and analogy to integers. Statement and proof of the Factor Theorem. Factorisation of $ax^2 + bx + c$, $a \neq 0$ where a, b, c are real numbers, and of cubic polynomials using the Factor Theorem.

Recall of algebraic expressions and identities. Further identities of the type:

$(x + y + z)^2 = x^2 + y^2 + z^2 + 2xy + 2yz + 2zx$, $(x \pm y)^3 = x^3 \pm y^3 \pm 3xy\,(x \pm y)$,

$x^3 + y^3 + z^3 - 3xyz = (x + y + z)\,(x^2 + y^2 + z^2 - xy - yz - zx)$ and their use in factorization of polynomials. Simple expressions reducible to these polynomials.

Linear Equations in Two Variables (Periods 12)

Recall of linear equations in one variable. Introduction to the equation in two variables. Prove that a linear equation in two variables has infinitely many solutions, and justify their being written as ordered pairs of real numbers, plotting them and showing that they seem to lie on a line. Examples, problems from real life, including problems on Ratio and Proportion and with algebraic and graphical solutions being done simultaneously.

Unit III: Coordinate Geometry (Periods 9)

The Cartesian plane, coordinates of a point, names and terms associated with the coordinate plane, notations, plotting points in the plane, graph of linear equations as examples; focus on linear equations of the type $ax + by + c = 0$ by writing it as $y = mx + c$ and linking with the chapter on linear equations in two variables.

Unit IV: Geometry

1. *Introduction to Euclid's Geometry* (Periods 6)

History – Euclid and geometry in India. Euclid's method of formalizing observed phenomenon into rigorous mathematics with definitions, common/obvious notions, axioms/postulates, and theorems. The five postulates of Euclid. Equivalent versions of the fifth postulate. Showing the relationship between axiom and theorem.

1. Given two distinct points, there exists one and only one line through them.
2. (Prove) Two distinct lines cannot have more than one point in common.

2. *Lines and Angles* (Periods 10)

1. (Motivate) If a ray stands on a line, then the sum of the two adjacent angles so formed is 180° and the converse.
2. (Prove) If two lines intersect, the vertically opposite angles are equal.
3. (Motivate) Results on corresponding angles, alternate angles, interior angles when a transversal intersects two parallel lines.
4. (Motivate) Lines, which are parallel to a given line, are parallel.
5. (Prove) The sum of the angles of a triangle is 180°.
6. (Motivate) If a side of a triangle is produced, the exterior angle so formed is equal to the sum of the two interior opposite angles.

3. *Triangles* (Periods 20)

1. (Motivate) Two triangles are congruent if any two sides and the included angle of one triangle is equal to any two sides and the included angle of the other triangle (SAS Congruence).
2. (Prove) Two triangles are congruent if any two angles and the included side of one triangle is equal to any two angles and the included side of the other triangle (ASA Congruence).
3. (Motivate) Two triangles are congruent if the three sides of one triangle are equal to three sides of the other triangle (SSS Congruence).
4. (Motivate) Two right triangles are congruent if the hypotenuse and a side of one triangle are equal (respectively) to the hypotenuse and a side of the other triangle.
5. (Prove) The angles opposite to equal sides of a triangle are equal.
6. (Motivate) The sides opposite to equal angles of a triangle are equal.
7. (Motivate) Triangle inequalities and relation between 'angle and facing side'; inequalities in a triangle.

4. *Quadrilaterals* (Periods 10)

1. (Prove) The diagonal divides a parallelogram into two congruent triangles.
2. (Motivate) In a parallelogram opposite sides are equal and conversely.
3. (Motivate) In a parallelogram opposite angles are equal and conversely.

4. (Motivate) A quadrilateral is a parallelogram if a pair of its opposite sides is parallel and equal.
5. (Motivate) In a parallelogram, the diagonals bisect each other and conversely.
6. (Motivate) In a triangle, the line segment joining the mid points of any two sides is parallel to the third side and (motivate) its converse.

