(A) Main Concepts and Results

- **Rectangle**
  (a) Area = length × breadth
  (b) Perimeter = 2 (length + breadth)
  (c) Diagonal = \(\sqrt{(\text{length})^2 + (\text{breadth})^2}\)

- **Square**
  (a) Area = (side)\(^2\)
  (b) Perimeter = 4 × side
  (c) Diagonal = \(\sqrt{2} \times \text{side}\)

- **Triangle with base \((b)\) and altitude \((h)\)**
  Area = \(\frac{1}{2} \times b \times h\)

- **Triangle with sides as \(a, b, c\)**
  (i) Semi-perimeter = \(\frac{a + b + c}{2} = s\)
  (ii) Area = \(\sqrt{s(s-a)(s-b)(s-c)}\) (Heron’s Formula)

- **Isosceles triangle, with base \(a\) and equal sides \(b\)**
Area of isosceles triangle = \( \frac{a}{4} \sqrt{4b^2 - a^2} \)

- **Equilateral triangle with side** \( a \)

  Area = \( \frac{\sqrt{3}}{4} a^2 \)

- **Parallelogram with base** \( b \) **and altitude** \( h \)

  Area = \( bh \)

- **Rhombus with diagonals** \( d_1 \) and \( d_2 \)
  
  (a) Area = \( \frac{1}{2} d_1 \times d_2 \)
  
  (b) Perimeter = \( 2\sqrt{d_1^2 + d_2^2} \)

- **Trapezium with parallel sides** \( a \) and \( b \), and the distance between two parallel sides as \( h \).

  Area = \( \frac{1}{2} (a + b) \times h \)

- **Regular hexagon with side** \( a \)

  Area = \( 6 \times \text{Area of an equilateral triangle with side} \ a \)

  \[ = 6 \times \frac{\sqrt{3}}{4} a^2 = \frac{3}{2} \sqrt{3} a^2 \]

(B) **Multiple Choice Questions**

Write the correct answer:

**Sample Question 1**: The base of a right triangle is 8 cm and hypotenuse is 10 cm. Its area will be

(A) 24 cm\(^2\)  (B) 40 cm\(^2\)  (C) 48 cm\(^2\)  (D) 80 cm\(^2\)

**Solution**: Answer (A)

**EXERCISE 12.1**

1. An isosceles right triangle has area 8 cm\(^2\). The length of its hypotenuse is

(A) \( \sqrt{32} \) cm  (B) \( \sqrt{16} \) cm  (C) \( \sqrt{48} \) cm  (D) \( \sqrt{24} \) cm
2. The perimeter of an equilateral triangle is 60 m. The area is
   (A) \(10\sqrt{3}\) m\(^2\)  (B) \(15\sqrt{3}\) m\(^2\)  (C) \(20\sqrt{3}\) m\(^2\)  (D) \(100\sqrt{3}\) m\(^2\)

3. The sides of a triangle are 56 cm, 60 cm and 52 cm long. Then the area of the triangle is
   (A) 1322 cm\(^2\)  (B) 1311 cm\(^2\)  (C) 1344 cm\(^2\)  (D) 1392 cm\(^2\)

4. The area of an equilateral triangle with side \(2\sqrt{3}\) cm is
   (A) 5.196 cm\(^2\)  (B) 0.866 cm\(^2\)  (C) 3.496 cm\(^2\)  (D) 1.732 cm\(^2\)

5. The length of each side of an equilateral triangle having an area of \(9\sqrt{3}\) cm\(^2\) is
   (A) 8 cm  (B) 36 cm  (C) 4 cm  (D) 6 cm

6. If the area of an equilateral triangle is \(16\sqrt{3}\) cm\(^2\), then the perimeter of the triangle is
   (A) 48 cm  (B) 24 cm  (C) 12 cm  (D) 36 cm

7. The sides of a triangle are 35 cm, 54 cm and 61 cm, respectively. The length of its longest altitude
   (A) \(16\sqrt{5}\) cm  (B) \(10\sqrt{5}\) cm  (C) \(24\sqrt{5}\) cm  (D) 28 cm

8. The area of an isosceles triangle having base 2 cm and the length of one of the equal sides 4 cm, is
   (A) \(\sqrt{15}\) cm\(^2\)  (B) \(\sqrt{\frac{15}{2}}\) cm\(^2\)  (C) \(2\sqrt{15}\) cm\(^2\)  (D) \(4\sqrt{15}\) cm\(^2\)

9. The edges of a triangular board are 6 cm, 8 cm and 10 cm. The cost of painting it at the rate of 9 paise per cm\(^2\) is
   (A) Rs 2.00  (B) Rs 2.16  (C) Rs 2.48  (D) Rs 3.00

(C) Short Answer Questions with Reasoning

Write True or False and justify your answer:

Sample Question 1 : If \(a\), \(b\), \(c\) are the lengths of three sides of a triangle, then area of a triangle \(= \sqrt{s(s - a)(s - b)(s - c)}\), where \(s = \) perimeter of triangle.