5. *Area* (Periods 4)

Review concept of area, recall area of a rectangle.

1. (Prove) Parallelograms on the same base and between the same parallels have the same area.
2. (Motivate) Triangles on the same base and between the same parallels are equal in area and its converse.

6. *Circles* (Periods 15)

Through examples, arrive at definitions of circle related concepts, radius, circumference, diameter, chord, arc, subtended angle.

1. (Prove) Equal chords of a circle subtend equal angles at the centre and (motivate) its converse.
2. (Motivate) The perpendicular from the centre of a circle to a chord bisects the chord and conversely, the line drawn through the centre of a circle to bisect a chord is perpendicular to the chord.
3. (Motivate) There is one and only one circle passing through three given non-collinear points.
4. (Motivate) Equal chords of a circle (or of congruent circles) are equidistant from the centre(s) and conversely.
5. (Prove) The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
6. (Motivate) Angles in the same segment of a circle are equal.
7. (Motivate) If a line segment joining two points subtends equal angle at two other points lying on the same side of the line containing the segment, the four points lie on a circle.
8. (Motivate) The sum of the either pair of the opposite angles of a cyclic quadrilateral is 180^0 and its converse.

7. *Constructions* (Periods 10)

1. Construction of bisectors of a line segment and angle, 60°, 90°, 45° angles etc, equilateral triangles.
2. Construction of a triangle given its base, sum/difference of the other two sides and one base angle.
3. Construction of a triangle of given perimeter and base angles.

Unit V: Mensuration

1. *Areas* (Periods 4)

Area of a triangle using Heron's formula (without proof) and its application in finding the area of a quadrilateral.

2. *Surface Areas and Volumes* **(Periods 10)**

Surface areas and volumes of cubes, cuboids, spheres (including hemispheres) and right circular cylinders/cones.

Unit VI: Statistics and Probability

1. *Statistics* **(Periods 13)**

Introduction to Statistics: Collection of data, presentation of data – tabular form, ungrouped/grouped, bar graphs, histograms (with varying base lengths), frequency polygons, qualitative analysis of data to choose the correct form of presentation for the collected data. Mean, median, mode of ungrouped data.

2. *Probability* **(Periods 12)**

History, Repeated experiments and observed frequency approach to probability. Focus is on empirical probability. (A large amount of time to be devoted to group and to individual activities to motivate the concept; the experiments to be drawn from real-life situations, and from examples used in the chapter on statistics).

Appendix

1. *Proofs in Mathematics*

What a statement is; when is a statement mathematically valid. Explanation of axiom/postulate through familiar examples. Difference between axiom, conjecture and theorem. The concept and nature of a 'proof' (emphasize deductive nature of the proof, the assumptions, the hypothesis, the logical argument) and writing a proof. Illustrate deductive proof with complete arguments using simple results from arithmetic, algebra and geometry (e.g., product of two odd numbers is odd etc.). Particular stress on verification not being proof. Illustrate with a few examples of verifications leading to wrong conclusions – include statements like "every odd number greater than 1 is a prime number". What disproving means, use of counter examples.

2. *Introduction to Mathematical Modelling*

The concept of mathematical modelling, review of work done in earlier classes while looking at situational problems, aims of mathematical modelling, discussing the broad stages of modelling – real-life situations, setting up of hypothesis, determining an appropriate model, solving the mathematical problem equivalent, analyzing the conclusions and their real-life interpretation, validating the model. Examples to be drawn from ratio, proportion, percentages, etc.

Class X

Units

I. Number Systems
II. Algebra
III. Trigonometry
IV. Coordinate Geometry
V. Geometry
VI. Mensuration
VII. Statistics and Probability

Appendix: 1. Proofs in Mathematics
2. Mathematical Modelling

Unit I: Number Systems

Real Numbers **(Periods 15)**

Euclid's division lemma, Fundamental Theorem of Arithmetic – statements after reviewing work done earlier and after illustrating and motivating through examples. Proofs of results – irrationality of $\sqrt{2}, \sqrt{3}, \sqrt{5}$, decimal expansions of rational numbers in terms of terminating/non-terminating recurring decimals.

Unit II: Algebra

1. *Polynomials* **(Periods 6)**

Zeros of a polynomial. Relationship between zeros and coefficients of a polynomial with particular reference to quadratic polynomials. Statement and simple problems on division algorithm for polynomials with real coefficients.

2. *Pair of Linear Equations in Two Variables* **(Periods 15)**

Pair of linear equations in two variables. Geometric representation of different possibilities of solutions/inconsistency.

Algebraic conditions for number of solutions. Solution of pair of linear equations in two variables algebraically – by substitution, by elimination and by cross multiplication. Simple situational problems must be included. Simple problems on equations reducible to linear equations may be included.

3. *Quadratic Equations* **(Periods 15)**

Standard form of a quadratic equation $ax^2 + bx + c = 0$, $(a \neq 0)$. Solution of quadratic equations (only real roots) by factorization and by completing the square, i.e., by using quadratic formula. Relationship between discriminant and nature of roots.

Problems related to day-to-day activities to be incorporated.

4. *Arithmetic Progressions* (AP) **(Periods 8)**

Motivation for studying AP. Derivation of standard results of finding the n^{th} term and sum of first n terms.

Unit III: Trigonometry

1. *Introduction to Trigonometry* **(Periods 18)**

Trigonometric ratios of an acute angle of a right-angled triangle. Proof of their existence (well defined); motivate the ratios, whichever are defined at 0° and 90°. Values (with proofs) of the trigonometric ratios of 30°, 45° and 60°. Relationships between the ratios.

Trigonometric Identities: Proof and applications of the identity $\sin^2 A + \cos^2 A = 1$. Only simple identities to be given. Trigonometric ratios of complementary angles.

2. *Heights and Distances* **(Periods 8)**

Simple and believable problems on heights and distances. Problems should not involve more than two right triangles. Angles of elevation/depression should be only 30^0, 45^0, 60^0.

Unit IV: Coordinate Geometry

Lines (In two-dimensions) **(Periods 15)**

Review the concepts of coordinate geometry done earlier including graphs of linear equations. Awareness of geometrical representation of quadratic polynomials. Distance between two points and section formula (internal). Area of a triangle.

Unit V: Geometry

1. *Triangles* **(Periods 15)**

Definitions, examples, counterexamples of similar triangles.

1. (Prove) If a line is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.
2. (Motivate) If a line divides two sides of a triangle in the same ratio, the line is parallel to the third side.
3. (Motivate) If in two triangles, the corresponding angles are equal, their corresponding sides are proportional and the triangles are similar.

4. (Motivate) If the corresponding sides of two triangles are proportional, their corresponding angles are equal and the two triangles are similar.
5. (Motivate) If one angle of a triangle is equal to one angle of another triangle and the sides including these angles are proportional, the two triangles are similar.
6. (Motivate) If a perpendicular is drawn from the vertex of the right angle to the hypotenuse, the triangles on each side of the perpendicular are similar to the whole triangle and to each other.
7. (Prove) The ratio of the areas of two similar triangles is equal to the ratio of the squares on their corresponding sides.
8. (Prove) In a right triangle, the square on the hypotenuse is equal to the sum of the squares on the other two sides.
9. (Prove) In a triangle, if the square on one side is equal to sum of the squares on the other two sides, the angles opposite to the first side is a right triangle.

2. *Circles* (Periods 8)

Tangents to a circle motivated by chords drawn from points coming closer and closer to the point.

1. (Prove) The tangent at any point of a circle is perpendicular to the radius through the point of contact.
2. (Prove) The lengths of tangents drawn from an external point to a circle are equal.