Solution : False. Since in Heron’s formula,
HERON’S FORMULA

\[ s = \frac{1}{2}(a + b + c) \]

\[ = \frac{1}{2} \text{ (perimeter of triangle)} \]

**EXERCISE 12.2**

Write **True** or **False** and justify your answer:

1. The area of a triangle with base 4 cm and height 6 cm is 24 cm².
2. The area of \( \Delta ABC \) is 8 cm² in which \( AB = AC = 4 \text{ cm} \) and \( \angle A = 90^\circ \).
3. The area of the isosceles triangle is \( \frac{5}{4} \sqrt{11} \text{ cm}^2 \), if the perimeter is 11 cm and the base is 5 cm.
4. The area of the equilateral triangle is \( 20\sqrt{3} \text{ cm}^2 \) whose each side is 8 cm.
5. If the side of a rhombus is 10 cm and one diagonal is 16 cm, the area of the rhombus is 96 cm².
6. The base and the corresponding altitude of a parallelogram are 10 cm and 3.5 cm, respectively. The area of the parallelogram is 30 cm².
7. The area of a regular hexagon of side ‘a’ is the sum of the areas of the five equilateral triangles with side \( a \).
8. The cost of levelling the ground in the form of a triangle having the sides 51 m, 37 m and 20 m at the rate of Rs 3 per m² is Rs 918.
9. In a triangle, the sides are given as 11 cm, 12 cm and 13 cm. The length of the altitude is 10.25 cm corresponding to the side having length 12 cm.

**(D) Short Answer Questions**

**Sample Question 1:** The sides of a triangular field are 41 m, 40 m and 9 m. Find the number of rose beds that can be prepared in the field, if each rose bed, on an average needs 900 cm² space.

**Solution:** Let \( a = 41 \text{ m}, \ b = 40 \text{ m}, \ c = 9 \text{ m} \).

\[ s = \frac{a + b + c}{2} = \frac{41 + 40 + 9}{2} = 45 \text{ m} \]
Area of the triangular field
\[ = \sqrt{s(s-a)(s-b)(s-c)} \]
\[ = \sqrt{45(45-41)(45-40)(45-9)} \]
\[ = \sqrt{45 \times 4 \times 5 \times 36} = 180 \text{ m}^2 \]

So, the number of rose beds \( = \frac{180}{0.09} = 2000 \)

**Sample Question 2**: Calculate the area of the shaded region in Fig. 12.1.

**Solution**: For the triangle having the sides 122 m, 120 m and 22 m:

\[ s = \frac{122 + 120 + 22}{2} = 132 \]

Area of the triangle \( = \sqrt{s(s-a)(s-b)(s-c)} \)
\[ = \sqrt{132(132-122)(132-120)(132-22)} \]
\[ = \sqrt{132 \times 10 \times 12 \times 110} \]
\[ = 1320 \text{ m}^2 \]

For the triangle having the sides 22 m, 24 m and 26 m:

\[ s = \frac{22 + 24 + 26}{2} = 36 \]

Area of the triangle \( = \sqrt{36(36-22)(36-24)(36-26)} \)
\[ = \sqrt{36 \times 14 \times 12 \times 10} \]
\[ = 24\sqrt{105} \]
\[ = 24 \times 10.25 \text{ m}^2 \text{ (approx.)} \]
\[ = 246 \text{ m}^2 \]

Therefore, the area of the shaded portion \( = (1320 - 246) \text{ m}^2 \)
\[ = 1074 \text{ m}^2 \]
EXERCISE 12.3

1. Find the cost of laying grass in a triangular field of sides 50 m, 65 m and 65 m at the rate of Rs 7 per m².

2. The triangular side walls of a flyover have been used for advertisements. The sides of the walls are 13 m, 14 m and 15 m. The advertisements yield an earning of Rs 2000 per m² a year. A company hired one of its walls for 6 months. How much rent did it pay?