3. *Constructions* (Periods 8)

1. Division of a line segment in a given ratio (internally).
2. Tangent to a circle from a point outside it.
3. Construction of a triangle similar to a given triangle.

Unit VI: Mensuration

1. *Areas Related to Circles* (Periods 12)

Motivate the area of a circle; area of sectors and segments of a circle. Problems based on areas and perimeter/circumference of the above said plane figures.

(In calculating area of segment of a circle, problems should be restricted to central angle of 60°, 90° and 120° only. Plane figures involving triangles, simple quadrilaterals and circle should be taken.)

2. *Surface Areas and Volumes* (Periods 12)

1. Problems on finding surface areas and volumes of combinations of any two of the following: cubes, cuboids, spheres, hemispheres and right circular cylinders/cones. Frustum of a cone.

2. Problems involving converting one type of metallic solid into another and other mixed problems. (Problems with combination of not more than two different solids be taken.)

Unit VII: Statistics and Probability

1. *Statistics* **(Periods 15)**

Mean, median and mode of grouped data (bimodal situation to be avoided). Cumulative frequency graph.

2. *Probability* **(Periods 10)**

Classical definition of probability. Connection with probability as given in Class IX. Simple problems on single events, not using set notation.

Appendix

1. *Proofs in Mathematics*

Further discussion on concept of 'statement', 'proof' and 'argument'. Further illustrations of deductive proof with complete arguments using simple results from arithmetic, algebra and geometry. Simple theorems of the "Given ……… and assuming… prove ……..". Training of using only the given facts (irrespective of their truths) to arrive at the required conclusion. Explanation of 'converse', 'negation', constructing converses and negations of given results/statements.

2. *Mathematical Modelling*

Reinforcing the concept of mathematical modelling, using simple examples of models where some constraints are ignored. Estimating probability of occurrence of certain events and estimating averages may be considered. Modelling fair instalments payments, using only simple interest and future value (use of AP).

Higher Secondary Stage

General Guidelines

(i) All concepts/identities must be illustrated by situational examples.

(ii) The language of 'word problems' must be clear, simple and unambiguous.

(iii) Problems given should be testing the understanding of the subject.

(iv) All proofs to be produced in a manner that allow the learner to see flow of reasons. Wherever possible, give more than one proof.

(v) Motivate results, wherever possible. Prove explicitly those results where a short and clear argument reinforces mathematical thinking and reasoning. There must be emphasis on correct way of expressing the arguments.

Class XI

Units

I. Sets and Functions
II. Algebra
III. Coordinate Geometry
IV. Calculus
V. Mathematical Reasoning
VI. Statistics and Probability

Appendix: 1. Infinite Series,
2. Mathematical Modelling

Chapters with Time Allocation

1.1	Sets	Periods 12
1.2	Relations and Functions	Periods 14
1.3	Trigonometric Functions	Periods 18
2.1	Principle of Mathematical Induction	Periods 06
2.2	Complex Numbers and Quadratic Equations	Periods 10
2.3	Linear Inequalities	Periods 10
2.4	Permutations and Combinations	Periods 12
2.5	Binomial Theorem	Periods 08
2.6	Sequence and Series	Periods 10
3.1	Straight Lines	Periods 09
3.2	Conic Sections	Periods 12
3.3	Introduction to Three-dimensional Geometry	Periods 08
4.1	Limits and Derivatives	Periods 18
5.1	Mathematical Reasoning	Periods 08
6.1	Statistics	Periods 10
6.2	Probability	Periods 15
	Total Periods	180

Unit I: Sets and Functions

1. *Sets*

Sets and their representations. Empty set. Finite and Infinite sets. Equal sets. Subsets. Subsets of the set of real numbers especially intervals (with notations). Power set. Universal set. Venn diagrams. Union and Intersection of sets. Difference of sets. Complement of a set.

2. *Relations and Functions*

Ordered pairs, Cartesian product of sets. Number of elements in the cartesian product of two finite sets. Cartesian product of the reals with itself (upto $R \times R \times R$).

Definition of relation, pictorial diagrams, domain, co-domain and range of a relation. Function as a special kind of relation from one set to another. Pictorial representation of a function, domain, co-domain and range of a function. Real valued function of the real variable, domain and range of these functions, constant, identity, polynomial, rational, modulus, signum and greatest integer functions with their graphs. Sum, difference, product and quotients of functions.

3. *Trigonometric Functions*

Positive and negative angles. Measuring angles in radians and in degrees and conversion from one measure to another. Definition of trigonometric functions with the help of unit circle. Truth of the identity $\sin^2 x + \cos^2 x = 1$, for all x. Signs of trigonometric functions and sketch of their graphs. Expressing $\sin(x+y)$ and $\cos(x+y)$ in terms of $\sin x$, $\sin y$, $\cos x$ and $\cos y$. Deducing the identities like following:

$$= \frac{\tan x \pm \tan y}{1 \mp \tan x \tan y}, \cot(x \pm y) = \frac{\cot x \cot y \mp 1}{\cot y \pm \cot x},$$

$$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}, \cos x + \cos y = 2\cos\frac{x+y}{2}\cos\frac{x-y}{2},$$

$$\sin x - \sin y = 2\cos\frac{x+y}{2}\sin\frac{x-y}{2}, \cos x - \cos y = -2\sin\frac{x+y}{2}\sin\frac{x-y}{2}.$$

Identities related to $\sin 2x$, $\cos 2x$, $\tan 2x$, $\sin 3x$, $\cos 3x$ and $\tan 3x$. General solution of trigonometric equations of the type $\sin\theta = \sin\alpha$, $\cos\theta = \cos\alpha$ and $\tan\theta = \tan\alpha$. Proofs and simple applications of sine and cosine formulae.

Unit II: Algebra

1. *Principle of Mathematical Induction*

Processes of the proof by induction, motivating the application of the method by looking at natural numbers as the least inductive subset of real numbers. The principle of mathematical induction and simple applications.

2. *Complex Numbers and Quadratic Equations*

Need for complex numbers, especially $\sqrt{-1}$, to be motivated by inability to solve every quadratic equation. Brief description of algebraic properties of complex numbers. Argand plane and polar representation of complex numbers. Statement of Fundamental Theorem of Algebra, solution of quadratic equations in the complex number system.

3. *Linear Inequalities*

Linear inequalities. Algebraic solutions of linear inequalities in one variable and their representation on the number line. Graphical solution of linear inequalities in two variables. Solution of system of linear inequalities in two variables – graphically.

4. *Permutations and Combinations*

Fundamental principle of counting. Factorial n. Permutations and combinations, derivation of formulae and their connections, simple applications.

5. *Binomial Theorem*

History, statement and proof of the binomial theorem for positive integral indices. Pascal's triangle, general and middle term in binomial expansion, simple applications.

6. *Sequence and Series*

Sequence and Series. Arithmetic progression (A. P.), arithmetic mean (A.M.). Geometric progression (G.P.), general term of a G. P., sum of n terms of a G.P., geometric mean (G.M.), relation between A.M. and G.M. Sum to n terms of the special series: $\sum n, \sum n^2 \text{and} \sum n^3$.

Unit III: Coordinate Geometry

1. *Straight Lines*

Brief recall of 2D from earlier classes. Slope of a line and angle between two lines. Various forms of equations of a line: parallel to axes, point-slope form, slope-intercept form, two-point form, intercepts form and normal form. General equation of a line. Distance of a point from a line.

2. *Conic Sections*

Sections of a cone: Circles, ellipse, parabola, hyperbola, a point, a straight line and pair of intersecting lines as a degenerated case of a conic section. Standard equations and simple properties of parabola, ellipse and hyperbola. Standard equation of a circle.

3. *Introduction to Three-dimensional Geometry*

Coordinate axes and coordinate planes in three dimensions. Coordinates of a point. Distance between two points and section formula.