3. From a point in the interior of an equilateral triangle, perpendiculars are drawn on the three sides. The lengths of the perpendiculars are 14 cm, 10 cm and 6 cm. Find the area of the triangle.

4. The perimeter of an isosceles triangle is 32 cm. The ratio of the equal side to its base is 3 : 2. Find the area of the triangle.

5. Find the area of a parallelogram given in Fig. 12.2. Also find the length of the altitude from vertex A on the side DC.

6. A field in the form of a parallelogram has sides 60 m and 40 m and one of its diagonals is 80 m long. Find the area of the parallelogram.

7. The perimeter of a triangular field is 420 m and its sides are in the ratio 6 : 7 : 8. Find the area of the triangular field.

8. The sides of a quadrilateral ABCD are 6 cm, 8 cm, 12 cm and 14 cm (taken in order) respectively, and the angle between the first two sides is a right angle. Find its area.

9. A rhombus shaped sheet with perimeter 40 cm and one diagonal 12 cm, is painted on both sides at the rate of Rs 5 per m². Find the cost of painting.

10. Find the area of the trapezium PQRS with height PQ given in Fig. 12.3

![Fig. 12.2](image_url)

![Fig. 12.3](image_url)
(E) Long Answer Questions

Sample Question 1: If each side of a triangle is doubled, then find the ratio of area of the new triangle thus formed and the given triangle.

Solution: Let \(a, b, c\) be the sides of the triangle (existing) and \(s\) be its semi-perimeter.

Then, \(s = \frac{a + b + c}{2}\)

or, \(2s = a + b + c\) \hspace{1cm} (1)

Area of the existing triangle = \(\sqrt{s(s-a)(s-b)(s-c)} = \Delta\), say

According to the statement, the sides of the new triangle will be \(2a, 2b\) and \(2c\). Let \(S\) be the semi-perimeter of the new triangle.

\(S = \frac{2a + 2b + 2c}{2} = a + b + c\) \hspace{1cm} (2)

From (1) and (2), we get

\(S = 2s\) \hspace{1cm} (3)

Area of the new triangle

\(= \sqrt{S(S-2a)(S-2b)(S-2c)}\)

Putting the values, we get

\(= \sqrt{2s(2s-2a)(2s-2b)(2s-2c)}\)

\(= \sqrt{16s(s-a)(s-b)(s-c)}\)

\(= 4\sqrt{s(s-a)(s-b)(s-c)} = 4\Delta\)

Therefore, the required ratio is 4:1.

EXERCISE 12.4

1. How much paper of each shade is needed to make a kite given in Fig. 12.4, in which ABCD is a square with diagonal 44 cm.
2. The perimeter of a triangle is 50 cm. One side of a triangle is 4 cm longer than the smaller side and the third side is 6 cm less than twice the smaller side. Find the area of the triangle.

3. The area of a trapezium is 475 cm$^2$ and the height is 19 cm. Find the lengths of its two parallel sides if one side is 4 cm greater than the other.

4. A rectangular plot is given for constructing a house, having a measurement of 40 m long and 15 m in the front. According to the laws, a minimum of 3 m, wide space should be left in the front and back each and 2 m wide space on each of other sides. Find the largest area where house can be constructed.

5. A field is in the shape of a trapezium having parallel sides 90 m and 30 m. These sides meet the third side at right angles. The length of the fourth side is 100 m. If it costs Rs 4 to plough 1 m$^2$ of the field, find the total cost of ploughing the field.

6. In Fig. 12.5, $\triangle ABC$ has sides $AB = 7.5$ cm, $AC = 6.5$ cm and $BC = 7$ cm. On base $BC$ a parallelogram $DBCE$ of same area as that of $\triangle ABC$ is constructed. Find the height $DF$ of the parallelogram.

7. The dimensions of a rectangle $ABCD$ are 51 cm $\times$ 25 cm. A trapezium $PQCD$ with its parallel
sides QC and PD in the ratio 9 : 8, is cut off from the rectangle as shown in the Fig. 12.6. If the area of the trapezium PQCD is \( \frac{5}{6} \)th part of the area of the rectangle, find the lengths QC and PD.

![Fig. 12.6](image1)

8. A design is made on a rectangular tile of dimensions 50 cm × 70 cm as shown in Fig. 12.7. The design shows 8 triangles, each of sides 26 cm, 17 cm and 25 cm. Find the total area of the design and the remaining area of the tile.

![Fig. 12.7](image2)