Unit IV: Calculus

Limits and Derivatives

Derivative introduced as rate of change both as that of distance function and geometrically, intuitive idea of limit. Definition of derivative, relate it to slope of tangent of the curve, derivative of sum, difference, product and quotient of functions. Derivatives of polynomial and trigonometric functions.

Unit V: Mathematical Reasoning

Mathematically acceptable statements. Connecting words/phrases — consolidating the understanding of "if and only if (necessary and sufficient) condition", "implies", "and/or", "implied by", "and", "or", "there exists" and their use through variety of examples related to real life and Mathematics. Validating the statements involving the connecting words – difference between contradiction, converse and contrapositive.

Unit VI: Statistics and Probability

1. *Statistics*

Measure of dispersion; mean deviation, variance and standard deviation of ungrouped/grouped data. Analysis of frequency distributions with equal means but different variances.

2. *Probability*

Random experiments: Outcomes, sample spaces (set representation). Events: Occurrence of events, 'not', 'and' & 'or' events, exhaustive events, mutually exclusive events. Axiomatic (set theoretic) probability, connections with the theories of earlier classes. Probability of an event, probability of 'not', 'and' & 'or' events.

Appendix

1. *Infinite Series*

Binomial theorem for any index, infinite geometric series, exponential and logarithmic series.

2. *Mathematical Modelling*

Consolidating the understanding developed up to Class X. Focus on modelling problems related to real-life (like environment, travel, etc.) and connecting with other subjects of study where many constraints may really need to be ignored, formulating the model, looking for solutions, interpreting them in the problem situation and evaluating the model.

Class XII

Units

I. Relations and Functions
II. Algebra
III. Calculus
IV. Vectors and Three-Dimensional Geometry
V. Linear Programming
VI. Probability

Appendix: 1. Proofs in Mathematics
2. Mathematical Modelling

Chapters with Time Allocation

1.1	Relations and Functions	**Periods 10**
1.2	Inverse Trigonometric Functions	**Periods 12**

2.1	Matrices	Periods 18
2.2	Determinants	Periods 20
3.1	Continuity and Differentiability	Periods 18
3.2	Applications of Derivatives	Periods 10
3.3	Integrals	Periods 20
3.4	Applications of the Integrals	Periods 10
3.5	Differential Equations	Periods 10
4.1	Vectors	Periods 10
4.2	Three-dimensional Geometry	Periods 12
5.1	Linear Programming	Periods 12
6.1	Probability	Periods 18
	Total Periods	**180**

Unit I: Relations and Functions

1. *Relations and Functions*

Types of relations: Reflexive, symmetric, transitive and equivalence relations. One to one and onto functions, composite functions, inverse of a function. Binary operations.

2. *Inverse Trigonometric Functions*

Definition, range, domain, principal value branches. Graphs of inverse trigonometric functions. Elementary properties of inverse trigonometric functions.

Unit II: Albegra

1. *Matrices*

Concept, notation, order, equality, types of matrices, zero matrix, transpose of a matrix, symmetric and skew symmetric matrices. Addition, multiplication and scalar multiplication of matrices, simple properties of addition, multiplication and scalar multiplication. Non-commutativity of multiplication of matrices and existence of non-zero matrices whose product is the zero matrix (restrict to square matrices of order 2). Concept of elementary row and column operations. Invertible matrices and proof of the uniqueness of inverse, if it exists; (Here all matrices will have real entries).

2. *Determinants*

Determinant of a square matrix (up to 3 × 3 matrices), properties of determinants, minors, cofactors and applications of determinants in finding the area of a triangle. Adjoint and inverse of a square matrix. Consistency, inconsistency and number of solutions of system of linear equations by examples, solving system of linear equations in two or three variables (having unique solution) using inverse of a matrix.

Unit III: Calculus

1. *Continuity and Differentiability*

Continuity and differentiability, derivative of composite functions, chain rule, derivatives of inverse trigonometric functions, derivative of implicit function. Concept of exponential and logarithmic functions and their derivatives. Logarithmic differentiation. Derivative of functions expressed in parametric forms. Second order derivatives. Rolle's and Lagrange's Mean Value Theorems (without proof) and their geometric interpretations.

2. *Applications of Derivatives*

Applications of derivatives: Rate of change, increasing/decreasing functions, tangents and normals, approximation, maxima and minima (first derivative test motivated geometrically and second derivative test given as a provable tool). Simple problems (that illustrate basic principles and understanding of the subject as well as real-life situations).

3. *Integrals*

Integration as inverse process of differentiation. Integration of a variety of functions by substitution, by partial fractions and by parts, only simple integrals of the type

$$\int \frac{dx}{x^2 \pm a^2}, \int \frac{dx}{\sqrt{x^2 \pm a^2}}, \int \frac{dx}{\sqrt{a^2 - x^2}}, \int \frac{dx}{ax^2 + bx + c}, \int \frac{dx}{\sqrt{ax^2 + bx + c}},$$

$$\int \frac{(px+q)}{ax^2 + bx + c} dx, \int \frac{(px+q)}{\sqrt{ax^2 + bx + c}} dx, \int \sqrt{a^2 \pm x^2} dx \text{ and } \int \sqrt{x^2 - a^2} dx$$ to be evaluated.

Definite integrals as a limit of a sum. Fundamental Theorem of Calculus (without proof). Basic properties of definite integrals and evaluation of definite integrals.

4. *Applications of the Integrals*

Applications in finding the area under simple curves, especially lines, arcs of circles/parabolas/ellipses (in standard form only), area between the two above said curves (the region should be clearly identifiable).

5. *Differential Equations*

Definition, order and degree, general and particular solutions of a differential equation. Formation of differential equation whose general solution is given. Solution of differential equations by method of separation of variables, homogeneous differential equations of first order and first degree. Solutions of linear differential equation of the type:

$$\frac{dy}{dx} + \mathrm{P}\,y = \mathrm{Q},$$ where P and Q are functions of x.

Unit IV: Vectors and Three-Dimensional Geometry

1. *Vectors*

Vectors and scalars, magnitude and direction of a vector. Direction cosines/ratios of vectors. Types of vectors (equal, unit, zero, parallel and collinear vectors), position vector of a point, negative of a vector, components of a vector, addition of vectors, multiplication of a vector by a scalar, position vector of a point dividing a line segment in a given ratio. Scalar (dot) product of vectors, projection of a vector on a line. Vector (cross) product of vectors.

2. *Three-dimensional Geometry*

Direction cosines/ratios of a line joining two points. Cartesian and vector equation of a line, coplanar and skew lines, shortest distance between two lines. Cartesian and vector equation of a plane. Angle between (i) two lines, (ii) two planes, (iii) a line and a plane. Distance of a point from a plane.

Unit V: Linear Programming

Introduction, related terminology such as constraints, objective function, optimization, different types of linear programming (L.P.) problems, mathematical formulation of L.P. problems, graphical method of solution for problems in two variables, feasible and infeasible regions, feasible and infeasible solutions, optimal feasible solutions (up to three non-trivial constrains).

Unit VI: Probability

Multiplication theorem on probability. Conditional probability, independent events, total probability, Baye's theorem. Random variable and its probability distribution, mean and variance of haphazard variable. Repeated independent (Bernoulli) trials and Binomial distribution.

Appendix

1. *Proofs in Mathematics*

Through a variety of examples related to mathematics and already familiar to the learner, bring out different kinds of proofs: direct, contrapositive, by contradiction, by counter-example.

2. *Mathematical Modelling*

Modelling real-life problems where many constraints may really need to be ignored (continuing from Class XI). However, now the models concerned would use techniques/results of matrices, calculus and linear programming.